The Light Environment of Plant Canopies

In Ch. 14 plant canopies are treated as big leaves. We did not worry about their structure or the details of how the leaves make up the canopy, we just assumed that we could find a canopy conductance for vapor and boundary layer conductances for heat and vapor. Combining these with the absorbed radiation and soil heat flux densities allowed us to compute canopy temperatures and transpiration rates. We even estimated carbon assimilation rates by knowing transpiration rate or light interception.

In this chapter we look in more detail at the light environment of plant canopies. Without knowing how the light is distributed on leaves within the canopy we could not use detailed photosynthesis models like the one presented in Ch. 14 to estimate canopy photosynthesis, but a study of the light environment of plant stands is useful for many other purposes as well. In this chapter we show how to compute the fraction of radiation intercepted by a canopy and the fraction transmitted to the soil. These are important for computing assimilation using simple models like Eq. (14.13), as well as for partitioning potential evapotranspiration between evaporation (from soil) and transpiration (from leaves). We also show how to compute the change in spectral composition of light as it is transmitted and reflected by the canopy. These spectral changes have important implications in predicting responses of organs or organisms which are triggered by a specific ratio of red to far-red radiation and in radiometric remote sensing.

15.1 Leaf Area Index and Light Transmission Through Canopies

We use the cumulative hemi-surface area index (HSAI) L to measure the optical pathlength of radiation from the top of the canopy downward. The hemi-surface area index is one-half the surface area of leaves per unit ground area. For thin, flat leaves, the hemi-surface area index is the same as the leaf area index (LAI), which is the silhouette (one-sided) area of leaves per unit ground surface area. For more complicated shapes, like conifer needles or branches, the hemi-surface area index is not equal to the silhouette leaf area index. For example, conifer needles shaped...
like cylinders have a hemi-surface area index equal to \( \pi/2 \) times the silhouette leaf area index. Silhouettes or "projected" leaf area index is not consistent in canopies with leaves that have complex shapes because the details of the projection are important and often not recorded. For example, some needles are shaped like nemophiles and are twisted along their length a variable number of rotations, and others may have a cross section like 1/4 of a cylinder and lay flat on a planimeter with more than one orientation. Throughout this chapter we consider flat leaves and use the terms HAI and LAI interchangeably. At the top of the canopy, \( L = 0 \). With increasing depth into the canopy, \( L \) increases and is equal to the leaf area index of the canopy \( L_c \) below the canopy. For a given canopy there exists a relationship between \( L \) and the physical distance one would measure in the canopy, but the relationship is not necessarily a simple one.

Much can be learned about the role of canopy architecture in determining the relation between leaf radiative properties and canopy radiative properties by considering canopies to consist of a statistical distribution of flat leaves. You may recall from Ch. 1 that leaves typically have absorptivities of about 0.5 (Table 11.5) and canopies typically have absorptivities of about 0.8 (Table 11.2); this difference is related to the architecture of the canopy. Separating the effects of leaf spectral properties from the effect of canopy architecture is important because most leaves have similar reflectance and transmittance spectra, even though they depend on wavelength, but canopy architecture can vary widely with species, environmental condition, and time.

A idealized canopy can be constructed above some horizontal ground area of size \( A \) by randomly placing horizontal, black (leaves that do not reflect or transmit radiation) leaves each of area \( a \) above the ground. If one black leaf is randomly placed over the area \( A \) the probability that a random ray will hit this leaf is \( a/A \). The fraction of the area \( A \) that would be in shadow by a uniform light beam at the zenith is also \( a/A \). If this incident beam of light is thought of as being composed of a great many very small rays of light, then a fraction \( a/A \) of these tiny rays would intersect the leaf and the fraction \( (1 - a/A) \) would pass by the leaf unscattered. If a second leaf is placed randomly over the area \( A \) the probability that a light ray will not be intercepted by either leaf is \((1 - a/A)^2\), because the placement of the second leaf is independent of the placement of the first leaf. If \( N \) leaves are placed randomly above the area \( A \), then the probability that none of the leaves will intercept a light ray is \((1 - a/A)^N\). This describes a binomial distribution and accommodates the fact that many of the shadows of these horizontal leaves will overlap on the ground surface, \( A \). The quantity \((1 - a/A)^N\) can be thought of as the transmittance of light from the zenith through this canopy of black, horizontally-distributed leaves. Alternatively, \((1 - a/A)^N\) can be considered to be the fraction of the ground area illuminated by the incident beam. At this point, it is important to recognize that these probabilities apply to both transmittances and area fractions. Binomial probability distributions are not part of the assumption to simplify \( a \) is much smaller than \( A \) for many realistic examples, since \( a/A \rightarrow 0 \), that is, the leaf area index. For distribution with the Poisson horizontal leaves is independent of the leaf no matter what direction does a quick look at vegetation density horizontally; the horizontal shadowed or "black" leaves no matter whether there is a range of sizes, the compared to the area, \( A \) fraction of the plane from a particular this fraction in the extinction \( K_0(\psi) \) as the mean beam canopy divided by the black leaves. Clearly, for \( \psi \) can be the azimuth angle of \( \psi \) estimating \( K_0(\psi) \), but numerically distributed at a good assumption for \( \psi \) therefore assumes \( K_0(\psi) \) for various leaf-area "black" leaves. If flat leaves in a canopy in space, then the frasty angle \( \psi \) that penetrates the canopy is \( (1 - a/A)^N \), where \( K_0(\psi) \) is the cut leaves are stomped (not often still be approximated simplified by a lumped are less efficient in use distributed. Row crops - only 70 to 80 percent is randomly distributed in The fraction of the canopy is \( 1 - K_0(\psi) \). This factor and the photosynthesis, \( L_A \) is accepted by the canopy.
of canopy architecture in deter-
mining the effects of leaf-spectral prop-
nerties and canopy radiative properties. This is important because most
photon transmittance spectra, even though
vegetative architecture can vary widely with time.

Uberti above some horizontal ground horizon, black leaves that do not
reach above a horizontal plane. If
the area A the probability that
A, the fraction of the area A that
light beam at the zenith is also A/A.
The light as being composed of a great
number of light rays, each of which
must be thought of as passing randomly over the
to be intercepted by either leaf if
of the second leaf is independent of
are placed randomly above the
distribution and accommodates
these leaves horizontal leaves will overlap
light (1 - e/A)^n can be thought of as
zenith through the canopy of black,
leaf. Alternatively, (1 - e/A)^n can
be found around the incident light
leaves that are not intercepted by the
transmitted fraction. Binomial probability

\[
K_0(\psi) \sim \exp(-K_0(\psi)L_z)
\]

where \(K_0(\psi)\) is the canopy extinction coefficient just described. When
leaves are clumped (not randomly distributed), canopy transmittance can
be approximated by an exponential function of \(L_z\), but \(L_z\) is
multiplied by a clumping factor \(\Omega\) to account for the fact that leaves
are less efficient in covering the ground when they are randomly
distributed. Rows with leaves clumped in the rows may intercept
only 70 to 80 percent of the radiation they would if their leaves were
randomly distributed in space.

