Vegetation–atmosphere interaction and surface conductance at leaf, canopy and regional scales

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Abstract

The problem of areally averaging descriptions of land–atmosphere energy and mass exchange is common to both the leaf–canopy and canopy–region scale translations. This paper attempts a unified treatment. The starting point is a review of both the basic and equilibrium-departure forms of the Penman–Monteith or combination equation (CE) for the latent and sensible heat fluxes at an evaporating surface; the equilibrium-departure form expresses the evaporation as a linear combination of two other scalar fluxes, those of available energy and saturation deficit. Next, tools for scale translation are established, including (1) the basic conservation requirement that net scalar fluxes average linearly over the land surface and (2) the matching of model form between scales. These tools are applied to the CE for latent and sensible heat fluxes $F_E$ and $F_H$, leading to a 'flux matching' averaging scheme based on term-by-term matching of the linearly additive scalar flux terms in the CE for $F_E$. Several variations of this scheme are discussed. For $F_E$, all variations satisfy the scalar conservation requirement that fluxes average linearly, but for $F_H$, none satisfies this requirement exactly, except when long-wave radiative coupling is ignored. By considering a range of canopy–region scale translations, errors in the averaging of $F_H$ are shown to be small in practice. Finally, three averaging schemes (the flux-matching scheme and two schemes based on parallel sums of elemental aerodynamic and surface conductances, respectively) are compared for leaf–canopy scale translations, by constructing a sequence of analytic model canopies at increasing levels of detail. For every canopy model considered, all three averaging schemes give nearly identical definitions of the bulk canopy conductance for typical dry canopies. However, schemes other than the flux matching scheme can become inconsistent over partially wet surfaces, yielding undefined or negative bulk conductances.
List of symbols

c radiation extinction coefficient
\(c_p\) isobaric specific heat of air
\(c_{P_0}\) proportionality coefficient \(f_p/\phi_{eq}\) for Canopies III and IV
\(c_{II}\) proportionality coefficient \(g_{II}/g_m\) for Canopy II
\(C\) concentration of arbitrary scalar entity
\(D\) potential saturation deficit: \(D = Q_{sat}(\Theta) - Q\)
\(f, F\) scalar flux at small, large scale
\(F_{Ec}, F_{Es}\) vegetation, soil latent heat flux at canopy scale
\(g, G\) conductance at small, large scale
\(g_{av}\) maximum elemental aerodynamic conductance (as \(\Lambda \to 0\))
\(g_{st}\) maximum stomatal conductance
\(g_{p}\) slope \(dg_{st}/df_p\) for Canopy III (all \(f_p\)) and Canopy IV (\(f_p \to 0\))
\(g_{o}\) slope \(dg_{st}/df_{o}\) for Canopies III and IV: \(g_{o} = g_{p}/c_{P_0}\)
\(h\) height of well-mixed slab
\(\rho\) ratio \(g_{st}/(g_{st} + g_r) = r_{st}/r_{at}\)
\(Q\) specific humidity
\(Q_{sat}\) saturation specific humidity
\(r, R\) resistance at small, large scale
\(r_{st}\) \(1/g_{st}\)
\(R\) region enclosed by surface \(S\)
\(S\) closed bounding surface (\(S_L\), part coincident land–atmosphere interface; \(S_A\), part in air; \(S_i\), area of element \(i\))
\(t\) time
\(T\) temperature
\(U\) wind speed
\(v, V\) arbitrary dependent variable at small, large scale
\(x, X, y, Y\) arbitrary independent variables at small, large scale
\(X, Y, Z\) variables defined and used in the Appendix
\(z\) height above evaporating surface

Greek letters

\(\Gamma\) dry adiabatic lapse rate
\(\delta_{eq}, \Delta_{eq}\) equilibrium saturation deficit at small, large scale
\(\epsilon\) dimensionless slope of \(Q_{sat}(T): \epsilon = (\lambda/c_p)\partial Q_{sat}/\partial T\)
\(\zeta\) coordinate traversing leaf area index
\(\Theta\) potential temperature: \(\Theta = T + \Gamma z\)
\(\lambda\) latent heat of vaporisation of water
\(\Lambda\) leaf area index
\(\rho\) density of air
\(\tau\) canopy transmission
\(\phi_{eq}, \Phi_{eq}\) equilibrium evaporation rate in energy units at small, large scale
\(\phi_{4}, \Phi_{4}\) isothermal available energy flux at small, large scale

Subscripts denoting transferred entities

\(A\) available energy (sensible plus latent heat)
\(C\) arbitrary scalar
\(D\) saturation deficit
\(E\) latent heat
\(H\) sensible heat
1. Introduction

Land–atmosphere interactions are strongly heterogeneous processes, at practically every spatial scale from individual stomata to the whole Earth. Therefore, theories or models for energy, mass, and momentum fluxes between land surfaces and the atmosphere always involve some kind of implicit or explicit spatial averaging. These models usually express fluxes in terms of transfer coefficients over various exchange pathways, such as aerodynamic or surface resistances. They are also usually nonlinear in the spatially changing independent variables, a good example being the combination equation for evaporation. Because of the nonlinearities, it is not always easy to relate transfer coefficients defined at different scales; for instance, to use empirical knowledge of conductances at a small scale to infer an averaged conductance for use in a bulk model at a larger scale. The 'scaling problem' in land–atmosphere interaction is essentially the problem of using information about exchange processes at one scale, such as empirical transfer coefficients, to characterise the same processes at a larger (or smaller) scale.

Three spatial scales of great practical importance are the single energy-exchanging element, the homogeneous land-surface patch, and the heterogeneous landscape of linear scale up to a few hundred kilometres. For brevity, these will be called the leaf, canopy and regional scales, respectively. The two primary scale translations in practice are therefore leaf–canopy and canopy–region. Leaf–canopy scaling has been an issue for several decades, following the postulate by Monteith (1965) that a canopy acts as a ‘big leaf’ with a canopy conductance equal to the parallel sum of the leaf stomatal conductances (references to later work are given in context). Canopy–region scaling has advanced in part through the need for spatially averaged lower
boundary conditions in large-scale atmospheric models. Using the idea of a 'blending height' above which the flow is approximately spatially uniform, several workers have proposed canopy–region scaling rules for momentum transfer, by quantifying the regionally averaged momentum roughness length (Mason, 1988; Claussen, 1991). A similar treatment of heat exchange was given by Wood and Mason (1991), whereas Blyth et al. (1993) considered coupled sensible and latent heat exchange on the basis of heuristic scaling arguments for transfer resistances.

This paper considers some features of the areal averaging problem for energy exchanges, applicable to both the leaf–canopy and canopy–region scale translations. The approach is based on the Penman–Monteith or combination equation (CE) for the latent and sensible heat fluxes at an evaporating surface. The key requirements are (1) scalar conservation, which implies that net fluxes of conserved scalars must average linearly across a land surface, and (2) the matching of model form between scales, which is a prerequisite if model parameters (such as surface conductances) are to be interpreted similarly at different scales. The plan of the paper is as follows. Sections 2, 3 and 4 develop a unified scheme by laying out some basic properties of the CE (Section 2), establishing some general constraints and procedures for areal averaging (Section 3), and then spatially averaging the latent and sensible heat fluxes (Section 4). In Section 5, the scheme is applied to canopy–region scale translations, to illustrate the complications introduced by radiative coupling (the dependence of available energy on surface temperature through the outward longwave radiation component). Section 6 considers leaf–canopy translations, particularly to derive analytic expressions for canopy–scale conductances in terms of leaf-scale properties. Conclusions appear in Section 7.

A complete list of symbols is provided at the beginning of the paper. Much of the paper is independent of any particular scale translation such as leaf–canopy or canopy–region. Therefore, fluxes and conductances at the smaller and larger scales are denoted \( f \) and \( F \), respectively, with a subscript to identify the transferred entity. Conductances and resistances (with the dimensions of velocity and inverse velocity, respectively) are \( g \) and \( r = 1/g \) at smaller scale, and \( G \) and \( R = 1/G \) at larger scale. ‘Isothermal’ quantities (defined as if the surface temperature were equal to air temperature and all else remained unchanged) are denoted by Greek letters: thus \( \phi \) and \( \Phi \) denote isothermal fluxes at smaller and larger scales. It is sometimes convenient to denote an areal average explicitly by angle brackets: for instance, \( F = \langle f \rangle \) and \( G = \langle g \rangle \). These averages are not necessarily linear.

