Name:

Instructions:

1. Write your name above.

2. No baseball caps are allowed (turn it backwards if you have one on).

3. Write your answers in the space provided or on the back side of any page. If you attach additional sheets of paper, be sure to indicate that answers are continued and carefully identify any work that carries over.

4. After you have completed the exam, sign the Honor Code statement below.

HONOR CODE PLEDGE: I affirm that I have neither given nor received aid on this exam.

Signature:

Multiple Choice (2 points per question)

1. Theory suggests that the supply curve of labor is:
   (a) upward-sloping.
   (b) horizontal.
   (c) downward-sloping.
   (d) downward-sloping for some wage levels, and upward-sloping for others.

2. The labor market demand curve for labor is [BLANK]-sloping and [BLANK] than the horizontal sum of individual demand curves.
   (a) upward; flatter
   (b) upward; steeper
   (c) downward; steeper
   (d) downward; flatter

3. In the context of the retirement model, an increase in a worker’s wage rate, ceteris paribus, will
   (a) delay retirement if the substitution effect is dominant.
   (b) increase the years a worker is retired by both the income and substitution effects.
   (c) induce earlier retirement since there is no income effect.
   (d) expedite retirement since only an income effect is present.
4. The intertemporal substitution hypothesis suggests that

(a) labor force participation rates and hours worked increase when wages are higher as people substitute time over the life cycle to take advantage of changes in the price of leisure.

(b) wage increases over the lifecycle generate income effects which cause workers to spend less time working and more time enjoying leisure during their “prime years.”

(c) the age earnings profile is “U-shaped,” since earnings first decrease and later increase as a worker ages.

(d) there exists only a substitution effect when comparing the wage profiles of two different workers.

5. Relatively more curved isoquants represent combinations of inputs which are [BLANK] in the production process and have a relatively [BLANK] elasticities of substitution.

(a) substitutes; small

(b) substitutes; large

(c) complements; small

(d) complements; large

6. A firm whose production technology involves the use of perfect complements will demand

(a) only labor when the wage rate exceeds the rental price of capital.

(b) the same quantity of labor and capital, regardless of input prices.

(c) more capital only when more labor is demanded, and vice versa.

(d) only labor when the rental price of capital exceeds the wage rate.

7. Consider a firm where production depends on two inputs - labor and capital - with prices w and r, respectively. Initially the firm faces market prices of w = 6 and r = 4. These prices then shift to w = 4 and r = 2. The prices changes will cause

(a) an increase in the quantity of labor used because of the substitution effect.

(b) a decrease in the quantity of capital used because of the dominant scale effect.

(c) a decrease in the quantity of labor used because of the scale effect.

(d) an increase in the quantity of capital used by both the scale and substitution effects.

8. A firm that experiences an increase in the rental price of capital will

(a) utilize more labor if the substitution effect dominates.

(b) utilize more capital if the scale effect dominates.

(c) not change its input combination regardless of which effect dominates.

(d) utilize less labor if the substitution effect dominates.
9. If the cross-price elasticity of demand between two inputs is -1.46, we can conclude that the inputs are
(a) substitutes.
(b) normal goods.
(c) **complements.**
(d) inferior goods.

10. Monopsonies are characterized
(a) as markets with upward-sloping labor supply curves.
(b) by firms which must take P and w as given.
(c) as markets which have only one seller of the output.
(d) **by firms which face upward-sloping labor supply curves.**

11. Non-discriminating monopsonists hire [BLANK] workers than a competitive firm would and pay [BLANK].
(a) more; a worker his or her reservation wage.
(b) fewer; all workers the same wage.
(c) more; all workers the same wage.
(d) fewer; a worker his or her reservation wage.

12. Firms with monopoly power hire workers up to the point where
(a) \( P \cdot MP_E = w \).
(b) \( MR = MC \).
(c) \( MR \cdot MP_E = w \).
(d) \( \frac{w}{r} = \frac{MP_K}{MP_E} \).

