Multiple Choice (2 points per question)

1. In the context of the retirement model, an increase in pension benefits will
   (a) encourage workers to delay retirement because of a dominant income effect.
   (b) induce earlier retirement if the substitution effect dominates.
   (c) increase the years a worker is retired by both the income and substitution effects.
   (d) delay retirement since only an income effect is present.

2. The labor market demand curve for labor is [BLANK]-sloping and [BLANK] than the horizontal sum of individual demand curves.
   (a) upward; steeper
   (b) downward; flatter
   (c) upward; flatter
   (d) downward; steeper

3. A firm is more responsive to wage changes in the [BLANK], which suggests the long-run labor demand curve [BLANK] the short-run labor demand curve.
   (a) short-run; is flatter than
   (b) short-run; has the same slope as
   (c) long-run; is flatter than
   (d) long-run; has the same slope as
4. Consider a firm where production depends on two inputs - labor and capital - with prices \( w \) and \( r \), respectively. Initially the firm faces market prices of \( w = 6 \) and \( r = 4 \). These prices then shift to \( w = 4 \) and \( r = 2 \). The prices changes will cause

(a) a decrease in the quantity of capital used because of the dominant scale effect.
(b) an increase in the quantity of labor used because of the substitution effect.
(c) an increase in the quantity of capital used by both the scale and substitution effects.
(d) a decrease in the quantity of labor used because of the scale effect.

5. Suppose that Jack’s wage rate is $25 and that his marginal product of time in the household sector is $10 per hour. Suppose also that Jill’s wage rate is $30 and that her marginal product of time in the household sector is $15 per hour. We know that

(a) Jill should specialize in the household sector because of her comparative advantage in that task.
(b) it doesn’t matter who specializes in the tasks, because neither Jack nor Jill has a comparative advantage in either activity.
(c) Jill should specialize in both activities because of her absolute advantage in both household production and labor market productivity.
(d) Jack should specialize in the household sector because of his comparative disadvantage in labor market productivity.

Short Answer/Essay (30 total points): Clearly state your answers in the space provided. Show your work.

1. (7 points) Suppose there are two inputs in the production function - labor and capital - and that the existing technology permits one machine to do the work of three persons. The firm wants to produce 100 units of output. Suppose that the price of capital is $750 per machine per week and that the weekly salary of each worker is $300.

(a) (2 points) Which combination of inputs should the firm use?

Since the firm can hire either one machine or three workers and produce the same quantity of output, labor and capital are perfect substitutes in the production process. Therefore, the firm will use only the cheaper input.

In this case, hiring one machine would cost the firm $750. Hiring three workers, who could produce the same output as the one $750 machine, would cost \( 3 \cdot $300 = $900 \). Therefore, the firm will hire \( E_1 = 0 \) workers and as many units of capital as required to produce 100 units of output.

(b) (2 points) Suppose that a worker’s weekly salary is $225. Which input combination should the firm then use?
Again, since the inputs to production are perfect substitutes, only the relative prices of using capital and labor need to be considered. One machine will still cost the firm $750, but now three workers will cost $225 = $675. Thus, the firm will hire \( K_2 = 0 \) machines and as many workers as required to produce 100 units of output.

(c) (3 points) What is the elasticity of labor demand as the wage falls from $300 to $225? Explain what this means in the context of this firm’s production technology.

The formula for elasticity,

\[
\delta = \frac{\% \Delta E}{\% \Delta w} = \frac{\Delta E}{E_1} \cdot \frac{w_1}{\Delta w},
\]

requires only one piece of information from parts (a) and (b). Specifically, since \( E_1 = 0 \),

\[
\delta = \frac{\Delta E}{0} \cdot \frac{300}{-75} = \infty,
\]

This suggests that the firm is completely responsive changes in the relative prices of labor and capital, which is consistent with the inputs being perfect substitutes.