The fraction of incident beam radiation intercepted by the canopy
is 1 - \(K_0(\psi)\). This fraction is available for scattering, transpiration,
and for photosynthesis. The fraction of beam radiation that is not
intercepted by the canopy \(\sigma_0(\psi)\) reaches the soil surface and is available

distributions are not particularly easy to work with so we make another
assumption to simplify the math; namely, that the area of a single leaf
is much smaller than the ground area, \(A\). This certainly is reasonable
for many realistic examples. If the mathematical limit of \((1 - e/A)^n\) is
taken as \(e/A \rightarrow 0\), then \((1 - e/A)^n \approx \exp(-Na/A)\), where \(Na/A\)
is the leaf area index. Essentially this amounts to replacing the binomial
distribution with the Poisson distribution. Interestingly, this result for
horizontal leaves is independent of the incident angle of the light, because
the shadow of a horizontal leaf cast onto a horizontal plane is the area of
the leaf no matter what direction the light comes from.

A quick look at vegetation reveals that leaves are almost never oriented
totally horizontally; they have a distribution of orientations. However,
the horizontal shadows of black leaves can always be treated as "horizon-
tal leaves" no matter what the leaf orientation. Although these shadows
have a range of sizes, they all are random relative to each other and small
compared to the area \(A\). This is all that is required for exponential ex-
trapolation to hold. Therefore, no matter what the orientation distribution
of leaves, the fraction of the leaf \(K_0(\psi)\) that is projected onto the horizontal
plane from a particular zenith angle \(\psi\) can be calculated. We refer to
this fraction as the extinction coefficient, \(K_0(\psi)\). One can also think of
\(K_0(\psi)\) as the mean beam flux density on an average illuminated leaf in
the canopy divided by the beam flux density on the horizontal plane above
the canopy. Clearly, for a canopy of perfectly horizontal leaves, \(K_0(\psi) = 1.0\).
The azimuth angle of the incident radiation may also be important in
estimating \(K_0(\psi)\), but we deal only with canopies that have leaves sy-
metrical about the azimuth (compass direction), which is a
good assumption for almost all canopies. The extinction coefficient
is therefore assumed independent of solar azimuth. Later we calculate
\(K_0(\psi)\) for various leaf-angle distributions and remove the assumption of
"black" leaves.

If leaves in a canopy of leaf area index \(I\), are randomly distributed in
space, then the fraction \(\sigma_0(\psi)\) of incident beam radiation from zenith
angle \(\psi\) that penetrates the canopy is

\[
\sigma_0(\psi) \sim \exp(-K_0(\psi)L_z) \text{ (15.1)}
\]
for evaporating water from the soil. A simple way to partition potential evapotranspiration (PET) between potential transpiration and potential soil evaporation uses \( r \). Potential transpiration is \( 1 - r \) times PET, and potential evaporation is \( r \) times PET. A canopy that covers the ground reasonably well has a leaf area index of perhaps three. If \( K(\psi) = 0.6 \), then, from Eq. (15.1), \( r = \exp(-0.6 \times 3) = 0.17 \); so 17 percent of the radiation is intercepted by the soil surface and 83 percent is intercepted by the canopy. If both canopy and soil surface were wet, so that evapotranspiration were at the potential rate, then 83 percent of the evapotranspiration would come from the canopy and 17 percent from the soil.

15.2 Detailed Models of Light Interception by Canopies

Our purpose here is to find equations that allow us to account for the major variations in PAR and near-infrared (NIR) fluxes on leaves in a canopy. The most obvious variations result from shading of some leaves by others. We therefore consider two classes of leaves, those that are shaded, and those that are sunlit. Average PAR or NIR flux densities for each of these classes can be derived. More detailed models subdivide each of these classes to account for the leaf angle distribution and position in the canopy of leaves, but we do not consider those now. Goudriaan (1988) shows how to derive a model with more radiation classes.

The calculation of an extinction coefficient requires calculating the area of an average projection from some direction \( \psi \) onto the horizontal, and this is not an easy thing, except by some geometrical reasoning. If all of the leaves in a canopy were vertical, but with random azimuthal orientations, then the distribution function for leaf area in the canopy would be the same as the distribution function for area on the vertical surface of a vertical cylinder. The ratio of the area projected onto the horizontal from the direction \( \psi \) to the hemi-surface area of a cylinder (length \( L_c \) and diameter \( D_c \)) is the extinction coefficient and is given by

\[
K_{0c}(\psi) = \frac{L_c D_c \tan \frac{\psi}{2} \frac{D_c}{L_c}}{2 \tan \frac{\psi}{2}} = \frac{2 \tan \frac{\psi}{2}}{\pi} \tag{15.2}
\]

where \( \psi \) is the zenith angle of the sun. Similarly, a crop might have leaves with leaf inclination angles similar to the distribution of angles on the surface of a sphere. Taking the ratio of the area of the projection of a sphere (radius \( r \)) onto a horizontal surface to the hemi-surface area of the sphere gives

\[
K_{0s}(\psi) = \frac{\pi r^2 \sin \psi}{2 \pi r^2} = \frac{1}{2 \cos \psi} \tag{15.3}
\]

A canopy with a spherical leaf angle distribution does not need to look like a ball. Imagine cutting the surface of a sphere into many little pieces, then moving these pieces about the volume occupied by the canopy while maintaining the zenith and azimuth orientations of each piece. The resulting canopy would have more vertical area than \( 1 \) area of a sphere is vertical \( r \) would be present in the canopy approximation to real plant growth. An extinction coefficient derived, but the most useful distribution generalizes the sphere or ellipsoid as the ratio of a sphere to an ellipsoid is (Campbell, 1986)

\[
K_{0e}(\psi) = \frac{2}{\pi}
\]

Here, the parameter \( x \) is the radius of a horizontal distribution, \( x = 1 \); for a vertical leaf angle, \( x \) approaches \( \infty \). The simple \( K_{0s} \)'s and all of the \( r \) angle density for three different distributions is given by Ca distribution has more vertical foliage uniformly among side horizontal angles and vertical angles. If we were to plot Fig. 15.1, we would be in a called Dirac delta function.

![Figure 25.1. Inclination angle more horizontal the leaves are.](image-url)
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A simple way to partition potential transpiration and potential evaporation is 1 = r times PET and A canopy that covers the ground up to about 60\% of its area. If \( K(\phi) = 0.6, \text{ then } x = 3, \) so that about 17\% of the photosynthesis and 83\% percent of the evaporation is percent from the soil.

Reception by

As that allow us to account for the absorbed (NIR) fluxes on leaves in a result from shading of some leaves of some classes of leaves, those that are transmittance FAR or NIR flux densities for some detailed models subtract each angle distribution and position in the database using data. Goodrich et al. (1998) are radiation classes.