Short Answer/Essay (66 total points): Clearly state your answers in the space provided. Show your work.

1. **(21 points)** Suppose Maggie is offered an hourly wage of \( w \), her non-labor income is \( V \), consumption \((C)\) is measured in dollars, and she has 40 hours to work \((H)\) or leisure \((L)\). Suppose also that Maggie derives utility from consumption and leisure according to the utility function:

\[
U = 15C^{\frac{4}{5}}L^{\frac{1}{5}}
\]

Her marginal utility functions are:

\[
MU_L = 3C^{\frac{3}{5}}L^{-\frac{2}{5}}
\]
\[
MU_C = 12C^{-\frac{1}{5}}L^{\frac{1}{5}}
\]
(a) (7 points) State and interpret the utility-maximization rule that Maggie uses to determine the optimal number of hours she should work. Then use that rule to determine how many hours Maggie will work if she consumes $800 in consumption goods and is offered a wage of $20 per hour. Then, use the same rule to solve for the optimal number of hours worked when consumption decreases to $480 and wages decrease to $10 per hour. Graph (and label) the budget constraints, these two consumption bundles, and the corresponding indifference curves in the space provided. Be sure to label the intercepts and the slope of the budget constraints.

The optimal level of consumption and leisure occurs at the tangency of the budget constraint and indifference curve. At the tangency, the slopes of the two lines is equal, so

\[
\frac{MU_L}{MU_C} = w
\]

\[
\Rightarrow \frac{MU_L}{w} = MU_C
\]

The condition can be interpreted as follows: at the tangency, the additional utility enjoyed from spending an additional dollar on leisure is exactly equal to the additional utility enjoyed from spending an additional dollar on consumption. In other words, the individual is indifferent between consuming more leisure or consumption. Using this condition, we can solve for the optimal amount of leisure, and from that, hours of work:

\[
\frac{3C^4L^4 - 12C^{-2}L^5}{-2} = 20 \Rightarrow \frac{C}{4L} = 20 \Rightarrow \frac{800}{4L} = 20 \Rightarrow 80L = 800 \Rightarrow L^* = 10
\]

\[
C_1 = $800, w_1 = $20, H_1^* = T - L_1^* = 40 - 10 = 30
\]

Similarly,

\[
\frac{3C^4L^4 - 12C^{-2}L^5}{-2} = 10 \Rightarrow \frac{C}{4L} = 10 \Rightarrow \frac{480}{4L} = 10 \Rightarrow 40L = 480 \Rightarrow L^* = 12
\]

\[
C_2 = $480, w_2 = $10, H_2^* = T - L_2^* = 40 - 12 = 28
\]

(b) (5 points) Calculate and interpret Maggie’s elasticity of labor supply when the wage rate changes from $20 to $10 per hour. Is her responsiveness to wage changes elastic or inelastic? Does this suggest a dominant income or substitution effect for Maggie? Explain.

Elasticity is calculated as:

\[
\sigma = \frac{\Delta H}{H_1} \cdot \frac{w_1}{\Delta w}
\]

\[
= \frac{28 - 30}{30} \cdot \frac{20}{10 - 20}
\]

\[
= 0.13
\]
This suggests that when wages decrease by 1%, hours of work will decrease by 0.13%. Because $|\sigma| < 1$, this is considered inelastic labor supply, which means Maggie is relatively unresponsive in terms of hours worked to wage changes. A positive elasticity (which suggests that the decreased opportunity cost of labor outweighs Maggie’s desire to decrease her consumption of leisure when her wealth decreases) of labor supply indicates that Maggie’s substitution effect dominates in this example.

(c) (6 points) Finally, on the original graph, illustrate this wage change graphically (with appropriate labels slope, intercepts, etc). Maggie’s hours worked will now decrease. Explain using income/substitution effects. Decompose the graphical change in hours worked into income and substitution effects. [Note: Your graph should correspond to your answer(s) in part (a).]