2. (23 Points) Consider a perfectly competitive firm with the following production function:

\[
q = 30K^{\frac{1}{3}}E^{\frac{2}{3}}
\]

The corresponding marginal product of labor and marginal product of capital functions are:

\[
 MP_E = 20K^{\frac{1}{3}}E^{-\frac{1}{3}}
 MP_K = 10K^{-\frac{2}{3}}E^{\frac{2}{3}}
\]

Suppose that \( r = $80 \) and that the firm’s output sells in the market at a price of $2.

(a) (4 points) Assume initially that the firm is in the short-run. State and interpret the short-run profit-maximizing input condition. If the firm has hired 10 machines, how many workers should it employ if workers are paid $20 per hour?

The short-run profit-maximizing condition, in all its glory, is as follows:

The firm continues hiring workers up to the point where \( w = VMP \). In other words, the firm will continue to hire workers as long as the cost of hiring them (the wage rate) offsets the revenue generated by hiring another worker (the value of marginal product). When the two are equal, the optimal number of workers has been hired. \( E^* \) is only profit-maximizing if the marginal product curve has begun to decrease, and if the \( VAP(E^*) \)
$ > \text{VMP}(E^*)$. These two caveats imply that the firm will not stop hiring when the next potential worker would add more revenue (if MP is still increasing, the next worker is even more productive) and that profits will not be negative.

This rule can be used to solve for $E^*$ when $K = 10$ and $w = \$20$. Specifically,

$$w = \text{VMP}$$

$$20 = 2 \cdot 20^{\frac{4}{3}} E^{-\frac{4}{3}}$$

$$1 = \frac{2K^{\frac{1}{3}}}{E^{\frac{1}{3}}}$$

$$E^{\frac{1}{3}} = 2K^{\frac{1}{3}}$$

$$\left[ E^{\frac{1}{3}} \right]^3 = \left[ 2K^{\frac{1}{3}} \right]^3$$

$$E = 8K$$

$$E = 8(10)$$

$$E = 80$$

(b) (3 points) Suppose the firm is instead operating in the long-run (you may assume this for the duration of this problem) and currently employing 5 workers and 40 machines at the current input prices of $w = \$20$ and $r = \$80$. Should it make any changes to its input combination? In other words, state whether or not this is a profit-maximizing input combination, and if not, how should it adjust $K$ and $E$?

In the long run, the profit-maximizing input rule states that the slopes of the isocost and isoquant must be equal. In other words,

$$- \frac{\text{MP}_E}{\text{MP}_K} = \frac{w}{r}$$

Using this firm’s production function,

$$\frac{20K^{\frac{1}{3}} E^{-\frac{1}{3}}}{10K^{-\frac{2}{3}}E^{\frac{1}{3}}} = \frac{20}{80}$$

$$\frac{20K^{\frac{1}{3}} K^{\frac{2}{3}}}{10E^{\frac{2}{3}}E^{\frac{1}{3}}} = \frac{20}{80}$$

$$\frac{2K}{E} = \frac{20}{80}$$

$$160K = 20E$$

$$8K = E$$

[Note that this result is the same as the one in part (a).] Therefore, for every machine the firm hires, its production technology requires that eight workers be employed to operate the machines. Five workers and 40 machines is, thus, not profit-maximizing. Note that the opposite ($E = 40, K = 5$) would be profit-maximizing, which means the firm must hire fewer machines and/or more workers in order to profit-maximize.
(c) (3 points) If the firm continues to operate in the long run and decreases its capital stock from $K = 10$ (part a) to $K = 9$ when wages decrease from $\$20$ to $\$15$, how many workers should it employ? Use this answer, coupled with your response to part (a), to construct the firm’s long run demand for labor (note that the answer to part (a) is the same whether you use the long- or short-run profit-maximization rule).