Coefficient of scattering that requires calculating the relative effectiveness of \( \psi \) onto the horizontal, by some geometrical reasoning. If critical, but with random azimuthal action for leaf area is the canopy function for area on the vertical to of the area projected onto the hemi-surface area of a cylinder, action coefficient and it is given by

\[
\psi = \frac{2 \tan \psi}{\pi} \tag{15.2}
\]

Similarly, a crop might have far to the distribution of angles on 90\% of the area of the projection of the surface to the hemi-surface area of

\[
= \frac{1}{2 \cos \psi} \tag{15.3}
\]

Distribution does not need to look at a sphere into many little pieces, volume occupied by the canopy by orientations of each piece. The resulting canopy would have a spherical angle distribution. There would be more vertical area than horizontal area, because more of the surface area of a sphere is vertical than horizontal, but leaves of all inclinations would be present in the canopy. A spherical angle distribution is a good approximation to real plant canopies.

An extinction coefficient for a conical leaf distribution could also be derived, but the most useful distribution is ellipsoidal. The ellipsoidal distribution generalizes the spherical, but allows the sphere to be flattened or elongated. The ratio of projected area to hemi-surface area for an ellipsoid is (Campbell, 1980):

\[
K_{\infty}(\psi) = \frac{\sqrt{\psi^2 + \tan^2 \psi}}{x^2 + 1.774(x^2 + 1.182)^{0.5}} \tag{15.4}
\]

Here, the parameter \( x \) is the ratio of average projected areas of canopy elements on horizontal and vertical surfaces. For a spherical leaf angle distribution, \( x = 1; \) for a vertical distribution, \( x = 0; \) and for a horizontal leaf canopy, \( x \) approaches infinity. Equation (15.4) therefore gives all of the simple \( K_{\infty}(\psi) \) s and all of the ones in between. Figure 15.1 shows leaf angle density for three different values of \( x. \) The equation for these distributions is given by Campbell (1980). As mentioned, the spherical distribution has more vertical than horizontal area, but spreads the area fairly uniformly among almost all angles. As \( x \) increases the peak shifts toward horizontal angles and as \( x \) decreases the peak shifts toward vertical angles. If we were to plot the horizontal and vertical distributions on Fig. 15.1, they would be infinitely narrow, and infinitely tall spikes (called Dirac delta functions) at 0 and 90°.

![Figure 15.1: Inclination angle density for three canopies. The larger \( x \) is, the more horizontal the leaves are.](image-url)
Figure 15.2 shows extinction coefficients as a function of beam zenith angle for a range of $x$ values. Note that extinction in horizontal canopies has no zenith angle dependence, but for all other canopies, zenith angles below about 60° have extinction coefficients below unity, while at zenith angles greater than 60°, the extinction coefficient is greater than unity.

By using these values of extinction coefficient in Eq. (15.1), we can show how canopy structure (in terms of leaf angle distribution) influences radiation transmission and interception. This is done in Fig. 15.3 for a canopy with a leaf area index of one. Since the extinction coefficient has no angle dependence in a horizontal-leaf canopy, the transmission does not depend on zenith angle for horizontal canopies. When $L_r = 1$, and $K_n(\psi) = 1$, Eq. (15.1) gives $\exp(-1) = 0.37$. All other canopies transmit more and intercept less radiation at small zenith angles than do horizontal canopies. As large zenith angles, canopies with inclined leaves intercept more radiation than do canopies with horizontal leaves. A canopy with completely vertical elements would intercept no radiation if the solar beam were directly overhead at 0°. Obviously no real canopy has absolutely vertical leaves, but this limiting case can help to understand and verify the equations.

Measured values of $x$ for a number of crops are given in Table 15.1. It can be seen from the table that natural canopies tend to be more horizontal than vertical and that the spherical distribution ($x = 1$) approximates many of the canopies. If no information is available about the angle distri-

![Figure 15.2](image)

**Figure 15.2.** The extinction coefficient $K_n(\psi)$ as a function of zenith angle for $x$ values representing various leaf angle distributions.

**Table 15.1.** Values of $x$ for various crop canopies.

<table>
<thead>
<tr>
<th>Crop</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ryegrass</td>
<td>0.67</td>
</tr>
<tr>
<td>Maize</td>
<td>0.73</td>
</tr>
<tr>
<td>Rye</td>
<td>0.83</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.96</td>
</tr>
<tr>
<td>Barley</td>
<td>1.22</td>
</tr>
<tr>
<td>Timothy</td>
<td>1.12</td>
</tr>
<tr>
<td>Sorghum</td>
<td>1.42</td>
</tr>
<tr>
<td>Lucerne</td>
<td>1.54</td>
</tr>
<tr>
<td>Hybrid swede</td>
<td>1.25</td>
</tr>
<tr>
<td>Sugar beet</td>
<td>1.46</td>
</tr>
<tr>
<td>Rape</td>
<td>1.95</td>
</tr>
</tbody>
</table>
$I(x)$ as a function of zenith angle for various distributions.

![Graph showing transmission as a function of zenith angle and leaf angle distribution.](image)

**Figure 15.3.** Fraction of the incident solar beam reaching the ground below a canopy with LAI = 1, for different leaf angle distributions.

The inclination of leaves in the canopy is often assumed to be spherical. Values of $x$ are not particularly intuitive for understanding leaf orientation. Mean leaf inclination angle is more easily understood. The mean leaf inclination angle can be approximated as $\cos^{-1}(K_x,0))$ using Eq. (15.4). The mean leaf inclination angles are therefore about 75°, 60°, and 34° for $x$ values of 0.6, 1.0, and 5.0. This approximation of mean leaf inclination

### Table 15.1. Values of the leaf angle distribution parameter $x$ for various crop canopies (from Campbell and van Evert, 1994)

<table>
<thead>
<tr>
<th>Crop</th>
<th>$x$</th>
<th>Crop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ryegrass</td>
<td>0.67-2.47</td>
<td>Cucumber</td>
</tr>
<tr>
<td>Maize</td>
<td>0.76-2.53</td>
<td>Tobacco</td>
</tr>
<tr>
<td>Barley</td>
<td>0.8-1.27</td>
<td>Potatoes</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.96</td>
<td>Horde Wheat</td>
</tr>
<tr>
<td>Barley</td>
<td>1.20</td>
<td>Sunflower</td>
</tr>
<tr>
<td>Timothy</td>
<td>1.17</td>
<td>White clover</td>
</tr>
<tr>
<td>Sorghum</td>
<td>1.43</td>
<td>Strawberry</td>
</tr>
<tr>
<td>Lucerne</td>
<td>1.54</td>
<td>Soybean</td>
</tr>
<tr>
<td>Hayseed</td>
<td>1.28-1.81</td>
<td>More</td>
</tr>
<tr>
<td>Sugar beet</td>
<td>1.48-1.88</td>
<td>2. artichoke</td>
</tr>
<tr>
<td>Rape</td>
<td>1.92-2.11</td>
<td></td>
</tr>
</tbody>
</table>
angle is not exact but it is close. For example, the spherical leaf angle distribution has a true mean leaf inclination angle of 57°, rather than 60°.