2. Generic properties of the combination equation

The combination equation (CE) is one of the best known and most widely used equations in micrometeorology. This section summarises some of its properties for later reference. The CE describes the partition of the energy balance \( f_A = f_E + f_H \) at an evaporating surface, where \( f_E \) and \( f_H \) are the surface-to-air fluxes of latent and sensible heat and \( f_A \) the air-to-surface flux of available energy (the net irradiance less thermal storage and photosynthetic energy usage; however, thermal storage and
photosynthetic contributions are not considered explicitly hereafter). The CE is usually written to describe \( f_E \), but a complementary form exists for \( f_H \). These are:

\[
f_E = \frac{\epsilon \phi_A + \rho \lambda D (g_{aH} + g_r)}{\epsilon + (g_{aH} + g_r)(r_{aE} + r_s)}
\]

(1)

\[
f_H = \frac{g_{aH}(r_{aE} + r_s)\phi_A - \rho \lambda D g_{aH}}{\epsilon + (g_{aH} + g_r)(r_{aE} + r_s)}
\]

(2)

Here \( \phi_A \) is the isothermal available energy flux, equal to \( f_A \) with the outward longwave flux evaluated at air temperature rather than surface temperature (Monteith, 1973); \( g_{aH} = 1/r_{aH} \) and \( g_{aE} = 1/r_{aE} \) are the aerodynamic conductances for heat and water vapour transfer, respectively; \( g_s = 1/r_s \) is the stomatal conductance; \( g_r \approx 1/230 \text{ ms}^{-1} \) is the radiative conductance (Monteith, 1973); \( D = Q_{sat}(\Theta) - Q \) is the potential saturation deficit of the ambient air, with \( \Theta \) potential temperature and \( Q \) specific humidity; \( \epsilon = (\lambda/c_p) dQ_{sat}/dT \) is the dimensionless slope of the saturation specific humidity \( Q_{sat}(T) \) as a function of temperature \( T \); \( \rho \) is air density; \( \lambda \) is the latent heat of vaporisation of water; \( c_p \) is the isobaric specific heat of air. Conductances \( g \) and resistances \( r = 1/g \) are interchanged freely according to convenience, conductances being better for parallel sums \( (g = g_1 + g_2) \) and resistances for series sums \( (r = r_1 + r_2) \). The reason for the replacement of the available energy \( f_A \) with its isothermal counterpart \( \phi_A \) is to accommodate ‘radiative coupling’, the dependence of the outward longwave part of the net irradiance upon surface temperature. The radiative conductance \( g_r \), which acts in parallel with the aerodynamic conductance \( g_{aH} \) for heat, then accounts for the difference in longwave fluxes from bodies at air and surface temperature. In terms of \( \phi_A \), the surface energy balance \( f_A = f_E + f_H \) becomes

\[
\phi_A = f_E + \left(\frac{g_{aH} + g_r}{g_{aH}}\right)f_H
\]

(3)

The difference between \( \phi_A \) and \( f_A \) is \( (g_r/g_{aH})f_H \), so that \( \phi_A \) exceeds \( f_A \) when \( f_H \) is positive, and vice versa.

It is convenient to define the total thermal resistance \( r_{tH} = (g_{aH} + g_r)^{-1} \) (the parallel sum of the aerodynamic and radiative pathways) and the total latent heat resistance \( r_{tE} = r_{aE} + r_s \) (the series sum of the aerodynamic and surface pathways). Eqs. (1)–(3) then take the more compact forms

\[
f_E = \frac{\epsilon r_{tH} \phi_A + \rho \lambda D}{r_d}
\]

(4)

\[
f_H = p \left(\frac{r_{tE} \phi_A - \rho \lambda D}{r_d}\right)
\]

(5)

\[
\phi_A = f_E + p^{-1}f_H
\]

(6)
where
\[
\begin{align*}
    r_d &= \epsilon r_{tH} + r_{tE} = \epsilon r_{tH} + r_{aE} + r_s \\
    p &= r_{tH} / r_{aH} = g_{atH} / (g_{atH} + g_r)
\end{align*}
\] (7)

The resistance \( r_d \), the denominator in the CE, is closely related to the resistance controlling the flux of saturation deficit (see below), and will therefore be called the 'deficit resistance'. The ratio \( p \) accounts for radiative coupling; it is typically in the range 0.5–0.9 (lower for surfaces of lower roughness or in lighter wind speeds). If \( p = 1 \), then radiative coupling is absent.

An important thermodynamic property of an evaporating surface is the 'equilibrium evaporation rate', \( \phi_{eq} \). This quantity can be defined in several different ways, the most fundamental being the long-term equilibrium evaporation rate into an enclosed, well-mixed volume from an evaporating surface supplied with energy (McNaughton and Jarvis, 1983). Its value can be found by considering an enclosed, well-mixed slab of air with fixed height \( h \), containing an evaporating lower surface with fixed resistances, supplied with energy at a fixed \( \phi_A \) (McNaughton and Jarvis, 1983). The conservation equation for the saturation deficit \( D \) in the slab is
\[
dD/dt = f_D/h
\]

where
\[
f_D = (\phi f_H - f_E)/\rho \lambda
\] (8)
is the flux of saturation deficit. Using the CE for \( f_E \) and \( f_H \), it follows that
\[
f_D = (\delta_{eq} - D)(p_e + 1) / r_d
\] (9)

where
\[
\delta_{eq} = \frac{p\phi_r(r_s + r_{aE} - r_{aH})}{(p_e + 1)\rho \lambda}
\] (10)

and \( r_d \) is given by Eq. (7). The deficit conservation equation then becomes
\[
dD/dt = (\delta_{eq} - D)(p_e + 1)/(r_d h),
\] a first-order linear ordinary differential equation with an exponentially decaying solution \( D(t) \), tending as \( t \to \infty \) to \( f_D = 0 \) and \( D = \delta_{eq} \). Physically, \( D = \delta_{eq} \) is the 'equilibrium saturation deficit' attained in the enclosed slab (denoted by a Greek letter because it is derived from \( \phi_A \)), and \( r_d/(p_e + 1) \) is the resistance linking \( \delta_{eq} - D \) to the flux \( f_D \) (hence the name 'deficit resistance' for \( r_d \)).

When \( D = \delta_{eq} \) is substituted into the CE, Eq. (4), the associated equilibrium evaporation rate is found to be
\[
\phi_{eq} = \frac{p\phi_r\phi_A}{p_e + 1} = \frac{\epsilon r_{tH}\phi_A}{\epsilon r_{tH} + r_{aH}}
\] (11)

As equilibrium evaporation represents a fundamental, thermodynamically determined state, it is appropriate to recast the CE to express the fluxes \( f_E \) and \( f_H \) as perturbations about their equilibrium values. This is easily done by using the fact that the fluxes \( \phi_A \) (or \( \phi_{eq} \)) and \( f_D \) are linear combinations of \( f_E \) and \( f_H \), defined by
Eqs. (6) and (8). Therefore \( f_E \) and \( f_H \) can likewise be expressed as linear combinations of \( \phi_{eq} \) and \( f_D \), yielding

\[
f_E = \phi_{eq} + \frac{\rho \lambda (D - \delta_{eq})}{r_d} \tag{12}
\]

\[
f_H = \phi_{eq} + \frac{p \rho \lambda (\delta_{eq} - D)}{r_d} \tag{13}
\]

These can be called the 'equilibrium departure' forms of the CE. They constitute building blocks for the averaging scheme described below.

3. Generic constraints upon areal averaging

Constraints must be established to define larger-scale surface properties from their smaller-scale counterparts. Two generic tools for imposing consistent constraints are discussed here: (1) the primary physical constraint, from scalar conservation requirements, that net fluxes of conserved scalars must average linearly over the land surface; (2) the matching of model form across scales.

3.1. Constraints required by scalar conservation

The scalar conservation equation \( \frac{\partial C}{\partial t} + \mathbf{U} \cdot \nabla C = -\nabla \cdot \mathbf{f}_C \) (where \( C \) is the concentration of a conserved scalar, \( \mathbf{f}_C \) the sum of turbulent and molecular flux density vectors, and \( \mathbf{U} \) the mean velocity vector) provides a primary physical constraint on areal averaging. For modelling purposes, this equation is always integrated spatially over air volumes \( R \) such as the air space in a canopy layer, a general circulation model grid cell or the entire convective boundary layer. Let \( R \) be bounded partly by the land–atmosphere interface, so that its closed bounding surface \( S = S_L + S_A \) consists of a part \( S_L \) coincident with the land–atmosphere interface (vegetation or soil) and a part \( S_A \) in the air. Using the divergence theorem, the conservation equation can be integrated over \( R \):

\[
\frac{\partial}{\partial t} \int_R \int_S C d\mathbf{x} = \int_S U_n C d\mathbf{S} + \int_{S_A} f_C dS + \int_{S_L} f_C dS \tag{14}
\]

where \( U_n \) and \( f_C \) are the inward-normal components of \( \mathbf{U} \) and \( \mathbf{f}_C \) on \( S \), and \( x \) is a position coordinate. The three terms on the right-hand side correspond respectively to advection, atmospheric flux divergence and the scalar input into \( R \) by surface fluxes. If this last term is written as \( S_L F_C \), with \( F_C = \langle f_C \rangle \) an areally averaged flux density over the surface \( S_L \), then Eq. (14) requires that

\[
F_C = \frac{1}{S_L} \int_{S_L} f_C dS = \sum_{i=1}^{n} a_i f_{Ci} \tag{15}
\]

where \( S_L \) consists of several patches \( i (i = 1 \) to \( n) \) with areas \( S_i \), normalised areas
\[ a_i = \frac{S_i}{S_L} \text{ and flux densities } F_{Ci}. \]

Eq. (15) says that scalar mass conservation requires elemental surface flux densities to be averaged linearly, with area weighting, across land surfaces. Areal averages of other land–atmosphere interaction parameters (surface temperatures, conductances and so on) need to be consistent with this constraint to satisfy mass conservation. No equivalent linear-averaging constraint exists for these other parameters.

Eq. (15) applies to net fluxes of conserved scalars, in particular, \( f_E \) and \( f_H \) and their sum \( f_A = f_E + f_H \), the available energy flux. Eq. (15) also applies to net irradiances (quantum fluxes) in any spectral waveband, as these are also net scalar fluxes. However, it does not apply to inward and outward irradiances \( (f_{S1}, f_{S7} \) for shortwave radiation, \( f_{L1}, f_{L7} \) for longwave) in the case when multiple scattering occurs between the surfaces \( S_i \), as in a plant canopy or other non-planar surface. The reason is that incoming and outgoing quantum fluxes do not obey conservation principles individually in a multiple-scattering environment: a single quantum scattered from surface element \( S_1 \) to element \( S_2 \) is both outgoing for \( S_1 \) and incoming for \( S_2 \), so that the contribution of a single quantum to (say) \( f_{S1} \) can be counted many times. As the isothermal available energy flux \( \phi_A = f_A + f_{L1}(\Theta_0) - f_{L1}(\Theta) \) includes terms dependent on outward longwave fluxes \( f_{L1} \), both \( \phi_A \) and \( \phi_{eq} \) obey Eq. (15) only approximately in the multiple-scattering case. However, the approximation is fairly good because \( f_{L1}(\Theta_0) - f_{L1}(\Theta) \) is usually small, and becomes exact when \( g_r = 0 \) and \( p = 1 \), so that \( \phi_A = f_A \), a net scalar flux.