The substitution effect suggests that there exists a positive relationship between the wage rate and hours worked. Because less is earned for each hour worked now that wages have decreased, the opportunity cost of consuming leisure has decreased, thus increasing the demand for it (Q to R on the graph).

The income effect states that this wage decrease has caused total income/wealth to decrease, and since leisure is a normal good, less will be demanded when people have less money on which to spend it. Thus, in this case, the demand for leisure will decrease, and the income effect predicts more hours will be spent working (P to Q on the graph).

The income and substitution effects do not agree, but since $H^*$ did decrease (from 30 to 28), the substitution effect dominates Maggie’s decision, as illustrated above.

Note that I would have accepted “change ambiguously” in the stem of this question since the income and substitution effects have opposite effects on labor; in Maggie’s specific case, hours of work decrease.
(d) (3 points) Calculate the minimum wage offer that would make Maggie enter the labor market if, in addition to her current non-labor income, she were to receive welfare payments in the amount of $280.

This problem requires that you solve for Maggie’s reservation wage, the wage offer that makes her indifferent between working and not working. Thus, it is the slope of her indifference curve at the endowment point, V. The slope of his indifference curve is given by:

\[
\frac{MU_L}{MU_C} = \frac{C}{4L}
\]

At the endowment point, i.e. not working, Maggie’s consumption level is made up of her non-labor income, plus welfare benefits in the amount of $280, and \( L = T \). Therefore, at the beginning of the problem when \( C = $800 \) and \( w = $20 \),

\[
\frac{MU_L}{MU_C} = \bar{w} = \frac{800 - (20)(30) + 280}{4 \cdot 40} = $3
\]

Maggie will not accept a wage offer less than her reservation wage of $3.

2. (21 Points) Consider a perfectly competitive firm with the following production function:

\[ q = 10K^{\frac{1}{2}}E^{\frac{1}{2}} \]

The corresponding marginal product of labor and marginal product of capital functions are:

\[ MP_E = 5K^{\frac{1}{2}}E^{-\frac{1}{2}} \]
\[ MP_K = 5K^{-\frac{1}{2}}E^{\frac{1}{2}} \]

Suppose that \( r = $80 \) and that the firm’s output sells in the market at a price of $8.

(a) (4 points) Assume initially that the firm is in the short-run. State and interpret the short-run profit-maximizing input condition. If the firm has hired 15 machines, how many workers should it employ if workers are paid $20 per hour? How many workers should the firm hire if the wage rate decreases to $10 per hour, all else constant?

The short-run profit-maximizing condition, in all its glory, is as follows:

The firm continues hiring workers up to the point where \( w = VMP \). In other words, the firm will continue to hire workers as long as the cost of hiring them (the wage rate)
offsets the revenue generated by hiring another worker (the value of marginal product). When the two are equal, the optimal number of workers has been hired. \( E^* \) is only profit-maximizing if the marginal product curve has begun to decrease, and if the \( \text{VAP}(E^*) > \text{VMP}(E^*) \). These two caveats imply that the firm will not stop hiring when the next potential worker would add more revenue (if MP is still increasing, the next worker is even more productive) and that profits will not be negative.

This rule can be used to solve for \( E^* \) when \( K = 15 \) and \( w = $20 \) (and \( w = $10 \)). Specifically,

\[
\begin{align*}
    w & = \text{VMP} \\
    20 & = 8 \cdot 5K^{\frac{1}{2}}E^{-\frac{1}{2}} \\
    1 & = \frac{2K^{\frac{1}{2}}}{E^{\frac{1}{2}}} \\
    E^{\frac{1}{2}} & = 2K^{\frac{1}{2}} \\
    \left[ E^{\frac{1}{2}} \right]^2 & = \left[ 2K^{\frac{1}{2}} \right]^2 \\
    E & = 4K \\
    E & = 4(16) \\
    E & = 60
\end{align*}
\]

Therefore, \( w_1 = $20, E_1^* = 60 \)

The same approach can be used to solve for \( E_2^* \).

\[
\begin{align*}
    w & = \text{VMP} \\
    10 & = 8 \cdot 5K^{\frac{1}{2}}E^{-\frac{1}{2}} \\
    E^{\frac{1}{2}} & = 4K^{\frac{1}{2}} \\
    E & = 16K,
\end{align*}
\]

and \( w_2 = $10, E_2^* = 240 \).