The fraction of beam radiation that is transmitted through the canopy without interception \( t_0(\varphi) \) is given by Eq. (15.1) with \( K_{\psi}(\varphi) \) from Eq. (15.4) (or one of the simpler equations for \( K_{\psi}(\varphi) \) if the distribution is horizontal, vertical, or spherical) using the appropriate sun zenith angle (Eq. (11.1)).

15.3 Transmission of Diffuse Radiation

The diffuse radiation comes from all directions, and is attenuated differently from beam radiation, which comes from just one direction. Diffuse radiation can be thought of as many beams and a diffuse transmission coefficient for the canopy can be calculated from

\[
t_d = 2 \int_0^{\pi/2} t_0(\varphi) \sin \psi \cos \varphi \, d\varphi.
\]

(15.5)

For horizontal leaves, \( t_0(\varphi) \) is not dependent on \( \psi \), and so \( t_d = t_0 \), but for the other leaf angle distributions \( t_0(\varphi) \) does depend on \( \psi \) and the integration in Eq. (15.5) must be carried out numerically. When the integration is done numerically, it is found that \( t_d \) does not decrease exponentially with \( L \), as it does for beam radiation (except for horizontal leaves). In order to obtain a useful approximation for models, an exponential equation can be fit to the values obtained and allow \( K_{\psi} \), the extinction coefficient for black leaves in diffuse radiation, to vary with leaf area index. Figure 15.4 shows the result based on a numerical integration of Eq. (15.5), assuming

\[
\begin{array}{c|c}
\text{Leaf Area Index} & K_{\psi} \\
\hline
0.1 & 1.0 \\
0.2 & 0.9 \\
0.3 & 0.8 \\
0.4 & 0.7 \\
0.5 & 0.6 \\
0.6 & 0.5 \\
0.7 & 0.4 \\
0.8 & 0.3 \\
0.9 & 0.2 \\
1.0 & 0.1 \\
\end{array}
\]

Figure 15.4. Apparent extinction coefficient for diffuse radiation in canopies differing in leaf angle distribution.

15.4 Reflection of Light by Plant Canopies

The transmission of light through a uniform evercast sky (no horizontal leaves, \( K_{\psi} = 1 \) is around 0.7. This is an ir note that diffuse radiation distributed relatively uniformly for a particular layer in the on the leaves at some depth \( L \) density estimated on the h and the diffuse flux density

15.5 Reflection of Light by

For a canopy of randomly LAI so large that the soil h the canopy, the canopy here by

\[
R = 1 - (1 - \alpha) e^{-\alpha}
\]

where \( \alpha \) is the leaf absorp horizontal leaves, in the F \( \alpha = 0.2 \). \( R_{\psi,\varphi} = 0.38 \) This canopy reflection co
Reflection of Light by Plant Canopies

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a uniform overcast sky (no zenith angle dependence of sky radiance). For horizontal leaves, \( \Phi_y = 1 \), but for a spherical canopy, with \( L = 3, \Phi_y \) is around 0.7. This is an important point which we return to later.

Note that diffuse radiation, unlike beam radiation from the sun, is distributed relatively uniformly over all leaves with various orientations for a particular layer in the canopy. Thus the diffuse flux density incident on a leaf at some depth \( z \) in the canopy is the same as the diffuse flux density estimated on the horizontal at the same depth using Eq. (15.5) and the diffuse flux density above the canopy.

15.4 Light Scattering in Canopies

The leaves in plant canopies are not black, of course, and do transmit and reflect radiation. Goudsmit (1977) has shown that the transmission and reflection of radiation when the leaves are not assumed black can still be approximated using an exponential model (Eq. (15.1)), but with a modification to \( K_\theta \). If the absorptivity of leaves for radiation is \( \alpha \), then the total beam radiation (direct and down scattered) transmitted through the canopy is depth \( L \) is

\[ \alpha = \exp(-\int K_\theta d\psi) \].

It can be seen that when \( \alpha = 1 \) (black leaves) this equation is the same as Eq. (15.1) and when \( \alpha \) is small radiation will be attenuated minimally. The transmission of light through the leaves therefore gives an additional amount of radiation under the canopy. Equation (15.5) is too approximation, and Goudsmit (1977) has shown (his Table 5, p. 27) that Eq. (15.6) works well for a range of sun zenith angles, canopy architectures, and leaf absorptivity values. For a canopy with a spherical leaf angle distribution, Eq. (15.5) works well for sun zenith angles less than 65°. The transmission of diffuse radiation by the canopy is predicted by a similar equation, but with \( K_\phi \) as the extinction coefficient. Typical values for \( \alpha \) are \( \alpha = 0.8 \) for PAR and \( \alpha = 0.2 \) for NIR radiation. For total solar radiation, absorptivity is the mean of the values for PAR and NIR, so \( \alpha = 0.5 \).

15.5 Reflection of Light by Plant Canopies

For a canopy of randomly located, horizontally oriented leaves with a LAI so large that the soil has negligible effect on radiation reflected from the canopy, the canopy hemispherical reflection coefficient, \( \rho_{ref} \), is given by

\[ \rho_{ref} = \frac{1 - \sqrt{\alpha}}{1 + \sqrt{\alpha}} \] (15.5)

where \( \alpha \) is the leaf absorptivity. This means that for a dense canopy of horizontal leaves, in the PAR (\( \alpha = 0.8 \)), \( \rho_{ref} = 0.056 \); in the NIR (\( \alpha = 0.2 \)), \( \rho_{ref} = 0.38 \); and in the solar (\( \alpha = 0.5 \)), \( \rho_{ref} = 0.17 \). This canopy reflection coefficient for solar radiation actually is not a
reliable estimate. Equation (15.6) accommodates multiple scattering in the canopy and is only appropriate where reflectivity and transmissivity are constant with wavelength. If the reflection coefficient is averaged over a wavelength band where spectral reflectivity and transmissivity vary considerably with wavelength, it does not hold for the visible and near-infrared portions of the solar spectrum so that Eq. (15.7) is unreliable. This can best be understood with a simple example, as shown in Example 15.1.

Example 15.1. Estimate the transmission of solar radiation through two filters, stacked on top of each other, using the following two methods.
1. Assume an average transmission for the solar wavelength band (τ_S).
2. Use visible (VIS) and near-infrared (NIR) transmissions separately.
Assume 1/2 of the solar radiation is in NIR and 1/2 is in VIS, the visible transmittance (τ_v) is 0.5, and near-infrared transmittance (τ_n) is 0.9.

Solution.
1. Using τ_S:
   \[ τ_S = 0.5(0.5) + 0.5(0.9) = 0.65 \]
2. Using τ_v and τ_n with τ_S = 0 and τ_S = 0.9:
   Visible
   \[ τ_v = 0.5 \times 0.5 = 0.25 \]
   Near-infrared
   \[ τ_n = 0.5 \times 0.9 = 0.45 \]
   Solar
   \[ τ_S = 0.5(0.5) + 0.5(0.8) = 0.40 \]

Therefore, averaging multiple transmissions or reflections, as happens in plant canopies, over wavelength bands with different spectral properties causes errors; in this example a factor of two.