### 3.2. Constraints required by matching model form

Let us now consider the areal averaging of an arbitrary dependent variable \( v \) across the land–atmosphere interface, where \( v \) is a specified function \( h(x, y) \) of the independent variables \( x, y \) (it suffices to show two variables, but there may be more). For instance, \( v = h(x, y) \) might represent some form of the CE. The function \( h(x, y) \) constitutes a model for \( v \) at a small scale. We seek a corresponding model \( V = h(X, Y) \) at a larger scale, where \( V, X \) and \( Y \) are area averages over a heterogeneous surface. If \( h(x, y) \) is nonlinear in \( x \) and \( y \) (as are all forms of the CE), it is not possible to use linear averages to define \( X, Y \) and \( V \), and simultaneously to retain the same functional form \( h \) at both smaller and larger scales; for instance, \( \langle x/y \rangle \neq \langle x \rangle/\langle y \rangle \) where the angular brackets denote linear averaging.

It is usually desirable to preserve the same form of model for \( v \) or \( V \) across scales, a requirement often taken as implicit, but stated explicitly by McNaughton (1994). This requires that the function \( h \) be the same at both scales. We can also prescribe the averaging operation defining \( V = \langle v \rangle \), the simplest choice being a linear average; as shown above, this choice is a physical requirement if \( v \) is a net scalar flux. Then it follows that

\[ V = h(X, Y) = \sum a_i v_i = \sum a_i h(x_i, y_i) \] (16)

where \( h \) is the same known function on both sides, and \( a_i \) the area fractions of the heterogeneous surface elements. Eq. (16) is one of two constraints (relationships) needed to define \( X \) and \( Y \) in terms of their patch values \( x_i \) and \( y_i \) (if \( n \) independent
variables are involved, $n$ relationships are required. The extra constraint(s) can be supplied in one of several ways, as follows:

1. If $h(x, y)$ can be broken into additive terms $h_1(x, y) + h_2(x, y)$, then Eq. (16) has two terms on each side. Matching these individually gives $V_1 = h_1(X, Y) = \sum a_i h_i(x_i, y_i)$ and likewise for $V_2$, thus completely determining $X$ and $Y$. The assumption that the terms can be matched individually amounts to a requirement that the ratios among $V_1$, $V_2$ and $V$ be the same for the large-scale model as for the summed small-scale models, usually a reasonable requirement if it is reasonable to average $v$ linearly in the first place. If all of $V$, $V_1$ and $V_2$ are net scalar fluxes, then all must average linearly and this requirement is a physical necessity. This term-by-term matching route has been widely used (Raupach, 1991; Lhomme, 1992; McNaughton, 1994), though it has not always been carefully justified.

2. If an ad hoc assumption is made about how (say) $X$ is defined (usually as a linear average of $x$), then $Y$ can be found from Eq. (16). Unlike route (1), this route does not necessarily preserve ratios among $V$, $V_1$ and $V_2$ in the case where $h = h_1 + h_2$. This can lead to inconsistencies, as shown below.

3. If $h(x, y)$ becomes independent of $y$ in a limit such as $y \to 0$ or $y \to \infty$, then matching $V = h(X)$ with $v = h(x)$ defines $X$, and $Y$ follows from the full match for $h(X, Y)$. This route, used by Finnigan and Raupach (1987), can be subject to difficulties similar to those for route (2).

Three further points need to be made. First, for a procedure of this kind to produce a physically consistent areally averaged model, a linear average for $v$ must be physically realistic. This is guaranteed (from a conservation viewpoint) if $v$ is a net scalar flux, but not if $v$ is another quantity such as a concentration, conductance, albedo or emissivity. For example, Lhomme (1992) assumed that surface temperatures average linearly, and obtained an averaging scheme that did not satisfy the conservation requirement, Eq. (15), for sensible and latent heat fluxes. Second, once the averaged independent variables $X$ and $Y$ have been defined for consistency with a model for one dependent variable $V = h(X, Y)$, then they will remain consistent for algebraic rearrangements of that model, but are not consistent (in general) for models of other dependent variables. Thus, if $X$ and $Y$ are large-scale conductances and a matching procedure based on the CE is used to define them, these conductances can be used in any form of the CE at large scale, but cannot automatically be used for other purposes, such as modelling momentum exchange or radiometric surface temperature. Third, there is no assurance that it is actually possible to write matched models for both $v$ and $V$ (that is, models with the same function $h$); an example where a complete match is impossible appears later. The possibility of writing a fully consistent, matched model at both smaller and larger scales must be explored case by case.

4. Flux-matching averaging schemes

4.1. Latent heat flux

We begin by areally averaging the equilibrium-departure form of the CE for latent
heat flux, Eq. (12). A linear average is appropriate because $F_E$ is a net scalar flux, and term-by-term matching is appropriate both to preserve model form and also because the equilibrium-departure CE expresses $F_E$ as a linear combination of two other scalar fluxes, $\phi_A$ and $f_D$. Forming the average $F_E$ as in Eq. (16), we obtain:

$$F_E = \Phi_{eq} + \frac{\rho \lambda (D_i - \Delta_{eq})}{R_d} = \sum a_i \left[ \phi_{eqi} + \frac{\rho \lambda (D_i - \Delta_{eqi})}{r_{di}} \right].$$

(17)

where the sum runs across a series of heterogeneous surface elements distinguished by the subscript $i$.

Analytic progress requires the simplification that all surface elements are exposed to the same ambient deficit $D = \langle D \rangle = D_i$; in other words, that the volume $R$ in Eq. (14) is well mixed. This is a good approximation for microscale land surface units in the daytime atmospheric boundary layer, and also applies for leaves in a plant canopy if the deficit changes across individual leaf boundary layers are large compared with changes through the canopy air space. If a well-mixed assumption cannot be made, then the problem of averaging over land surface elements cannot be separated from the problem of how $D_i$ varies across the surface, which is a dispersion or advection–diffusion problem for the scalar $D$. Techniques for solving this problem have been developed for many of the important situations encountered in scaling from leaf to region, including plant canopies (Raupach, 1989; Dolman and Wallace, 1991), leading-edge advection (Philip, 1987) and flow over hills (Raupach et al., 1992). However, to trace the structure of the scaling problem alone, it is helpful to avoid dispersion complications by making the well-mixed assumption.

Eq. (17) contains three terms which can be matched individually on grounds given above. Taking $D = \langle D \rangle = D_i$, we obtain

$$\Phi_{eq} = \sum a_i \phi_{eqi}$$

(18)

$$1/R_d = \sum a_i / r_{di}$$

(19)

$$\Delta_{eq}/R_d = \sum a_i \delta_{eqi} / r_{di}$$

(20)

thus defining the spatially averaged quantities, $\Phi_{eq}$, $R_d$ and $\Delta_{eq}$ in terms of elemental quantities. Although these three quantities are sufficient to calculate the spatially averaged $F_E$, it is also useful to develop expressions for spatial averages of the four resistances $R_{ith}$, $R_{aht}$, $R_{ahe}$ and $R_s$. To match model form across scales, these must relate to $R_d$ and $\Delta_{eq}$ in the same way that the equivalent small-scale quantities are related by Eqs. (7) and (10). This provides two constraints upon the four resistances, which can be written (using Eq. (11) at large scale to replace $\Phi_A$ with $\Phi_{eq}$)

$$R_d = \epsilon R_{ith} + R_{ahe} + R_s$$

(21)

$$\rho \lambda \Delta_{eq} = \Phi_{eq} (R_s + R_{ahe} - R_{aht})$$

(22)

Therefore two extra constraints are required. In the case where the aerodynamic resistances for heat and water vapour are the same (so that $r_{ahe} = r_{aht}$ and
$R_{aE} = R_{aH}$), Eqs. (22) and (20) define $R_s$ uniquely as

$$R_s = \frac{\rho \lambda \Delta \Phi_{eq}}{\Phi_{eq}} = \frac{R_d}{\Phi_{eq}} \sum a_i \phi_{eq} r_{si}$$

(23)

Thus, the area-averaged $R_s$ is given by an average of $r_{si}$ weighted at element scale with the factor $a_i \phi_{eq}/r_{di}$. Algebraic manipulation of Eqs. (19)-(22) shows that, with arbitrary $r_{AE}$ and $r_{aH}$, $\epsilon R_{tH} + R_{aH}$ is defined from $\epsilon r_{tH} + r_{aH}$ by exactly the same weighting factor. Therefore, a sensible way to define all of $R_{tH}$, $R_{aH}$, $R_{aE}$ and $R_s$ is

$$R_x = \frac{R_d}{\Phi_{eq}} \sum a_i \phi_{eq} r_{xi}$$

(24)

$x = H, A, aE, s$

so that this same weighting factor defines each resistance individually.

This averaging scheme for $F_E$ will be labelled $M_1$. It is defined by Eqs. (17)-(24) and is based on the equilibrium-departure CE. It is one of a class of schemes that can be called ‘flux-matching’ (M), because they are based on matching net scalar fluxes that must average linearly to satisfy scalar conservation. Scheme $M_1$ was proposed in a less general form (with $p = 1$ and $r_{AE} = r_{aH}$) by Raupach (1991, 1993).

