AMO 51/2.37 Let $h_{ij}$ denote the $(i,j)$th element of $H$, and $h_{ij}^k$ denote the $(i,j)$th element of $H^k$. Then

$$\text{Number of directed paths of length 2 from } i \text{ to } j$$

$$= \text{Number of nodes } \ell \text{ such that } (i, \ell) \in A \text{ and } (\ell, j) \in A$$

$$= \text{Number of nodes } \ell \text{ such that } h_{i\ell} = 1 \text{ and } h_{\ell j} = 1$$

$$= \sum_{\ell=1}^n h_{i\ell} h_{\ell j} = h_{ij}^2.$$ 

Suppose now that $h_{ij}^k = \text{number of distinct directed walks of } k \text{ arcs from } i \text{ to } j$. Then

$$\text{Number of distinct directed walks of } k + 1 \text{ arcs from } i \text{ to } j$$

$$= \text{Number of nodes } \ell \text{ such that there is a directed walk of } k \text{ arcs from } i \text{ to } \ell \text{ and } (\ell, j) \in A$$

By the induction hypothesis, this is equal to

$$\text{Number of nodes } \ell \text{ such that } h_{i\ell}^k = 1 \text{ and } h_{\ell j} = 1$$

$$= \sum_{\ell=1}^n h_{i\ell}^k h_{\ell j} = h_{ij}^{k+1}.$$