From Example 15.1, the visible and near-infrared wavelength bands should be treated separately because their spectral properties are so different. Thus a better estimate of the solar albedo is given by \( \rho'_{\text{S,NIR}} = 0.5(0.056) + 0.5(0.38) = 0.22 \), a value 29 percent larger than it obtained by substituting the average solar absorptivity into Eq. (15.7). This is one of the reasons that solar radiation must be divided into visible and near-infrared wavelength bands in environmental biophysics. Fortunately about one-half of the irradiance is in each band so approximate partitioning is simple.
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accommodates multiple scattering in...reflectivity and transmissivity...reflection coefficient is averaged over...reflectivity (and transmissivity) varies...then Eq. (15.7) is intractable. This...example shown as Example 15.1.

mission of solar radiation through two...using the following two methods...for the solar wavelength band (\( r_s \))...and (NIR) transmissions separately...in the NIR and 12 is VIS, the visible...infrared transmittance (\( r_s \)) is 0.9

\( r_s = 0.5(0.6) = 0.45 \)
\( = 0.45 \times 0.45 = 0.20 \).
\( \Rightarrow r_s = 0.9; \)
\( 0.0 \times 0.0 = 0.0 \)
\( 0.9 \times 0.9 = 0.81 \)
\( 0.9 = 0.5(0.61) = 0.40. \)

...as happens in...with different spectral properties...in two...and near-infrared wavelength bands...their spectral properties are so dif...value 29 percent higher than in ob...nner absorptivity into Eq. (15.7). This...must be divided into visible...in environmental biophysics. Fortu...in each band so approximate

Transmission of Radiation by Sparse Canopies—Soil Reflectance Effects

If the leaves are not horizontal, Goodfellow (1998) suggests that the beam reflection coefficient for a deep canopy can be approximated from

\( \rho_{\text{br}}(\psi) = \frac{2K_\text{br}(\psi)}{K_\text{br}(\psi) + 1} \rho \) \( \text{br} \). \( \text{(15.5)} \)

The reflection coefficient for diffuse radiation can be approximated by substituting \( L_\beta \) for \( K_\text{br}(\psi) \) in Eq. (15.5).

If the canopy is not dense, then the effect of the soil may be significant and the canopy reflection coefficient for beam irradiance becomes (Monteith and Unsworth, 1990)

\( \rho_{\text{br}}(\psi) = \frac{\rho_{\text{br}}(\psi) \exp(-2\sqrt{\rho} K_\text{br}(\psi)L_\beta)}{1 + \rho_{\text{br}}(\psi) \exp(-2\sqrt{\rho} K_\text{br}(\psi)L_\beta)} \) \( \text{(15.9)} \)

Neglecting second-order terms like \( \rho_{\text{br}}(\psi^2) \) and \( \rho_{\text{br}}(\psi) \rho_\text{s} \) results in the following simplified equation:

\( \rho_{\text{br}}(\psi) \approx \rho_{\text{br}}(\psi) - \rho\rho_\text{s} \exp(-2\sqrt{\rho} K_\text{br}(\psi)L_\beta) \) \( \text{(15.10)} \)

Equation (15.10) is a good approximation to Eq. (15.9) in the PAR, but in the NIR, relative discrepancies can approach five percent. The diffuse forms of Eqs. (15.9) and (15.10) have \( K_\text{br}(\psi) \) replaced by \( K_\beta \) and are represented by \( \rho_{\text{br}}(\psi) \rho_\text{s} \) soil reflectance.

15.6 Transmission of Radiation by Sparse Canopies—Soil Reflectance Effects

For a canopy with a high LAI, the transmission of beam radiation (including its scattered component) as a function of depth \( z \) in the canopy is given by Eq. (15.6). If the canopy is not dense, and the LAI is low, then radiation can be reflected from the soil and re-reflected from the leaves to enhance the downwelling radiation streams. Monteith and Unsworth (1990), give the following equation for determining the flux density of radiation under the canopy:

\( \tau_\text{br}(\psi) = \frac{\exp(-2\sqrt{\rho} K_\text{br}(\psi)L_\beta)}{\exp(-2\sqrt{\rho} K_\text{br}(\psi)L_\beta) - \exp(-2\sqrt{\rho} K_\text{br}(\psi)L_\beta) - \exp(-2\sqrt{\rho} K_\text{br}(\psi)L_\beta)} \) \( \text{(15.11)} \)

If the second order terms are again neglected, then Eq. (15.11) simplifies to Eq. (15.6), and this amounts to assuming that the ratio of upwelling to downwelling below \( L_\beta \) for a deep canopy is equivalent to the soil reflectance for a finite canopy. In the PAR, radiation, Eq. (15.6) may be a reasonable approximation to Eq. (15.11), depending on \( \rho \). But in the NIR, relative discrepancies of ten percent or more can occur. The beam radiation absorbed by the canopy can be approximated with

\( \alpha_{\text{br}}(\psi) = 1 - \rho_{\text{br}}(\psi) - \rho_{\text{br}}(\psi)(1 - \rho) \) \( \text{(15.12)} \)
while the beam radiation intercepted by the canopy is

\[ f_b = 1 - \tau_b(\phi) \]  

(15.13)

Clearly the absorptivity of the canopy depends on wavelength but the interception does not depend on wavelength.

15.7 Daily Integration

Equation (14.13) requires estimates of the fraction of radiation intercepted by the canopy, averaged over whole days. Fuchs et al. (1976) suggested that the interception of beam and diffuse radiation, averaged over whole days, can be approximated by the intercepted function for diffuse radiation because the sun traverses the whole sky over the period of the day. Tests with detailed models have shown this to be correct. Therefore the average transmission of canopies can be modelled over whole days using Eq. (15.6), with \( K_a(\phi) \) from Eq. (15.6).

Based on these observations, the daily fractional interception \( f_d \) can be computed from

\[ f_d = 1 - \exp(-K_a L_a). \]  

(15.14)

Absorption of PAR is about equal to interception, while absorption of total solar radiation is about 80 percent of interception (Campbell and van Event, 1994).

15.8 Calculating the Flux Density of Radiation on Leaves in a Canopy

The equations we have just derived can be used to compute the flux density of radiation on leaves within the canopy. Knowing the flux density on leaves is important for the purpose of computing photosynthesis and for calculating the radiation viewed by a remote sensor. Let \( Q_{ab} \) be the flux density of beam radiation on a horizontal surface at the top of the canopy and \( Q_{ab} \) be the flux density of diffuse radiation on the horizontal above the canopy. At a depth \( L \) in the canopy, three different flux densities can be calculated: the total beam, \( Q_b(\phi) \) (unintercepted beam plus down scattered beam); beam, \( Q_b(\phi) \) (unintercepted beam) and the diffuse flux, \( Q_d \). These are given by

\[ Q_b(\phi) = \tau_b(\phi) Q_{ab} \]  

(15.15)

\[ Q(\phi) = \tau(\phi) Q_{ab} \]  

(15.16)

\[ Q_d = 0. \]  

(15.17)

Here, \( \tau_b(\phi) \) and \( \tau(\phi) \) are given by Eq. (15.6) with the appropriate \( K \) for beam or diffuse radiation, and \( \tau(\phi) = \exp(-K_a(\phi)L_a) \).