In a recent comparison of several averaging schemes, McNaughton (1994) inferred another, similar flux-matching scheme by the alternative method of term-by-term matching the basic CE for $f_E$, Eq. (4), rather than the equilibrium-departure CE. In this case the matching equation is

$$F_E = \frac{\epsilon r_{tH} \phi_A + \rho \lambda \Delta}{R_d} = \sum a_i \left( \frac{\epsilon r_{tH} \phi_{Ai} + \rho \lambda \Delta}{r_{di}} \right)$$

(25)

Making the initial assumption

$$\phi_A = \sum a_i \phi_{Ai}$$

(26)

and carrying out a term-by-term match, we obtain Eq. (19) for $R_d$ and

$$R_x = \frac{R_d}{\phi_A} \sum a_i \phi_{Ai} r_{xi}$$

(27)

$x = H, aE$

for $R_{tH}$ and $R_{aE}$, of similar form to Eq. (24) but with $\phi_A$ replacing $\phi_{eq}$ in the weighting factor. Extra constraints are still needed to define all of $R_{tH}$, $R_{aH}$, $R_{aE}$ and $R_s$ in the general case with radiative coupling ($r_{tH} \neq r_{aH}$); as before, these can be chosen by generalising Eq. (27). This scheme will be denoted $M_2$; it is defined by Eqs. (19) and (25)-(27), and is founded on the basic CE. Schemes $M_2$ and $M_1$ are precisely identical when $p$ is constant over all surface elements, but give slightly different large-scale resistances otherwise. Both $M_1$ and $M_2$ produce the same large-scale flux $F_E$, exactly satisfying the conservation requirement of Eq. (15), that fluxes average linearly.

4.2. Sensible heat flux

In averaging the sensible heat flux, several alternatives are available which are not necessarily equivalent. First, the equilibrium-departure form of the CE for $f_H$,
Eq. (13), leads to a definition of $F_H$ under Scheme $M_1$:

$$F_H = \frac{\Phi_{eq}}{\epsilon} + \frac{P_\rho \lambda (\Delta_{eq} - \langle D \rangle)}{R_d} = \sum a_i \left[ \frac{\phi_{eq}}{\epsilon} + \frac{p_i \rho \lambda (\delta_{eq} - D_i)}{r_{di}} \right]$$

(28)

where $P$ is a spatial average (yet to be defined) of $p_i = g_{adi} / (g_{adi} + g_{ri})$. Second, the basic CE for $f_H$, Equation (5), yields $F_H$ under Scheme $M_2$:

$$F_H = P \left( \frac{R_t E_R A - \rho \lambda D}{R_d} \right) = \sum a_i p_i \left( \frac{r_t E_R A_i - \rho \lambda D}{r_{di}} \right)$$

(29)

Third, $F_H$ can be found from the large-scale surface energy balance

$$F_H = P (\Phi_A - F_E)$$

(30)

with $\Phi_A$ from Eq. (26) and $F_E$ from either of Schemes $M_1$ or $M_2$ (both of which give the same result, exactly satisfying conservation). This scheme for $F_H$ will be called $M_3$. We will compare Schemes $M_1$, $M_2$ and $M_3$ for $F_H$, but we must first consider the definition of $P$ in each scheme.

At this stage, a small difficulty appears. Under Scheme $M_1$, for instance, matching the second and third terms of Eq. (28) produces

$$P = \sum a_i p_i$$

(31)

Retaining the former definitions of $R_d$ and $\Delta_{eq}$, these represent different, incompatible definitions of $P$: the former requires $P$ to be independent of $\delta_{eq}$ and hence $\phi_{eq}$, whereas the latter requires the opposite. This is a case where complete matching across scales is impossible. The problem disappears in the case of no radiative coupling ($g_r = 0$, $p_i = 1$, $P = 1$), the only case dealt with explicitly by Raupach (1991, 1993) and McNaughton (1994). A similar problem exists under Scheme $M_2$, whereas Scheme $M_3$ leaves $P$ unresolved.

This formal difficulty is not a serious problem in practice, because the real-world range of $p$ extends only from about 0.4 to just less than unity, a very narrow proportional variation in comparison with most land–atmosphere exchange parameters (for instance, $g_s$, which varies from zero to infinity). Therefore we anticipate, subject to testing in the next section, that there will be only a small error induced by replacing $p_i$ with a suitable averaged, constant value $P$ in the summations on the right-hand sides of either Eq. (28) or (29). Such a replacement has the consequence that $F_H$ from the large-scale model is no longer precisely equal to $\sum a f_{Hi}$; that is, the scalar conservation requirement is no longer exactly met. Of many possible ways to define the average $P$, the following two have been found (by tests described in the next section) to be both simple and effective. First, $P$ can be taken as a simple area average:

$$P_1 = \sum a_d p_i$$

(32)

Second, $P$ can be defined so that the area-averaged values of $\Phi_{eq}$ and $\Phi_A$ (defined respectively by Eqs. (18) and (26)) satisfy the large-scale counterpart $\Phi_{eq} = P e \Phi_A (P e + 1)$ of the definition of equilibrium evaporation, Eq. (11). This
requires
\[ P_2 = \frac{\Phi_{eq}}{e(\Phi_A - \Phi_{eq})} \] (33)

Once \( P_1 \) is replaced by an average \( P \) (either \( P_1 \) or \( P_2 \)), then matching \( F_H \) under either Scheme M1 or M2 is straightforward and produces the same results for the averaged resistances as obtained for \( F_E \). Scheme M3 yields \( F_H \) directly once \( P \) is defined.

It is necessary to test empirically the approximation of using an average \( P \) in Eqs. (28)–(30) for \( F_H \), by evaluating the difference between \( F_H \) (calculated using the large-scale equations) and \( \sum a_i f_{Hi} \), a measure of the departure from exact scalar conservation in averaging \( F_H \). This is done in the next section, using the example of a canopy–region translation in a heterogeneous landscape.

4.3. Isothermal available energy flux

Areally averaging \( \Phi_A \) is straightforward when surface-to-surface multiple scattering is absent and \( \sum a_i = 1 \), as for canopy–region translations in level terrain (but not for leaf–canopy translations). In this case, the average \( \Phi_A \) is
\[ \Phi_A = F_{S1} - \langle \alpha_s \rangle F_{S1} + \langle \epsilon_s \rangle F_{L1} - \langle \epsilon_s \rangle \sigma \Theta^4 \] (34)
where \( f_{S1} \) and \( f_{L1} \) are the elemental incoming shortwave and longwave irradiances, \( \alpha_s \) the albedo, \( \epsilon_s \) the emissivity, and \( \sigma \) the Stefan–Boltzmann constant. In the absence of multiple scattering, Eq. (15) applies to \( f_{S1} \) and \( f_{L1} \), so we have
\[ F_{S1} = \sum a_i f_{S1,i}, \quad F_{L1} = \sum a_i f_{L1,i} \] (35)
where the last two equalities follow by matching the second and fourth terms of Eq. (34), respectively. A slightly different definition of \( \langle \epsilon_s \rangle \) (including weighting with \( f_{L1} \)) is implied by matching the third term, but the variation of both \( \epsilon_s \) and \( f_{L1} \) from element to element is usually small, so the errors introduced by the last of Eq. (35) are negligible.

5. Averaging from canopy to region

To test the averaging Schemes M1, M2 and M3, we consider the problem of averaging over microscale surface patches within a convective boundary layer (CBL). Here ‘microscale’ means that the patches are small enough that the overlying CBL is well mixed (Raupach, 1991); a criterion for this is that the patch length scale be much less than \( hU/w_* \), where \( h \) is CBL depth, \( U \) the bulk wind speed in the CBL and \( w_* \) the convective velocity scale. It is then appropriate to assume a common,
well-mixed saturation deficit $D$ for all patches. In this section, small-scale ($f, g, r$) and large-scale ($F, G, R$) quantities describe landscape patches and regional averages, respectively. The small-scale aerodynamic resistance describes transfer from the surface to the horizontally uniform flow above the blending height.

As a test, the schemes were applied to a series of hypothetical microscale-heterogeneous regions consisting of just two surface types, chosen from four possibilities: crop, desert, forest and lake. Thus, six surface pairings were considered in all: crop–desert, crop–forest, crop–lake, desert–forest, desert–lake and forest–lake. The four possible surface types, which span much of the observed range of wetness and roughness, were ascribed albedos ($\alpha_s$), roughness lengths ($z_0$) and patch-scale surface conductances ($g_s$) as shown in Table 1. The following common conditions were assumed to apply for all surface types: well-mixed CBL potential temperature $\Theta = 25^\circ C$, specific humidity $Q = 10 \text{ g kg}^{-1}$, downward shortwave and longwave irradiances $f_{SR} = 500 \text{ W m}^{-2}$ and $f_{LR} = 374 \text{ W m}^{-2}$, emissivity $\varepsilon_s = 0.99$. Aerodynamic resistances were estimated for the present purpose as $r_{ae} = \ln(z/z_0)/\ln(5z/z_0)/(k^2U)$ with $k = 0.4$, reference height $z = 50 \text{ m}$ and bulk wind speed $U = 5 \text{ m s}^{-1}$ (the factor five represents a typical ratio between momentum and scalar roughness lengths). For these conditions, the terms in the energy balance, Eq. (3), were calculated for each surface type and are given in Table 1. These values provide fluxes and other properties at patch scale.

For each pairing of surface types (say forest–lake), areally averaged latent and sensible heat fluxes were calculated for the complete range of surface fractions of forest (fraction $a_1$) and lake (fraction $a_2 = 1 - a_1$) as $a_1$ varies from zero (pure lake) to unity (pure forest). The calculations were done first by summing the patch fluxes ($F_E = \sum a_i f_{Ei}$ and likewise for $F_H$) to give the true areally averaged fluxes required by scalar conservation, and then using Schemes $M_1, M_2$ and $M_3$, each with $P$ defined both as $P_1$ and $P_2$ (Eqs. (32) and (33)).