(b) **3 points** Suppose the firm is instead operating in the long-run (you may assume this for the duration of this problem). State the firm’s profit-maximizing condition and use it to solve for the optimal number of employees that should be hired when the wage rate is $20, the price of capital is $80, and 15 machines have been rented. Additionally, solve for the profit-maximizing employment level if the quantity of capital used in the production process increases to 20 and the wage rate decreases to $10.

In the long run, the profit-maximizing input rule states that the slopes of the isocost and isoquant must be equal. In other words,

\[
\frac{\text{MP}_E}{\text{MP}_K} = \frac{w}{r}
\]
Using this firm’s production function,

\[
\frac{5K^{\frac{1}{2}}E^{\frac{1}{2}}}{5K^{\frac{1}{2}}E^{\frac{1}{2}}} = 20
\]

\[
\frac{K^{\frac{1}{2}}K^{\frac{1}{2}}}{E^{\frac{1}{2}}E^{\frac{1}{2}}} = 20
\]

\[
\frac{K}{E} = \frac{20}{80}
\]

\[
80K = 20E
\]

\[
4K = E
\]

Therefore, just like in the short-run, when \( K_1 = 15 \) and \( w_1 = $20 \), \( E_1^* = 60 \).

If the quantity of capital used increases to 20 and the wage rate decreases to $10, the same approach can be used to solve for \( E_2^* \).

\[
\frac{K}{E} = \frac{10}{80}
\]

\[
80K = 10E
\]

\[
8K = E, \text{ and}
\]

\( K_2 = 20, w_2 = $10 \) yields \( E_2^* = 160 \)

(c) (4 points) Use your answers from parts (a) and (b) to graph (and label) both the short- and long-run demand curves for labor. Also, explain the difference between the two curves (namely, the slopes) using what you know about elasticity, assumptions about the short- and long-run, etc.

This question was intended to illustrate the fact that firms are more responsive to wage changes in the long run because they are able to adjust the quantity of capital. However, the long run demand curve is steeper in this problem, which makes the explanation more difficult. A linear “curve” has a constantly changing elasticity, which is why the comparison between the elasticities cannot be made. Everyone receives full credit for the two points dedicated to the explanation of this graph.

(d) (3 points) Use your answers from part (b) to calculate and interpret this firm’s long-run elasticity of demand for labor when the wage rate decreases.
from $20 to $10. Is this considered an elastic or inelastic demand curve for labor?

Elasticity is calculated as:

$$\delta = \frac{\Delta E}{E_1} \cdot \frac{w_1}{\Delta w}$$

$$= \frac{160 - 60}{60} \cdot \frac{20}{10 - 20}$$

$$= -3.3$$

An elasticity of $-3.3\%$ is considered elastic, and it is interpreted as “a 1% decrease in wages causes a 3.3% decrease in the number of employees demanded by the firm.”

(e) (2 points) Use your understanding of the Marshallian Rules of Derived Demand to state and explain two factors that will cause a labor market demand curve to have the elasticity you calculated in part (d).

The Marshallian Rules of Derived Demand describe factors that will cause the demand for labor to be elastic. Specifically, any of the following four characteristics would make the demand for labor elastic:

- If the elasticity of substitution between labor and capital is larger, the firm has more flexibility in adjusting its production process. Therefore, when the price of labor increases, the firm can substitute capital for labor, and thus will substantially reduce the quantity of labor used in the production process.

