AMO 124/4.3 If there is no fixed cost, then
\[ c_{ij} = C_j(L_{i+1} + \cdots + L_j) \leq C_{i+1}L_{i+1} + \cdots + C_jL_j \]
Therefore, if in the shortest path we take arc \((i, j)\), we might as well take arcs \((i, i+1), (i+1, i+2), \ldots, (j-1, j)\) instead. Hence, there is a shortest path with arcs \((0, 1), (1, 2), \ldots, (n-1, n)\).

In the graph the costs are computed as, say
\[
\begin{align*}
c_{01} & = C_1L_1 + F_1 = 5 \times 100 + 1000 = 1500 \\
c_{02} & = C_2(L_1 + L_2) + F_2 = 6 \times 400 + 1200 = 3600 \\
c_{13} & = C_3(L_2 + L_3) + F_3 = 7 \times 500 + 1100 = 4600
\end{align*}
\]
All the costs are shown below:

- \(c_{01} = 1500, c_{02} = 3600, c_{03} = 5300, c_{04} = 9700, c_{05} = 18600, c_{06} = 23000\)
- \(c_{12} = 3000, c_{13} = 4600, c_{14} = 8800, c_{15} = 17400, c_{16} = 21600\)
- \(c_{23} = 2500, c_{24} = 6100, c_{25} = 13800, c_{26} = 17400\)
- \(c_{34} = 4300, c_{35} = 11400, c_{36} = 14600\)
- \(c_{45} = 7800, c_{46} = 10400\)
- \(c_{56} = 3400\)

This graph is acyclic, so we can use the recursion
\[
\begin{align*}
d_0 & = 0 \\
d_j & = \min \{ d_i + c_{ij} (i, j) \in A \} \forall j \neq 0.
\end{align*}
\]
The calculations are:
\[
\begin{align*}
d_1 & = 1500 \\
d_2 & = \min \{ d_0 + 3600(*), d_1 + 3000 \} = \min \{ 3600, 4500 \} = 3600 \\
d_3 & = \min \{ d_0 + 5300(*), d_1 + 4600, d_2 + 2500 \} = \min \{ 5300, 1500 + 4600, 3600 + 2500 \} = \min \{ 5300, 6100, 6100 \} = 5300 \\
d_4 & = \min \{ d_0 + 9700, d_1 + 8800, d_2 + 6100, d_3 + 4300(*) \} = \min \{ 9700, 10300, 10700, 9600 \} = 9600 \\
d_5 & = \min \{ d_0 + 18600, d_1 + 17400, d_2 + 13800, d_3 + 11400(*), d_4 + 7800 \} = \min \{ 18600, 18900, 17400, 16700, 17400 \} = 16700 \\
d_6 & = \min \{ d_0 + 23000, d_1 + 21600, d_2 + 17400, d_3 + 14600(*), d_4 + 10400, d_5 + 3400 \} = \min \{ 23000, 23100, 21000, 19900, 20000, 20100 \} = 19900
\end{align*}
\]
So the shortest 0 to 6 path is 0 → 3 → 6. That is, we store heights
• 1, 2, 3 on shelves meant for height 3;
• 4, 5, 6 on shelves meant for height 6.