Fig. 1 shows the results for the forest–lake pairing (the one with the largest errors in $F_H$) as a plot of $F_E$ and $F_H$ against the area fraction $a_1$. In each case, the scalar conservation requirement is a linear variation of the flux between its values for $a_1 = 0$ and $a_1 = 1$, from Eq. (15). For $F_E$, all schemes satisfy this requirement exactly, as required by Eqs. (17) and (25). For $F_H$, on the other hand, there are slight

<table>
<thead>
<tr>
<th></th>
<th>Desert</th>
<th>Crop</th>
<th>Lake</th>
<th>Forest</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_s$</td>
<td>0.3</td>
<td>0.15</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>$z_0(m)$</td>
<td>0.01</td>
<td>0.05</td>
<td>0.0001</td>
<td>2.0</td>
</tr>
<tr>
<td>$r_s(sm^{-1})$</td>
<td>2000</td>
<td>50</td>
<td>0.1</td>
<td>200</td>
</tr>
<tr>
<td>$\phi_A(W m^{-2})$</td>
<td>276.2</td>
<td>351.2</td>
<td>401.2</td>
<td>376.2</td>
</tr>
<tr>
<td>$f_S(W m^{-2})$</td>
<td>35.8</td>
<td>294.3</td>
<td>275.5</td>
<td>172.8</td>
</tr>
<tr>
<td>$f_L(W m^{-2})$</td>
<td>155.4</td>
<td>41.5</td>
<td>56.5</td>
<td>185.2</td>
</tr>
<tr>
<td>$\gamma_{AH}(m^{-1})$</td>
<td>107.8</td>
<td>73.5</td>
<td>241.6</td>
<td>19.4</td>
</tr>
<tr>
<td>$P = \delta_{ab} \div (\delta_{ab} + g_s)$</td>
<td>0.647</td>
<td>0.728</td>
<td>0.449</td>
<td>0.910</td>
</tr>
</tbody>
</table>

Table 1
Properties of patches used to test the flux-matching averaging scheme for canopy-region translations; albedo $\alpha_s$, roughness length $z_0$ and surface conductance $g_s$ are prescribed; other properties are calculated.
Fig. 1. Regionally averaged latent and sensible heat fluxes $F_E$ and $F_H$ over a forest–lake surface. Fluxes are calculated as linear averages, and also using averaging Schemes M1, M2 and M3, with choices $P_1$ and $P_2$ for $P$.

The differences between the averaging schemes (the mild curves) and conservation requirement $\sum a_i f_{H_i}$ (the straight line). Similar results are obtained for the other surface pairings. The complete set of results for all six pairings is summarised in Table 2, by giving the maximum percentage error, $100(F_H - \sum a_i f_{H_i})/(\sum a_i f_{H_i})$, in the estimates of $F_H$ from the various schemes. As shown in Fig. 1, this maximum error usually occurs close to $a_1 = 0.5$. The errors vary with surface pairing and the exact choice of averaging scheme; they are usually between 1% and 10% in magnitude.

The following conclusions complete the discussion of canopy–region translations:

1. Schemes M1, M2 and M3 satisfy the scalar conservation requirement of linear flux averaging exactly for $F_E$, as indicated above.

2. None of the schemes satisfies the conservation requirement exactly for $F_H$. However, maximum errors for all schemes are fairly small (typically 5% in magnitude). The largest error observed was $-15.7\%$ (incurred by all schemes, with $P = P_2$, for the forest–lake pairing).

Table 2

<table>
<thead>
<tr>
<th>Surface Pairing</th>
<th>$M_1$ with $P_1$</th>
<th>$M_2, M_3$ with $P_1$</th>
<th>$M_1, M_2, M_3$ with $P_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crop–desert</td>
<td>4.6</td>
<td>4.2</td>
<td>4.6</td>
</tr>
<tr>
<td>Desert–lake</td>
<td>$-9.5$</td>
<td>$-5.6$</td>
<td>$-10.3$</td>
</tr>
<tr>
<td>Desert–forest</td>
<td>1.7</td>
<td>1.4</td>
<td>2.1</td>
</tr>
<tr>
<td>Forest–crop</td>
<td>$-7.0$</td>
<td>$-6.6$</td>
<td>$-7.0$</td>
</tr>
<tr>
<td>Forest–lake</td>
<td>$-15.4$</td>
<td>$-8.0$</td>
<td>$-15.7$</td>
</tr>
<tr>
<td>Crop–lake</td>
<td>1.0</td>
<td>9.9</td>
<td>4.4</td>
</tr>
</tbody>
</table>
(3) The following exact equalities emerge from the calculations and are easily verified algebraically: Schemes M2 (from the basic CE) and M3 (from the large-scale energy balance) produce the same \( F_H \) irrespective of the choice of \( P \); and choosing \( P = P_2 \) produces the same \( F_H \) for all of Schemes M1, M2 and M3.

(4) The smallest errors in \( F_H \) are obtained by choosing \( P = P_1 \) and using Schemes M2 or M3 (which are equivalent). However, errors from other schemes are only a little larger.

(5) Similar calculations have been carried out with other expressions for \( P \) than Eqs. (32) and (33), leading to similar but slightly inferior results (details are omitted).

(6) When \( p = 1 \) (no radiative coupling), then \( P_1 = P_2 = 1 \) and all of Schemes M1, M2 and M3 become equivalent for both \( F_E \) and \( F_H \), exactly satisfying conservation requirements.

6. Averaging from leaf to canopy

We now turn to leaf–canopy scale translations, appropriately adjusting the notation: large-scale quantities \( (F, G, R) \) now apply to the canopy, and small-scale quantities \( (f, g, r) \) to leaves or other canopy elements. This section is focused on one key problem in leaf–canopy scaling: the relationship between the leaf stomatal conductance and the canopy-scale surface conductance. The leaf surface conductance can be regarded as essentially a physiological parameter (not externally prescribed, but dependent on many physical and biological variables in the soil–vegetation–atmosphere system). The problem is to identify the extent to which various possible versions of the canopy-scale surface conductance reflect the same physiological properties of the vegetation canopy as the leaf stomatal conductance, and the extent to which they are influenced by other, non-physiological properties of the system. This question has been debated for some time (e.g. Philip, 1966; Monteith, 1973; Tan and Black, 1976; Finnigan and Raupach, 1987; Raupach and Finnigan, 1988; Baldocchi et al., 1991), often via measurements on specific canopies or numerical calculations. The present effort is to approach the problem in a fairly general way.

6.1. Approach and simplifications

The approach is to construct a sequence of analytic model canopies at successively increasing levels of detail and realism. Relationships between elemental and bulk conductances can then be derived explicitly under various averaging schemes. In other words, the collapse from a multilayer to a single-layer canopy model (Raupach and Finnigan, 1988) is accomplished analytically, albeit with a highly simplified multilayer description. All the following analytic model canopies have the following features in common:

1. the total latent heat flux \( F_E \) from a vegetated surface is split into a vegetation (canopy) component \( F_{Ec} \) and a soil (ground) component \( F_{Eg} \), to be treated separately:

\[
F_E = F_{Ec} + F_{Eg}
\]
(2) A well-mixed assumption is made for the saturation deficit $D$ in the canopy, so that evaporation from all canopy elements takes place into air with a common $D$. This simplification is valid when deficit differences across individual leaf boundary layers are large compared with changes through the canopy air space. Relaxation of this restriction is a task for a numerical canopy model.

(3) The complications of radiative coupling are avoided by assuming $\bar{g}_r = 0$ and $p = 1$ for all surface elements. This means that Schemes $M_1$, $M_2$ and $M_3$ all reduce to the same generic flux-matching scheme (henceforth denoted $M$) which satisfies the conservation requirement of linear flux averaging exactly for both $F_E$ and $F_H$. Also, we will take $r_{aH}$ and $r_{aE}$ to be identical for all elements: $r_{aH} = r_{aE} = r_a$.

6.2. Averaging schemes for canopy evaporation

We first treat the vegetation (canopy) component $F_{Ec}$ of the total latent heat flux, working with the equilibrium-departure $CE$. In accord with usual conventions, element-scale fluxes (properly flux densities) are normalised with single-sided element (leaf) area, and canopy-scale fluxes with ground area; $\sum a_i$ is then the single-sided leaf area index $\Lambda$ (rather than unity as for canopy-region translations on flat terrain). Analysis is aided by replacing the sums in Eq. (17) with integrals over $\Lambda$, with $d\zeta$ being an element of leaf area index and $\zeta$ a coordinate traversing all the leaf elements over unit ground area, but excluding the ground; thus, the operator $\sum a_i$ becomes $\int d\zeta$. The order in which $\zeta$ traverses the canopy elements can be chosen according to convenience. It is customary for elements to be traversed in order of height (in which case $\zeta$ is the cumulative area index with height), but an alternative is for $\zeta$ to traverse the leaf elements in order of radiation receipt, so that the radiation distribution through the canopy is a smooth function of $\zeta$. Eq. (17) now becomes

$$F_{Ec} = \Phi_{eqc} + \frac{\rho \lambda (D - \Delta_{eqc})}{R_{dc}}$$

$$= \int_0^\Lambda \left[ \phi_{eqc}(\zeta) + \frac{\rho \lambda (D - \delta_{eqc}(\zeta))}{r_d(\zeta)} \right] d\zeta$$

where $R_{dc}$ is the canopy-scale average of $r_d$, $\Delta_{eqc}$ the canopy-scale equilibrium saturation deficit, and $\Phi_{eqc}$ the equilibrium evaporation corresponding to the isothermal available energy flux intercepted by the canopy per unit ground area. The subscript $c$ indicates a quantity defined for the canopy only, excluding the ground. The flux-matching scheme (M) defines the canopy-averaged quantities in Eq. (37) by matching terms:

$$\Phi_{eqc} = \int_0^\Lambda \phi_{eqc}(\zeta) d\zeta, \quad \frac{1}{R_{dc}} = \int_0^\Lambda \frac{d\zeta}{r_d(\zeta)}, \quad \frac{\Delta_{eqc}}{R_{dc}} = \int_0^\Lambda \frac{\delta_{eqc}(\zeta)}{r_d(\zeta)} d\zeta$$

analogous to Eqs. (18)–(20). The canopy-averaged surface and aerodynamic
conductances under Scheme M are given then by
\[
1/G^{(M)}_{ac} = R^{(M)}_{ac} = \frac{\rho \lambda A_{eq}}{\Phi_{eqc}}, \quad 1/G^{(M)}_{ac} = R^{(M)}_{ac} = \frac{R_{sc} - R_{sc}}{\epsilon + 1}
\]
from Eqs. (23) and (21) with \( p = 1 \) (alternative integral expressions can be obtained from Eq. (24)). These conductances apply to the vegetation canopy only, excluding the ground. In particular, \( G_{ac} \) is often called the canopy conductance to distinguish it from the total surface conductance \( G_s \) which describes both the canopy and the ground.

