You are the CEO of a startup company, and the table below shows

- the amount of expenses incurred by the startup in each month.
- the revenue received by the startup in each month.

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs</td>
<td>6921</td>
<td>6738</td>
<td>6176</td>
<td>6405</td>
<td>6935</td>
<td>4916</td>
<td>4410</td>
<td>4893</td>
<td>4057</td>
<td>4352</td>
<td>4813</td>
<td>4009</td>
</tr>
<tr>
<td>Revenues</td>
<td>1138</td>
<td>1202</td>
<td>1198</td>
<td>1272</td>
<td>8198</td>
<td>8015</td>
<td>8746</td>
<td>10945</td>
<td>11431</td>
<td>10965</td>
<td>10918</td>
<td></td>
</tr>
</tbody>
</table>

The company starts with 5000 dollars in cash (this is on top of first months’ revenues).

Obviously, in the short run you will have a cashflow problem, until the revenues kick in, and start outstripping the expenses. To deal with the cashflow problem, you can get a short-term loan, with interest rate \( srate \) in each month. This amount must be payed back in the next month.

Set up a linear program to determine the optimal borrowing strategy for the coming year. Your objective is minimizing the total interest payed during the whole year. For simplicity, you can assume that one dollar is worth the same during the whole year; ie. if you pay 100 dollars in interest in every month for the shortterm loans, then your total cost is 1200 dollars. (That is, we do not take into account the fact, that a dollar in June is worth more than one dollar in December, since in the intermediate 6 months we could invest it, and reap some profit.)

Solve the problem with AMPL and CPLEX, with

\( srate = 4\% \)

Now assume that you also can get a long-term loan, with interest rate \( lrate \) at the beginning of the year (ie. you can already use it to pay the expenses in month 1). You must pay this loan back after the 12th month, with the interest.

Solve this problem with AMPL and CPLEX with two datasets:

1. In the first,

\( lrate = 9\%, srate = 4\% \)

2. In the second,

\( lrate = 13\%, srate = 4\% \)
We make the following assumption:

All transactions happen in a fixed day of the month, and the revenues of that month, and the shortterm loan taken out in that month can already be used to pay the expenses of the same month, and also the shortterm loan taken out in the previous month.

Hints:

1. Before starting this problem, you should carefully review the sailboat production problem, and the skilled/trainee technicians problem.

2. The optimal cost when there is no long-term loan is 4924.11 dollars.

3. The optimal cost with the first dataset is 1779.7308 dollars, and we take out a short term loan only in months 5 and 6.

4. The optimal cost with the second dataset is 2419.612646 dollars, and we take out a short term loan only in months 4, 5 and 6.

5. To check the correctness of your model, you should first test it with some easy datafiles, so that you can figure out what the optimal solution is, just by inspection. One possibility with $n = 6$ months is:

<table>
<thead>
<tr>
<th>Month $i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenses</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1000</td>
<td>2000</td>
</tr>
<tr>
<td>Revenues</td>
<td>1000</td>
<td>2000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>