A subvector of a vector $a \in \mathbb{R}^n$ is a vector of the form

$$(a_k, a_{k+1}, \ldots, a_{\ell-1}, a_{\ell}),$$

where $\ell \geq k$.

You are given $a \in \mathbb{R}^n$.

1. Describe a DP based algorithm to find the subvector of $a$ in which the sum of elements is maximum. Prove that the complexity of your algorithm is $O(n)$.

2. Apply your algorithm to the vector

$$(−1, 2, 2, 3, −5, 1, −2, −2, −1, 1, 2, 5, −3).$$

3. Describe a graph in which finding the longest path from a certain node will be equivalent to computing the DP recursion.

4. Give a formal proof of the equivalence.