The flux-matching scheme will be compared with two other averaging schemes often used explicitly or implicitly for canopy averaging, concentrating on \( G_s \). One of these (which we will arbitrarily call Scheme J) is obtained by defining the canopy-scale \( G_a \) as a parallel sum over the individual-element conductances \( g_a \):
\[
G^{(J)}_a = \int_0^\zeta g_a(\zeta) d\zeta
\]

The corresponding \( G_{sc} \) is then found by inverting the CE, Eq. (4), written at canopy scale, to make \( G_{sc} \) the subject:
\[
1/G^{(J)}_{sc} = R^{(J)}_{sc} = \epsilon R_{HT}(\Phi_A/F_E - 1) - R_{ae} + \rho \lambda D/F_E
\]
where, in the approximation for this section, \( R_{HT} = R_{ae} = R_a \). The other scheme (to be called Scheme K) uses a parallel-sum bulk canopy conductance \( G_{sc} \):
\[
G^{(K)}_{sc} = \int_0^\lambda g_s(\zeta) d\zeta
\]

This ‘intuitive’ or ‘physiological’ definition of \( G_{sc} \) has been widely used. A corresponding \( G^{(K)}_a \) can be found by inverting the canopy-scale CE to make \( G_a \) the subject, but as our aim is to compare \( G_{sc} \) values, this is not pursued. Schemes J and K are not flux-matching schemes in the sense defined earlier, because they do not preserve a term-by-term match in the equilibrium-departure or basic CE forms.

6.3. Analytic model canopies

The sequence of model canopies can now be defined. To clarify the overall picture, a summary is given in Table 3.

6.3.1. Canopy I

The simplest possibility is to take \( g_s \) and \( g_a \) as constant (\( \zeta \)-independent) through the canopy. It immediately follows that \( G_{ac} = \Lambda g_a \) and \( G_{sc} = \Lambda g_s \) for each of Schemes M, J and K, so all these schemes produce identical, parallel-sum definitions of the bulk canopy and aerodynamic conductances. This result is true for any radiation distribution, described here by \( \phi_{eq}(\zeta) \).
Table 3
Properties of analytic model canopies

<table>
<thead>
<tr>
<th>Canopy</th>
<th>$g_s$</th>
<th>$g_a$</th>
<th>Radiation distribution</th>
<th>Bulk conductances</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Constant</td>
<td>Constant</td>
<td>Arbitrary</td>
<td>$G_{sc}^{(M)} = G_{ac}^{(J)} = G_{ac}^{(K)}$</td>
</tr>
<tr>
<td>II</td>
<td>Arbitrary</td>
<td>$g_a \propto g_s$</td>
<td>Arbitrary</td>
<td>$G_{sc}^{(M)} = G_{ac}^{(J)} = G_{ac}^{(K)}$</td>
</tr>
<tr>
<td>IIIa</td>
<td>$\propto f_P$</td>
<td>Constant</td>
<td>Exponential</td>
<td>$G_{sc}^{(M)} = G_{ac}^{(K)}$</td>
</tr>
<tr>
<td>IIIb</td>
<td>$\propto f_P$</td>
<td>Constant</td>
<td>Linear</td>
<td>$G_{sc}^{(M)} = G_{ac}^{(K)}$</td>
</tr>
<tr>
<td>IVa</td>
<td>$\frac{g_{sx} f_P}{f_P + f_p}$</td>
<td>Constant</td>
<td>Exponential</td>
<td>No equalities</td>
</tr>
<tr>
<td>IVb</td>
<td>$\frac{g_{sx} f_P}{f_P + f_p}$</td>
<td>Constant</td>
<td>Linear</td>
<td>No equalities</td>
</tr>
</tbody>
</table>

6.3.2. Canopy II

The severe restriction of uniform $g_s$ and $g_a$ through the canopy can be relaxed by allowing $g_s$ and $g_a$ to vary proportionally, so that $g_s(\zeta) = c_H g_a(\zeta)$. This is a qualitatively reasonable expectation, in that $g_s$ and $g_a$ usually both decrease with depth into the canopy. As for Canopy I, the radiation distribution can be arbitrary. A little algebra shows that Canopy II (like Canopy I) yields identical definitions for the bulk conductances $G_{sc}$ and $G_{ac}$ under each of the averaging Schemes M, J and K; these definitions all satisfy $G_{sc}/G_{ac} = g_s/g_a = c_H$. Therefore, the big-leaf model for canopy evaporation, with parallel-sum $G_{ac}$ and $G_{sc}$, is exact and self-consistent for well-mixed canopies with elemental $g_a$ proportional to $g_s$, and any radiation distribution.

6.3.3. Canopies III and IV

These canopies explore the consequences of two possible relationships between the stomatal conductance $g_s$ and the radiation field. The idea is that, as radiation fluctuates across canopy elements more than any other quantity influencing $g_s$, the next logical increase in complication in the sequence of analytic model canopies is to account explicitly for the link between the distributions of $g_s$ and radiation. Canopy III assumes that $g_s$ is proportional to the elemental flux $f_P$ of photosynthetically active radiation (PAR):

$$g_s(\zeta) = g'_P f_P(\zeta)$$  \hspace{1cm} (43)

whereas Canopy IV uses the more realistic hyperbolic relationship

$$g_s(\zeta) = \frac{g_{sx} f_P(\zeta)}{f_P(\zeta) + f_p}$$  \hspace{1cm} (44)

where $g'_P$, $g_{sx}$ and $f_p$ are constant through the canopy. A single distribution is used to specify both $f_P$ and $\phi_{eq}$, by assuming (roughly) that $f_P(\zeta)$ is proportional to $\phi_{eq}(\zeta)$ and thence to $\phi_{eq}(\zeta)$, so that $f_P(\zeta) = c_{PP} \phi_{eq}(\zeta)$. The proportionality coefficient is about two in practice (recognising that the assumption of proportionality is crude). To
examine the sensitivity of the results to this single radiation distribution, two choices are adopted: Canopies IIIa and IVa use the exponential distribution

\[ \frac{\phi_{eq}(\zeta)}{\phi_{eq}(\Lambda)} = e^{c(\zeta-\Lambda)} \]  

(45)

where \( c \) (approximately 0.6) is an extinction coefficient. Canopies IIIb and IVb use the linear distribution

\[ \frac{\phi_{eq}(\zeta)}{\phi_{eq}(\Lambda)} = \zeta/\Lambda \]  

(46)

The detailed analysis of Canopies III and IV is given in the Appendix. This shows that for Canopy III (with either radiation distribution), Schemes M and K give identical values for \( G_{sc} \), but the result from Scheme J is different. For Canopy IV, Schemes M, J and K all produce different \( G_{sc} \) values.

6.4. Graphical results

Figs. 2–5 indicate the behaviour of \( F_{Ec} \), and the values of \( G_{sc} \) and \( G_{ae} \) from various averaging schemes, for Canopies IIIa, IIIb, IVa and IVb. Table 4 summarises the parameters held constant for these calculations. The aerodynamic conductance \( g_a \) is assumed to decrease with increasing \( \Lambda \) as \( g_a = g_{ax}/(1 + \Lambda/\Lambda_x) \), to account roughly and empirically for the effect of increasing shelter between elements (but the conclusions do not depend on this assumption). Bearing in mind that the main objective is to compare different averaging schemes applied to the same canopy, rather than to intercompare different canopies, it is useful to choose the stomatal characteristics to give the model canopies roughly similar bulk evaporative fluxes. To do this, the parameter \( g'_a \) (which sets \( dG_{sc}/d\phi_{eq} \) and is defined in the Appendix) is given different values for Canopies III and IV. The value chosen for Canopy IV is in approximate

![Graph showing Canopy evaporation \( F_{Ec} \) and equilibrium and potential soil evaporation \( F_{Eq} \) for Canopies IIIa, IIIb, IVa and IVb. Parameters held constant are given in Table 4.](image-url)
Table 4
Parameters used in Figs. 2–5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_{ec}$</td>
<td>300 W m$^{-2}$</td>
</tr>
<tr>
<td>$c$</td>
<td>0.6</td>
</tr>
<tr>
<td>$T$, air pressure</td>
<td>20°C, 1000 mbar</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>2.20</td>
</tr>
<tr>
<td>$D/Q_{sat}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$g_a$</td>
<td>$g_{sat}/(1 + \Lambda/\Lambda_s)$ with $g_{sat} = 20$ mm s$^{-1}$ and $\Lambda_s = 4$</td>
</tr>
<tr>
<td>$g_{o}$</td>
<td>$0.10$ mm s$^{-1}$ (W m$^{-2}$)$^{-1}$ for Canopy III</td>
</tr>
<tr>
<td></td>
<td>$0.20$ mm s$^{-1}$ (W m$^{-2}$)$^{-1}$ for Canopy IV</td>
</tr>
<tr>
<td>$g_{sa}$ (Canopy IV only)</td>
<td>8.0 mm s$^{-1}$</td>
</tr>
</tbody>
</table>

accordance with the value 200 $\mu$mol m$^{-2}$ s$^{-1}$ approximately equal to 100 W m$^{-2}$ given for $f_P$ by Kelliher et al. (1995).