- When the demand for the firm’s output is more elastic, the firm will be more responsive to wage changes. In other words, when the price of labor increases, the marginal cost of production increases, and the firm will produce less output. A leftward shift of the supply curve will increase the price of output. If the demand curve for that output is flat (which means consumers are very responsive to changes in the price of the good), the resulting decrease in $Q^*$ will be relatively large. If consumers’ demand for the good decrease by a lot, the firm will have to cut back on its demand for labor by quite a bit, so the demand curve will be elastic.

- If labor is a large share of the firm’s total production costs, the demand for labor will be elastic. If the wage change affects a large portion of the total costs of production, the firm will be resistant to the wage increase (by cutting back on the labor it hires), because it changes total profits much.

- When the elasticity of supply of other inputs to production is large, the firm’s demand for labor will be elastic. More specifically, when the price of employment increases, the firm will choose to substitute away from labor toward a more capital-intensive production process. By doing so, the demand for capital increases. A rightward shift of the demand curve for labor will cause the price to increase. If the
supply curve for capital is flat (elastic), the price of capital will rise only moderately, and the firm’s increased demand for capital will be significant. Thus, the firm will make a considerable change in its production process and will demand much less labor.

(f) (5 points) Finally, describe the scale and substitution effects in the context of this firm’s response to the wage decrease from $20 to $10. State which effect is dominant, and decompose in the space below.

In this case, a wage decrease caused an increase in the quantity of labor demanded (which is predicted by both the scale and substitution effects) and an increase in the quantity of capital used. Therefore, there is a dominant scale effect.

The scale effect suggests that when the price of labor decreases, the firm’s marginal cost of production decreases. Since the firm was previously profit maximizing where MR = MC, a decrease in MC means the firm must increase its output in order to reach a new profit-maximizing output level. If Q∗ increases, the firm will hire more capital and more labor in order to achieve that goal. Therefore, the scale effect predicts that a wage decrease will result in the use of more capital and more labor, represented by the shift from P to Q in the graph below.

The substitution effect concerns relative prices. When r remained constant at $80 and the wage decreased from $20 to $10, labor became a relatively less expensive input in the production process. Therefore, the firm had an incentive to substitute away from capital, the now-more expensive input, toward labor. Thus, the scale effect predicts that demand for capital will decrease, represented by the shift from Q to R in the graph below.

3. (15 points) Use your answers from questions (1) and (2) to graph the perfectly competitive labor market.
(a) (5 points) Specifically, plot the supply curve using your answers from question (1) for wages of $20 and $10. Then, depict the long-run demand curve for labor using the same wages and your answers for question (2). Assume that there are 60 workers like Maggie in #1 and 30 firms identical to the one described in #2. After graphing the supply and demand curves (using specific (E,w) points), identify and label the equilibrium wage rate, \( w^* \), and employment level, \( E^* \).

The following information is needed in order to solve for the equilibrium wage and employment levels:

Supply:

\((E_1, w_1) = (30, 20)\) for each of 60 workers \(\Rightarrow (E_1, w_1) = (1800, 20)\) for the market

\((E_2, w_2) = (28, 10)\) for each of 60 workers \(\Rightarrow (E_2, w_2) = (1680, 10)\) for the market

Demand:

\((E_1, w_1) = (60, 20)\) for each of 30 firms \(\Rightarrow (E_1, w_1) = (1800, 20)\) for the market

\((E_2, w_2) = (160, 10)\) for each of 30 firms \(\Rightarrow (E_2, w_2) = (4800, 10)\) for the market

Plotting the two quantities of labor supplied, \((1800, 20)\) and \((1680, 10)\), and the two quantities of labor demanded, \((1800, 20)\) and \((4800, 10)\), reveals that when the wage rate is $10, 1800 employee hours are supplied to the labor market, and 1800 employee hours are demanded by firms in the market. By definition, this is equilibrium.