Using the $8K = E$ rule from above (valid only when $w = \$20$, $r = \$80$), when $K = 10$, $E^*_1 = 80$. Similarly, when $w = \$15$,

\[
\begin{align*}
2K &= 15 \\
\frac{E}{E^*_1} &= \frac{15}{80} \\
160K &= 15E \\
160 \cdot 9 &= 15E \\
96 &= E
\end{align*}
\]

Thus,

$K = 10, \ w = \$20, \ r = \$80: \ E = 80$

$K = 9, \ w = \$15, \ r = \$80: \ E = 96$

(d) (3 points) Use your answers from part (c) to calculate and interpret this firm’s elasticity of demand for labor when the wage rate decreases from $\$20$ to $\$15$. Is this considered an elastic or inelastic demand curve for labor?

Elasticity is calculated as:

\[
\delta = \frac{\Delta E}{E_1} \cdot \frac{w_1}{\Delta w}
\]

\[
= \frac{96 - 80}{80} \cdot \frac{20}{15 - 20}
\]

\[
= -0.8
\]

An elasticity of -0.8 is considered inelastic, and it is interpreted as “a 10% decrease in wages causes an 8% increase in the number of employees demanded by the firm.”

(e) (4 points) Use your understanding of the Marshallian Rules of Derived Demand to state and explain two factors that will cause a labor market demand curve to have the elasticity you calculated in part (d). Feel free to use graphs to assist your explanation if appropriate.

Since the Marshallian Rules of Derived Demand describe factors that will cause the demand for labor to be elastic, the opposite characteristics will cause the demand for labor to be inelastic, as in part (d).

Specifically, any of the following four characteristics would make the demand for labor inelastic:
• If the elasticity of substitution between labor and capital is smaller, the firm has less flexibility in adjusting its production process. Therefore, when the price of labor increases, the firm cannot substitute capital for labor, and thus will pay the higher cost of employment.

• When the demand for the firm’s output is less elastic, the firm will be less responsive to wage changes. In other words, when the price of labor increases, the marginal cost of production increases, and the firm will produce less output. A leftward shift of the supply curve will increase the price of output. If the demand curve for that output is steep (which means consumers are not very responsive to changes in the price of the good), the resulting decrease in $Q^*$ will be relatively small. If consumers’ demand for the good doesn’t decrease much, the firm will not have to cut back on its demand for labor much, so the demand curve will be inelastic.

• If labor is a small share of the firm’s total production costs, the demand for labor will be inelastic. If the wage change affects only a small portion of the total costs of production, the firm will absorb the wage increase, because it doesn’t change total profits much.

• When the elasticity of supply of other inputs to production is small, the firm’s demand for labor will be inelastic. More specifically, when the price of employment increases, the firm will choose to substitute away from labor toward a more capital-intensive production process. In doing so, the demand for capital increases. A rightward shift of the demand curve for labor will cause the price to increase. If the supply curve for capital is steep (inelastic), the price of capital will rise significantly, and the firm’s increased demand for capital will be moderate. Thus, the firm will not make a significant change in its production process and will not demand much less labor.

(f) (6 points) Finally, describe the scale and substitution effects in the context of this firm’s response to the wage decrease from $20 to $15. State which effect is dominant, and decompose in the space at the top of the page.

In this case, a wage decrease caused an increase in the quantity of labor demanded (which is predicted by both the scale and substitution effects) and a decrease in the quantity of capital used. Therefore, there is a dominant substitution effect.

The substitution effect concerns relative prices. When $r$ remained constant at $80 and the wage decreased from $20 to $15, labor became a relatively less expensive input in the production process. Therefore, the firm had an incentive to substitute away from capital, the now-more expensive input, toward labor. Thus, the scale effect predicts that demand for capital will decrease, represented by the shift from $Q$ to $R$ in the graph below.

The scale effect suggests that when the price of labor decreases, the firm’s marginal cost of production decreases. Since the firm was previously profit maximizing where $MR = MC$, a decrease in $MC$ means the firm must increase its output in order to reach a new profit-maximizing output level. If $Q^*$ increases, the firm will hire more capital and
more labor in order to achieve that goal. Therefore, the scale effect predicts that a wage decrease will result in the use of more capital and more labor, represented by the shift from P to Q in the graph below.