Fig. 2 shows the canopy evaporation $F_{Ec}$ plotted against $\Lambda$ (also shown are two estimates of the soil evaporation $F_{Es}$, discussed below). For all canopies, $F_{Ec}$ properly tends to zero as the canopy disappears ($\Lambda \to 0$) and tends to a value independent of $\Lambda$ as the canopy becomes very dense ($\Lambda \to \infty$). Evaporation from Canopies III and IV is not the same, which is to be expected as these model canopies have different stomatal properties. The effect of radiation distribution is seen in the differences between Canopies IIIa and IIIb, and between Canopies IVa and IVb: these differences are very small except in dense canopies (at large $\Lambda$), where many of the leaves in the canopy are not exposed to full radiation. For dense canopies, the smoother linear radiation distribution in Canopies IIIb and IVb gives a larger $F_{Ec}$ than the sharply peaked exponential distribution in Canopies IIIa and IVa.

The parallel-sum canopy conductances $G_{ec}^{(K)}$ are shown in Fig. 3. Just as for $F_{Ec}$, $G_{ec}^{(K)}$ tends to zero in the limit $\Lambda \to 0$, and becomes independent of $\Lambda$ in the limit

Fig. 3. Parallel-sum canopy conductance $G_{ec}^{(K)}$ for Canopies IIIa, IIIb, IVa and IVb.
\( \Lambda \to \infty \) (because of the light dependence of \( g_s \) and the shading of lower leaves in dense canopies). Values of \( G_{sc}^{(K)} \) are identical for Canopies IIIa and IIIb, but not for Canopies IVa and IVb. However, the dependence of \( G_{sc}^{(K)} \) on radiation distribution in Canopy IV (that is, the distinction between IVa and IVb) does not become significant until many leaves are shaded at \( \Lambda \gtrsim 3 \). As with \( FE_c \), the parallel-sum conductance at high \( \Lambda \) for Canopy IV is larger for the smooth linear radiation distribution in IVb than for the sharp exponential distribution in IVa.

Fig. 4 compares the three definitions of canopy conductance, \( G_{sc}^{(M)} \), \( G_{sc}^{(J)} \) and \( G_{sc}^{(K)} \), for Canopies IIIa, IIIb, IVa and IVb separately (also shown are bulk surface conductances including soil, to be discussed below). The main result from this figure is that, for the typical dry-canopy conditions assumed here, definitions M, J and K of \( G_{sc} \) yield very similar values in practice. (It is shown below that this similarity does not extend to wet canopies). There is strict equality between \( G_{sc}^{(M)} \) and \( G_{sc}^{(K)} \) for Canopies IIIa and IIIb, but \( G_{sc}^{(J)} \) is practically identical with these except for dense canopies (\( \Lambda \gtrsim 3 \)). For Canopy IV no strict equalities exist, but all three bulk \( G_{sc} \) values are almost equal except for \( \Lambda \gtrsim 3 \), where \( G_{sc}^{(M)} > G_{sc}^{(K)} > G_{sc}^{(J)} \). Even at large \( \Lambda \), the differences are less than 20%.

Fig. 5 shows the flux-matching bulk aerodynamic conductance \( G_{ac}^{(M)} \) and the parallel-sum conductance \( G_{ac}^{(J)} \) for Canopies IIIa, IIIb, IVa and IVb (\( G_{ac}^{(J)} \) is the same for all these canopies). As with \( G_{sc} \), the choice of averaging scheme matters.

Fig. 4. Bulk canopy conductances \( G_{sc}^{(M)}, G_{sc}^{(J)}, G_{sc}^{(K)} \), and bulk surface conductance \( G_{st}^{(M)} \) with equilibrium and potential soil evaporation \( F_{Es} \), for Canopies IIIa, IIIb, IVa and IVb. For Canopies IIIa and IIIb, \( G_{sc}^{(M)} \) and \( G_{sc}^{(K)} \) are identical.
little at low $\Lambda$. For dense canopies ($\Lambda \gtrsim 3$), the major cause of differences among values of $G_{ac}^{(M)}$ is, surprisingly, the radiation distribution. The formal reason is the different limiting behaviours of $R_{dc}$ for the exponential and linear radiation distributions (see the Appendix). Over the range of $\Lambda$ normally encountered, these differences are negligible at the low-$\Lambda$ end, and about a factor of two at the high-$\Lambda$ end.

6.5. Inclusion of soil evaporation

Philip (1957) and Gardner (1958) showed that soil (ground) evaporation $F_{E_g}$ is given to a good approximation by the lesser of a meteorologically determined value $F_{E_{gl}}$ and a soil-determined value $F_{E_{g2}}$, which govern soil evaporation when the soil is wet and dry, respectively. The switchover between these two phases occurs typically a few days after rainfall sufficient to saturate the soil. In the meteorologically determined phase, $F_{E_{gl}}$ can be described by a suitable version of the CE. In the soil-determined phase, $F_{E_{g2}}$ is governed mainly by the ability of the deeper soil layers to supply the soil near the surface with water, by the capillary movement of liquid water through the unsaturated zone.

If the CE (or another bulk model at canopy scale) is to describe both the canopy and the soil together, then the bulk surface conductance $G_s$ entering this model accounts for both canopy and soil evaporation, and therefore differs from the canopy conductance $G_{sc}$ discussed so far. The difference between $G_{sc}$ and $G_s$ is examined here for the meteorologically determined phase of soil evaporation ($F_{E_g} = F_{E_{gl}}$), for which it is reasonable to treat the soil as an evaporating surface describable by a CE (Shuttleworth and Wallace, 1985). Using the equilibrium-departure form, this can be written

$$F_{E_g} = \Phi_{eqg} + \frac{\rho \lambda (D - \Delta_{eqg})}{R_{dg}}$$

(47)

which is the counterpart for soil evaporation of Eq. (37) for canopy evaporation. The
subscript \( g \) indicates a quantity defined using conditions at the ground. The equilibrium evaporation at the ground surface, \( \Phi_{eqg} \), is \( \Phi_{eqg} = \tau \Phi_{eq} \) where \( \tau \) is the canopy transmission (\( \tau = e^{-cA} \) for an exponential radiation distribution in the canopy); in complement, the canopy equilibrium evaporation is \( \Phi_{eqc} = (1 - \tau) \Phi_{eq} \). As before, the air in the canopy is assumed to be well mixed with saturation deficit \( D \).

Fig. 2 includes the soil evaporation rate \( F_{Eq} \) from Eq. (47), in two scenarios: equilibrium soil evaporation (\( D = \Delta_{eqg} \) or soil aerodynamic conductance \( G_{ag} \rightarrow 0 \)) and potential soil evaporation (soil surface conductance \( G_{sg} \rightarrow \infty \)). It has been assumed (crudely) that \( G_{ag} = g_a \), the leaf aerodynamic conductance. The equilibrium \( F_{Eq} \) falls with increasing \( \Lambda \) because of the decreasing canopy transmission, whereas the potential \( F_{Eq} \) falls because \( g_a \) (hence \( G_{ag} \)) decreases with increasing \( \Lambda \) through the effect of shelter. (The rough parameterisation of the shelter effect in this work is given in Table 4.) Hence \( F_{Ec} \) and \( F_{Eq} \) have complementary behaviours with increasing \( \Lambda \), respectively increasing from zero and decreasing to zero, and the sum \( F_{Ec} + F_{Eq} \) varies much more slowly with \( \Lambda \) than either evaporation component alone.

The total (canopy plus soil) evaporation \( F_{Ec} + F_{Eq} \) can also be described by a CE, identical to Eqs. (37) or (47) but without \( c \) or \( g \) subscripts. Then, any of averaging Schemes M, J and K can be used to relate the total conductances \( G_a \) and \( G_s \) (pertaining to canopy plus soil) to the component conductances \( G_{ac} \) and \( G_{sc} \) (for the canopy) and \( G_{ag} \) and \( G_{sg} \) (for the soil). In the case of the flux-matching scheme (M), it follows immediately from Eq. (24) that the total surface conductance is

\[
G_{s(M)}^{(M)} = \frac{R_{dg} + R_{dc}}{(1 - \tau)R_{dg}R_{sc} + \tau R_{dc}R_{sg}}
\]

This prediction is shown in Fig. 4 for Canopies IIIa, IIIb, IVa and IVb, in the scenarios of equilibrium and potential \( F_{Eq} \). The general finding is that, with equilibrium soil evaporation, \( G_s \) varies relatively slowly with \( \Lambda \) (like the total evaporation \( F_{Ec} + F_{Eq} \)), in contrast to \( G_{sc} \) which increases from 0 as \( \Lambda \) increases from zero (like \( F_{Ec} \)). Hence, for equilibrium soil evaporation, \( G_s \) is a rather conservative quantity (see also Kelliher et al., 1995). With potential soil evaporation (from wet soil), \( G_s \) increases rapidly with decreasing \( \Lambda \), but is always positive and free of singular behaviour.

The nonsingular behaviour of \( G_{s(M)}^{(M)} \) for a soil–canopy system illustrates a general point. The flux-matching scheme always gives a well-behaved (positive and nonsingular) \( G_{s(M)}^{(M)} \), whereas imposed-conductance schemes often produce non-physical, singular averaged conductances if there is sufficient variability in the elemental conductances. The usual situation leading to large variability is when part of the system is wet, so that \( g_s \) is infinite for that part. For instance, the parallel-sum total surface conductance \( G_{s(K)}^{(K)} \) is infinite (singular) whenever a fraction of the surface, no matter how small, is wet.

7. Conclusions

The main outcomes of this work can be summarised as follows:

(1) Scale translations from leaf to canopy or canopy to region require averaging
schemes for defining larger-scale bulk conductances \( (G_a, G_s) \) from their smaller-scale counterparts \( (g_a, g_s) \). Tools available to define \( G_a \) and \( G_s \) rationally include the basic physical constraint that net scalar fluxes average linearly over the land surface, and constraints imposed by matching model form between scales.