At depth \( L \) in the canopy some leaves are sunlit and some leaves are in the shade. The flux density on a horizontal surface at the position of a sunlit leaf is \( Q_s(\phi) + Q_d \). The flux density on the leaves themselves will vary depending on their sunlit leaves can be shown

\[ Q_L(\phi) \]

where \( Q_L \) is the flux density of the beam. The flux density of the scattered flux from the sun

The down-scattered radiation \( Q_d(\phi) \):

The next problem is to find the fraction of the beam penetrating to depth \( L \) in the canopy, \( L \), divided by \( K \), the horizontal surface to act leaf area index, then

\[ L = \frac{\exp(-K)}{L}. \]

In the limit as \( L \to \infty \), this fraction of the beam intercepted by the canopy, \( f_c(\phi) \) of sunlit

\[ f_c(\phi) \]

The fraction of shaded leaves is

\[ L \]

and the shaded LAI is

15.9 Calculating Canopy Assimilation

Several methods are available from leaf photosynthesis, including methods of measuring the humidity. Norman (1976) suggested that an assimilation rate is the two contributions:

One reason this method can be used to estimate the response of leaf photosynthesis.
Calculation of Canopy Assimilation from Leaf Assimilation

will vary depending on their orientation, but the mean flux density on the sunlit leaves can be shown to be

$$Q_{su}(\psi) = K_u(\psi)Q_u + Q_L + Q_{sc}$$

where $Q_u$ is the flux density of down-scattered radiation from the solar beam. The flux density on shaded leaves is the diffuse flux plus the down-scattered flux from the solar beam:

$$Q_s = Q_d + Q_{sc}.$$

The down-scattered radiation is the difference between $Q_{su}(\psi)$ and $Q_s(\psi)$:

$$Q_{sc} = Q_{su}(\psi) - Q_s(\psi).$$

The next problem is to know what fraction of the leaf area at depth $L$ is sunlit. The probability of finding a sunlit leaf area index in thickness $dL$ at depth $L$ in the canopy is the product of the probability that a ray will penetrate to depth $L$ and the probability that it will be intercepted in the leaf layer, divided by $K_u(\psi)$ (the rate of projections of leaf area on a horizontal surface to actual leaf area). If $L^*$ is used to represent the sunlit leaf area index, then

$$L^* = \exp(-K_u(\psi)\mu L^*) \left[ 1 - \exp(-K_u(\psi)\mu L) \right].$$

In the limit as $dL$ becomes small, $K_u(\psi)\mu L^* \rightarrow K_u(\psi)\mu L$. The fraction $f_{su}(\psi)$ of sunlit leaves at depth $L$ is $L^*/L$, so

$$f_{su}(\psi) = \exp(-K_u(\psi)L^*).$$

The fraction of shaded leaves is $f_{su}(\psi) = 1 - f_{su}(\psi)$. If the LAI of the entire canopy is $L_{CI}$, then the sunlit LAI of the whole canopy $L^*$ in

$$L^* = \frac{1 - \exp(-K_u(\psi)L_{CI})}{K_u(\psi)}$$

and the shaded LAI is $L_{CI} - L^*$,

15.9 Calculating Canopy Assimilation from Leaf Assimilation

Several methods are available for calculating canopy photosynthetic rate from leaf photosynthetic rate based on the distribution of light over leaves, including methods that consider additional factors such as wind and humidity; Norman (1992) compared various simple methods for estimating canopy assimilation from leaf assimilation. The most robust method seems to divide the canopy into sunlit and shaded leaf classes, calculate the assimilation rate for representative members of each class, and sum the two contributions according to the fraction of leaf area in each class. One reason this method works so well is that it accommodates the nonlinear response of leaf assimilation to light. Light assimilation responses of
leaves (see Fig. 14.6) can vary with depth in the canopy and this variation can be accommodated by partitioning the canopy into several layers and estimating the sunlit and shaded leaf fractions in each layer. Usually this is not necessary and a single, representative light assimilation response curve can be used for the entire canopy. Obviously most of the sunlit leaves are near the top of the canopy and most of the shaded leaves are near the bottom; therefore, one minor adjustment might be to use slightly different light assimilation response curves for sunlit and shaded leaves. In our example we use a single light assimilation response relation for all the leaves in the canopy.

Example 15.2. Estimate the canopy photosynthetic rate at 10°C (light assimilation curve in Fig. 14.6) for a canopy with a spherical leaf angle distribution and hemi-surface area index of 3.0, incident PAR above the canopy on the horizontal of \( Q_s = 2000 \mu \text{mol} \text{ photons m}^{-2} \text{ s}^{-1} \) with 80 percent as beam and 20 percent as diffuse radiation, sun zenith angle \( \psi = 40^\circ \), and leaf absorptivity \( \alpha = 0.8 \).

Solution. The canopy net assimilation rate \( A_{\text{can}} \) is the sum of contributions of sunlit and shaded leaves. These two contributions are added separately because sunlit leaves will be light saturated while shaded leaves will be in the linear portion of the light assimilation relation; thus canopy assimilation is not proportional to average light levels:

\[
A_{\text{can}} = A_{\text{sunlit}} + A_{\text{shaded}} = A_{\text{sunlit}}(L_s - L_i)
\]

where \( A_{\text{sunlit}} \) is the \( \mu \text{mol} \text{ CO}_2 \text{ m}^{-2} \text{ (leaf hemi-surface area)} \text{ s}^{-1} \), and \( A_{\text{shaded}} \) is the \( \mu \text{mol} \text{ CO}_2 \text{ m}^{-2} \text{ (ground area)} \text{ s}^{-1} \), and, of course, \( L_s \) and \( L_i \) are (leaf hemi-surface area (ground area))^{-1}.

First the average PAR incident on shaded leaves needs to be estimated. At the top of the canopy shaded leaves receive the diffuse radiation from the sky, 400 \( \mu \text{mol} \text{ photons m}^{-2} \text{ (ground area)} \text{ s}^{-1} \). At the bottom of the canopy \( Q_{\text{sh}} = Q_s + Q_d \). From Fig. 15.4, \( K_s = 0.72 \), so:

\[
Q_d = \tau_d Q_s = Q_s \exp(-\sqrt{\frac{K_d}{L_s}})
\]

\[
= 400 \exp(-\sqrt{3.0})(0.72)(3.0)
\]

\[
= 58 \mu \text{mol photons m}^{-2} \text{ (ground area)} \text{ s}^{-1}.
\]

The diffuse PAR on a horizontal plane is 400 at the top and 58 \( \mu \text{mol} \text{ photons m}^{-2} \text{ (ground area)} \text{ s}^{-1} \) at the bottom. For diffuse radiation, the flux density on the horizontal is assumed the same as the flux density on a leaf if the leaf area is expressed on a hemi-surface area basis (leaf HSA). Thus top shaded leaves have a diffuse illumination of 400 and bottom leaves receive 58 \( \mu \text{mol} \text{ photons m}^{-2} \text{ (leaf hemi-surface area)} \text{ s}^{-1} \). These two values could be averaged to obtain 229 \( \mu \text{mol} \text{ photons m}^{-2} \text{ (leaf hemi-surface area)} \text{ s}^{-1} \), but it is known that the attenuation is exponential and not linear, so a more appropriate average is an exponentially-weighted
The Light Environment of Plant Canopies

In the canopy and this variation in the canopy into several layers and of fractions in each layer. Usually this significant light assimilation response type. Obviously most of the study and most of the shaded leaves are arrangement might be to use slightly different curves for sunlit and shaded leaves. The assimilation response relation for all

$$\text{PPH} = 100 \text{°C (light)}$$

a canopy with a spherical leaf angle index of $\theta = 30^\circ$, incident PAR above the levels of 2000 $\mu$mol photons $\text{m}^{-2} \text{s}^{-1}$ with an angle zenith angle = 0°.

