Let $G = (V, A)$ be a directed graph with $c_{ij}$ nonnegative arc-capacities. The capacity of a path is the minimum of the capacities of all the arcs in it. Say, if the path consists of arcs $(1, 2), (2, 3), (3, 4)$, then its capacity is the minimum of $c_{12}, c_{23}$ and $c_{34}$.

Describe an algorithm to find the path with maximum capacity from node 1 to all other nodes. Eg. if $G$ has nodes 1, 2, 3, 4 and arcs $(1, 2), (2, 4), (1, 3), (3, 4)$ with $c_{12} = 2, c_{24} = 3, c_{13} = 1000, c_{34} = 1$, then the maximum capacity path from 1 to 4 is through node 2, with capacity 2. Give a rigorous proof of its correctness, and prove an upper bound of $O(n^2)$ on its complexity. (Hint: you do not need to know anything about maximum flows to solve this problem; rather try to find an algorithm analogous to Dijkstra’s.)

Illustrate your algorithm on the example of Figure 1.