(2) When applied to the combination equation (CE) for latent and sensible heat fluxes \( F_E \) and \( F_H \), these tools lead to a flux matching averaging scheme, based on term-by-term matching of the linearly additive scalar flux terms in the CE for \( F_E \). Similar, but not identical, schemes are obtained from both the basic and equilibrium-departure forms of the CE. The schemes become identical in the absence of radiative coupling (the dependence of available energy on surface temperature through the outward longwave flux).

(3) All variants of the flux-matching scheme satisfy the scalar conservation requirement of linear flux averaging for \( F_E \). None satisfies this requirement exactly for \( F_H \), except when radiative coupling is ignored.

(4) Empirical tests for canopy-region scale translations show that the errors (departures from scalar conservation) for \( F_H \) are small over a wide range of heterogeneous surfaces, if the parameter \( p = g_{aH} / (g_{aH} + g_R) \) is averaged linearly (here \( g_{aH} \) is the aerodynamic conductance for sensible heat and \( g_R \) the radiative conductance, so \( p \) departs from unity as radiative coupling increases). This provides a robust averaging scheme for canopy-region scale translations, specifically for calculating regional-scale conductances for the purpose of inferring the regionally averaged fluxes \( F_E \) and \( F_H \).

(5) For leaf-canopy translations, the problem of deriving the canopy-scale surface conductance from its leaf-scale counterpart has been approached using the flux-matching scheme (M) and also two other schemes (J and K, based respectively on parallel-sum definitions of bulk aerodynamic and surface conductances). Definitions by all three schemes of the bulk canopy conductance \( G_{sc} \) (describing vegetation only) are compared by constructing a sequence of analytic model canopies, in which assumptions are made about the variations of the elemental aerodynamic and stomatal conductances through the canopy. In all cases, the canopy and soil evaporation are treated separately \( (F_E = F_{Ec} + F_{Ek}) \), the canopy air is assumed to be well mixed so that all leaves are exposed to the same saturation deficit, and the canopy is assumed to be dry (free water is absent). The specific assumptions and outcomes of this analysis have been summarised in Table 3. The general conclusion is that, for all model canopies, averaging Schemes M, J and K produce remarkably similar results for \( G_{sc} \) except at high canopy density \( (A \gtrsim 10) \). Hence, for typical dry canopies, the bulk canopy conductance obtained from either Scheme M or Scheme J is close to the parallel-sum value from Scheme K, and is therefore approximately a physiological parameter of the system.

(6) The flux-matching scheme readily yields the total canopy surface conductance \( G_s \), describing both canopy and soil evaporation. The resulting \( G_s \) is always positive and nonsingular even when the soil (or part of the canopy) is wet. In contrast, the definitions of \( G_s \) from Schemes J and K can exhibit singularities when elemental conductances vary strongly through the canopy, particularly when the soil or part of the canopy is wet.
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Appendix: Detailed analysis for Canopies III and IV

The exponential and linear radiation distributions, Eqs. (45) and (46), specify the elemental radiation fluxes $\phi_{eq}(\zeta)$ in terms of the elemental flux $\phi_{eq}(\Lambda)$ at the canopy top. The elemental $\phi_{eq}(\Lambda)$ is related to the bulk flux $\Phi_{eq}$ incident on the canopy (normalised with ground area) by $\phi_{eq}(\zeta) = d\Phi_{eq}(\zeta)/d\zeta$. This follows from scalar conservation if $\Phi_{eq}$ is the net flux of a conserved scalar, as it is under the approximation $\rho = 1$. Integration gives $\phi_{eq}(\Lambda) = c\Phi_{eq}$ for the exponential distribution, and $\phi_{eq}(\Lambda) = (2/\Lambda)\Phi_{eq}$ for the linear distribution. It is also appropriate to require the total flux absorbed by the canopy, $\Phi_{eq}c$, to be the same for both distributions; the difference between the two distributions is then only in the way that this total flux is spread over the canopy elements.

A.1. Canopy III

We take $g_a$ to be constant and $g_s$ to be proportional to $\phi_{eq}$. Then, following Eq. (43), $g_s(\zeta) = g'_s \phi_{eq}(\zeta) = g'_s \phi_{eq}(\zeta)$, where $g'_s = dgs/d\Phi_{eq}$ and $g'_s = g'_s / \phi_{eq}. \phi_{eq}$. Proportionality between $g_s$ and $\phi_{eq}$ implies that the elemental equilibrium deficit $\delta_{eq}$ is constant through the canopy, which is why this is a simple and useful case to consider. It follows from Eq. (38) that

$$\Delta_{eqc} = \delta_{eq} = \phi_{eq}/(\rho \lambda g_s) = 1/(\rho \lambda g'_s)$$

\hspace{1cm} (A1)

and from Eqs. (39) and (42) that $G_{sc}^{(M)} = G_{sc}^{(K)} = g'_s \Phi_{eq}. \phi_{eq}$. Thus, the canopy conductance defined under Scheme M is the same as the parallel-sum canopy conductance, Scheme K. However, $G_{sc}^{(I)}$ is different.

Evaluation of $R_{dc}$ requires the radiation distribution. Substituting $r_d(\zeta) = (e + 1)r_a + \frac{1}{g'_s \phi_{eq}(\zeta)}$ into the second of Eqs. (38) and carrying out the integration, we obtain

$$\frac{1}{R_{dc}} = \frac{1}{c(e + 1)r_a} \ln \left( \frac{1 + X}{e^{-c\Lambda} + X} \right)$$

\hspace{1cm} (A2)

for Canopy IIIa (exponential radiation distribution), and

$$\frac{1}{R_{dc}} = \frac{\Lambda}{(e + 1)r_a} \left[ 1 - X \ln \left( 1 + \frac{1}{X} \right) \right]$$

\hspace{1cm} (A3)

for Canopy IIIb (linear radiation distribution), with $X = 1/[(e + 1)r_a g'_s \phi_{eq}(\Lambda)]$. 
These equations yield $G_{ac}^{(M)}$ for Canopies IIIa and IIIb, from the second of Eqs. (39).

A.2. Canopy IV

Again, $g_a$ is taken to be constant. The stomatal conductance is now specified by a hyperbola, Eq. (44), which can be rewritten as

$$g_s(\zeta) = \frac{g_{sx}f_p(\zeta)}{f_p(\zeta) + g_{sx}/g'_p} = \frac{g_{sx}\phi_{eq}(\zeta)}{\phi_{eq}(\zeta) + g_{sx}/g'_p}$$  \hspace{1cm} (A4)

where $g_{sx}$ is the limiting (maximum) value of $g_s$ at high light (incorporating effects of physiological stresses other than light limitation). The quantity $f_p$ in Eq. (44) is written here as $g_{sx}/g'_p$, with $g'_p$ being the slope of $g_s(f_p)$ at $f_p = 0$, and $g'_p = g'_p/c_{ph}$ as for Canopy III. Eq. (A4) reverts to the $g_s$ form for Canopy III as $g_{sx} \to \infty$ ($r_{sx} \to 0$). A partial evaluation of $\Delta_{eq}$ and thence $G_{ac}^{(M)}$ can take place independently of the radiation distribution, by substituting Eq. (A4) into the last of Eqs. (38) and rearranging. This gives

$$G_{ac}^{(M)} = \frac{\phi_{eq}c}{\rho\Delta_{eq}} = \frac{r_{sx} + (\epsilon + 1)r_a}{r_{sx}R_{dc} + (\epsilon + 1)r_a/(g'_p\phi_{eq})}$$  \hspace{1cm} (A5)

which reverts to the Canopy III result, $g'_p\phi_{eq}$, as $r_{sx} \to 0$. However, it is no longer true that $G_{ac}^{(M)} = G_{ac}^{(K)}$. As the complexity of the model canopy has increased, this simple property has been lost. To complete the evaluation of $G_{ac}^{(M)}$, an expression for $R_{dc}$ is required. This depends on the radiation distribution, and is therefore different for Canopy IVa (exponential) and Canopy IVb (linear). Substituting Eq. (A4) into the second of Eqs. (38) and carrying out the integration with these radiation distributions, we obtain, for Canopies IVa and IVb, respectively:

$$\frac{1}{R_{dc}} = \frac{1}{c[(\epsilon + 1)r_a + r_{sx}]} \ln \left( \frac{1 + Y}{e^{-c\Lambda} + Y} \right)$$  \hspace{1cm} (A6)

$$\frac{1}{R_{dc}} = \frac{\Lambda}{(\epsilon + 1)r_a + r_{sx}} \left[ 1 - Y\ln\left(1 - \frac{1}{Y}\right) \right]$$  \hspace{1cm} (A7)

with $Y = 1/[(\epsilon + 1)r_a + r_{sx}]g'_p\phi_{eq}(\Lambda)]$. These are very similar to the corresponding results for Canopies IIIa and IIIb, Eqs. (A2) and (A3), and properly revert to those results in the limit $r_{sx} \to 0$. By substitution of Eqs. (A6) and (A7) into Eq. (A4), $G_{ac}^{(M)}$ is found for each radiation distribution. Similar forms are obtained for the parallel-canopy conductance $G_{ac}^{(K)}$, which evaluates to

$$G_{ac}^{(K)} = \frac{g_{sx}}{c} \ln \left( \frac{1 + Z}{e^{-c\Lambda} + Z} \right)$$  \hspace{1cm} (A8)

$$G_{ac}^{(K)} = \Lambda g_{sx} \left[ 1 - Z \ln\left(1 + \frac{1}{Z}\right) \right]$$  \hspace{1cm} (A9)
for Canopies IVa and IVb, with \( Z = \frac{g_{ss}/\phi_{eq}(A)}{g_{ss}^2} \) (so \( Z \) is \( Y \) with \( r_q \) omitted). Eq. (A8) has been derived previously by Saugier and Katerji (1991) and Dolman et al. (1991).

References