Since rate of canopy leaves is the sum of contributions. These two contributions are added be light saturated while shaded leaves and light assimilation relation; thus canopy average light levels:

$$\text{average light levels:}$$

$$\frac{\text{PAR}_{\text{ave}}}{L_0 - L_1} = \frac{1}{2} \text{PAR}_{\text{ave}} (L_0 - L_1)$$

(15.24)

leaf, semi-surface area) $m^{-2}$, and of area $m^{-2}$, and, of course, $L_0$ and $L_1$ area $m^{-2}$ and shaded leaves need to be estimated as receive the diffuse radiation from ground area $m^{-2}$. At the bottom of the canopy, $K_{\text{ave}} = 0.72$, so

$$\text{diffuse} \approx \sqrt{K_{\text{ave}}}$$

$$= 0.72 \sqrt{2} = 0.62 \text{ (ground area) m}^{-2}$$

plane is 400 at the top and 58, a total 600. But for diffuse radiation, the shade begins as the flux density on a semi-surface area basis (leaf FBA), so illumination of 400 and bottom (leaf semi-surface area) $m^{-2}$. There are $229 \mu$mol photons $\text{m}^{-2}$ (leaf hemi-

The attenuation is exponential and change is an exponentially-weighted

Calculating Canopy Assimilation from Leaf Assimilation

average:

$$\overline{Q} = \frac{\overline{Q}_{\text{ave}}}{L_0 - L_1} = \frac{\overline{Q}_{\text{ave}}}{\sqrt{K_{\text{ave}}(L_0 - L_1)}}$$

$$= \frac{177}{\text{mmol}} \text{m}^{-2} \text{(leaf hemsurface area)} \text{s}^{-1}$$

The sunlit radiation in zero at the top of the canopy and is given by $Q_{\text{ave}} = Q_{\text{ave}}(\theta) = Q_{\text{ave}}(\theta) - Q_{\text{ave}}(\theta)$. If the beam extinction coefficient is

$$K_{\text{ave}}(40^\circ) = \frac{1}{1 + 1.7745 + 1.8265} = 0.652,$$

then

$$Q_{\text{ave}} = 1000 \exp(-\sqrt{0.652} \times 0.652 \times 0.30) = 1600 \times (0.174)

= 278 \mu$mol photons $\text{m}^{-2} \text{ (ground area) s}^{-1}$

$$Q_{\text{ave}} = 1600 \exp(-0.652 \times 0.30) = 1600 \times (0.141)

= 226 \mu$mol photons $\text{m}^{-2} \text{ (ground area) s}^{-1}$

$$Q_{\text{ave}} = 278 - 226 = 52 \mu$mol photons $\text{m}^{-2} \text{ (ground area) s}^{-1}$

Therefore the average scattered radiation on leaves is $\frac{52 + 0}{2} = 26 $ $\mu$mol photons $\text{m}^{-2} \text{ (leaf hemsurface area) s}^{-1}$, and the PAR flux density absorbed by shaded leaves is

$$\overline{Q}_{\text{ave}} = 0.8 \times (177 + 26) = 0.8 \times 203

= 162 \mu$mol photons $\text{m}^{-2} \text{ (leaf hemsurface area) s}^{-1}$

where the overbar denotes an average over the depth of the canopy. The PAR flux density absorbed by sunlit leaves is given by

$$\overline{Q}_{\text{ave}} = \frac{1}{2} \text{PAR}_{\text{ave}} (L_0 - L_1)$$

$$= 0.8 \times 0.652 \times 1600 \times (0.141)

= 997 \mu$mol photons $\text{m}^{-2} \text{ (leaf hemsurface area) s}^{-1}$.

The sunlit LAI ($L_0$) and shaded LAI ($L_1$) are given by

$$L_0 = 1 - \exp(-K_{\text{ave}}(40^\circ)) = 1 - 0.141

= 0.652$$

$$K_{\text{ave}}(40^\circ) = 1.32 \mu$mol photons $\text{m}^{-2} \text{ (leaf hemsurface area) m}^{-2} \text{ (ground area)}$$

The leaf assimilation rates can be obtained from Fig. 14.6 using $\overline{Q}_{\text{ave}}$ for shaded leaves and $\overline{Q}_{\text{ave}}$ for sunlit leaves. The leaf assimilation rates in Fig.
14.6 are in units of \( \mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf surface area)} \text{ s}^{-1} \), and the units needed are \( \mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf hemi-surface area)} \text{ s}^{-1} \). Therefore

\[
A'_{\text{leaf}} = 11 \frac{\mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf surface area)} \text{ s}}{2 \text{m}^2 \text{(leaf surface area)}} \times \frac{\mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf hemi-surface area)} \text{ s}}{1 \text{m}^2 \text{(leaf hemi-surface area)}} = 22 \frac{\mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf hemi-surface area)} \text{ s}}{\text{leaf hemi-surface area}}.
\]

Similarly, at 162/2 = 81 \( \mu\text{mol photons m}^{-2}\text{(leaf surface area)} \text{ s}^{-1} \)

\[
A'_{\text{leaf}} = 6 \frac{\mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf hemi-surface area)} \text{ s}}{\text{m}^2 \text{(leaf surface area)}}.
\]

Therefore the canopy assimilation is given by

\[
A_c = 22 \frac{\mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf hemi-surface area)} \text{ s}}{\text{m}^2 \text{(ground area)}} \times 1.32 \frac{\text{m}^2 \text{(leaf hemi-surface area)}}{\text{m}^2 \text{(ground area)}} + 6 \frac{\mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf hemi-surface area)} \text{ s}}{\text{m}^2 \text{(ground area)}} \times 1.68 \frac{\text{m}^2 \text{(leaf hemi-surface area)}}{\text{m}^2 \text{(ground area)}} + 29.0 + 10.1 = 36.1 \frac{\mu\text{mol CO}_2 \text{ m}^{-2}\text{(ground area)} \text{ s}}{\text{m}^2 \text{(ground area)}}.
\]