(b) (5 points) Graphically and in words, illustrate above and describe below how the imposition of a minimum wage would affect this perfectly competitive market. Draw an “effective” minimum wage, describe how \( E \) and \( w \) would
change, and discuss the unemployment rate.

The only way a minimum wage is effective in ensuring workers are paid a wage higher than the market would achieve in equilibrium is to impose a price floor that is higher than $20. A hypothetical minimum wage has been drawn on the graph above. A perfectly competitive market is affected by a minimum wage such that all workers hired are paid the minimum wage, which is higher than \( w^* \). However, at the higher wage, firms hire fewer workers. As drawn, \( E_D \) workers will find jobs at the minimum wage, but \( E_S \) workers are willing to work at the minimum wage (which is greater than the original \( E^* \) individuals who were working before the minimum wage was imposed). Therefore, \( E_S - E_D \) workers will be without jobs, and the unemployment rate will be \( \frac{E_S - E_D}{E_S} \).

(c) (5 points) Ignoring the minimum wage from part (b), describe the effect a payroll subsidy would have on this market. Which curve(s) would shift, in which direction, and how would \( E^* \) and \( w^* \) change as a result of the subsidy? What would the unemployment rate be?

If the government offers firms in this market a payroll subsidy, firms would be willing to pay a higher wage to those workers already employed, because they would receive some amount of money back for each worker. Thus, the entire demand curve for labor shifts right. In the absence of something like a minimum wage, this increase in demand will result in a new equilibrium characterized by a higher employment rate and a higher equilibrium wage rate. Both worker and firm benefit from the subsidy, as the net cost of employment for the firm will decrease (\( w_2 + \text{subsidy} < w_1 \)), and the take-home wage for the employee will increase (\( w_1 < w_2 \)).

4. (9 points) Describe and illustrate graphically the hiring decisions of the following non-competitive labor markets. How does each outcome (\( E^* \) and \( w^* \)) compare to competitive labor markets?

(a) (3 points) Discriminating Monopsony

Discriminating monopsonies are able to hire workers according to their reservation wages. Therefore, they hire the first worker at the wage it requires to get him to enter the labor market. The next worker hired demands a higher wage, and the discriminating monopsonist hires him at that rate. This continues up to the point where the demand and supply curves of labor intersect. The discriminating monopsonist hires the same number of workers as a competitive firm, but only the last worker hired is paid the competitive wage. All workers hired previously are paid according to their (lower) reservation wages.
(b) (3 points) Non-Discriminating Monopsony

Non-discriminating monopsonies cannot identify workers’ reservation wages, so they pay all workers the same rate. However, as a result of this, the marginal cost of hiring the last worker is not the same as the wage paid to that worker (see example in class notes). The non-discriminating monopsonist hires where the marginal cost of hiring another worker (MC) is equal to the revenue generated by hiring that worker (D = VMP), not where supply equal demand. Thus, this type of firm hires fewer workers than a competitive firm. Additionally, since workers are willing to work at wages depicted by the supply curve, the firm pays all workers according to the supply curve, not demand (which would measure the worker’s worth to the firm). Therefore, a non-discriminating monopsonist hires fewer workers, and pays them less, than a competitive firm.

(c) (3 points) Monopoly

Since demand and marginal revenue are not the same for a monopolist, the additional revenue generated by the last worker hired (VMP for competitive firms and monopsonies) is \( MR \cdot MP = MRP \) (Marginal Revenue Product). The monopolist has no control over wages, and like a competitive firm, takes the market-clearing wage as given. Thus, a monopolist hires where MRP = w (similar to VMP = w for competitive firms), but since MRP < VMP (since MR < P), this profit-maximizing condition results in fewer workers hired, relative to competitive markets. Monopolists pay the same wage as competitive firms, but they hire fewer workers, as denoted above.