The approach used in Example 15.2 to scale leaf assimilation to canopy assimilation accommodates the nonlinearity in the 10°C light assimilation curve in Fig. 14.6. If we had ignored the fact that the 10°C light assimilation curve is not a straight line and used an average absorbed PAR for the entire canopy to scale up the leaf assimilation rate, how large would the error be? The average absorbed PAR for the canopy \( Q \) is the mean of sunlight and shaded absorbed PAR weighted by the leaf area of each:

\[
Q = \frac{Q_{\text{sun}} L_s + Q_{\text{sh}} (L_s - L_s^*)}{L_s} = 997 \times 1.32 + 162 \times 1.68 \over 3.0 = 529 \frac{\mu\text{mol photons}}{\text{m}^2 \text{(leaf hemi-surface area)} \text{ s}}.
\]

From Fig. 14.6, the leaf assimilation rate corresponding to the average absorbed PAR is 20 \( \mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf hemi-surface area)} \text{ s}^{-1} \), so the

\[
A_{\text{leaf}} = 2 \frac{\mu\text{mol CO}_2 \text{ m}^{-2}\text{(leaf hemi-surface area)} \text{ s}}{\text{leaf hemi-surface area}}.
\]

This is 54 percent larger than the actual canopy photosynthesis. The reason is that the light absorbed by the leaves is not the same as the light absorbed by the canopy. If the canopy is very dense, the light absorbed by the leaves may be less than the light absorbed by the canopy.
canopy assimilation rate is given by

\[
A_{\text{canopy}} = 20 \frac{\mu\text{mol CO}_2}{\text{m}^2\text{(leaf semi-surface area)}} \times \frac{\mu\text{mol CO}_2}{\text{m}^2\text{(ground area)}} \\
\geq 60 \frac{\mu\text{mol CO}_2}{\text{m}^2\text{(ground area)}}
\]

This is a 54 percent larger than the sunlit-shaded method in Example 15.2. This value of 60 \( \mu\text{mol CO}_2 \text{ m}^{-2} \text{(ground area)} \text{ s}^{-1} \) is approximately the canopy photosynthetic rate that would occur if the canopy were illuminated with entirely diffuse irradiance at 2000 \( \mu\text{mol photons m}^{-2} \text{ (ground area)} \text{ s}^{-1} \). This is a diffuse irradiance of about 1300 \( \mu\text{mol photons m}^{-2} \text{ (ground area)} \text{ s}^{-1} \) would result in about the same canopy photosynthetic rate (59 \( \mu\text{mol CO}_2 \text{ m}^{-2} \text{ (ground area)} \text{ s}^{-1} \)) as 2000 \( \mu\text{mol photons m}^{-2} \text{ (ground area)} \text{ s}^{-1} \) with 83 percent beam and 17 percent diffuse. This means the diffuse irradiance is more efficient for photosynthesis than beam irradiance.

Light assimilation responses are not always as nonlinear as the 10°C curve in Fig. 14.4; for example, the 30°C curve in Fig. 14.6. Comparing the canopy assimilation prediction from the sunlit-shaded method with the average-APAR method results in 39 \( \mu\text{mol CO}_2 \text{ m}^{-2} \text{ (ground area)} \text{ s}^{-1} \) for both methods; this occurs because of the linearity of the 30°C curve. Considering the realistic leaf assimilation rate at 30°C from Fig. 14.6, it may be surprising to find the canopy assimilation rates for 20 and 30°C are nearly equal. This occurs because leaves at 30°C have higher photosynthetic rates on sunlit leaves and lower rates on shaded leaves, because of the larger dark respiration. Essentially the highest maximum leaf photosynthetic rate occurs at a higher dark respiration cost. Furthermore, the canopy architecture limits the fraction of leaves that can be sunlit.

Leaf stomatal conductance can be scaled to a canopy conductance by using the same method as outlined above for photosynthetic rates if stomatal conductances for sunlit and shaded leaves are known. Using stomatal conductances appropriate for the leaf assimilation rates plotted in Fig. 14.6 under humid atmospheric conditions, the canopy conductance for Example 15.2 can be estimated from an equation like Eq. (15.24) to be 0.5(1.32) + 0.1(1.68) = 1.0 mild water \( \text{m}^{-2} \text{ (ground area)} \text{ s}^{-1} \). Because sunlit LAI approaches a maximum as LAI increases \( \xi \), has a maximum of about 1.5 for high LAI canopy with \( \xi \approx 40 \) and the mean shaded stomatal conductance decreases as LAI increases, this sunlit-shaded approach clearly shows why canopy conductances tend to reach maximum values that might be expected to be related to sunlit leaf area index.
15.10 Remote Sensing of Canopy Cover and IPAR

Remote sensing is a name associated with inferring characteristics of surfaces from measurements of radiance. In environmental biophysics, remote sensing usually refers to the interpretation of radiometric measurements made above soil-vegetation systems from towers, aircraft, or satellites. A more general term is indirect measurement, which refers to any measurement made without directly contacting an object. Technically, our eyes indirectly sense the environment around us so an absurd interpretation might infer that all information obtained with our eyes (e.g., reading a ruler) could be considered remote sensing; however, this is not what we mean. In environmental biophysics, some examples of remote sensing include the following.

1. Infrared thermometer measurements of soil surface temperature.
2. Measuring soil or canopy roughness using the backscattered radiation from a laser (these systems are called LIDAR).
3. Estimating the water content of the top 5 cm layer of soil using passive microwave measurements of surface temperature and emissivity.
4. Estimating total forest-canopy water content to infer vegetation biomass using RADAR.
5. Inferring canopy cover, leaf area index, or intercepted photosynthetically active radiation (IPAR) from measurements of visible (VIS) and near-infrared (NIR) reflected radiance.

Another indirect measurement that is common in environmental biophysics, but not generally referred to as remote sensing, is the indirect measurement of canopy architecture. This is discussed briefly in a later section of this chapter.

Some of the fundamental characteristics of remote sensing data can be understood using knowledge of canopy architecture by considering the relation between canopy cover, IPAR, and reflected VIS and NIR radiation. In previous sections we discussed the penetration of radiation through canopies, the reflection of radiation from canopies and the distribution of radiation over the surface of leaves. Although all this is relevant to remote sensing, a second consideration also is required; that is, the portion and characteristics of the canopy and soil that occupy the field-of-view (FOV) of the sensor. As mentioned in Ch. 10, bidirectional reflectance factors (BRF) involve two directions; the direction of the source (usually the sun) and the direction of the receiver (a sensor). To simplify the analysis that follows, we do not consider finite solid angles of view, but only consider particular directions as though the radiation were composed of parallel rays all from that direction. Essentially this amounts to using data from a narrow FOV sensor that is calibrated to read out the flux density emanating from the target surface by making the output proportional to the radiance times the FOV of the sensor.

Remote Sensing of Canopy Cover and IPAR

For our purposes, the bidirectional reflectance factor (BRF) surface can be defined as follows:

$$\text{BRF} = \text{flux density leaving if}$$

$$\text{flux density in}$$

The flux density incident on the sensor is a reference condition at the target surface. Lambertian surface as possible wavelength bands such (BRFs), and the view of the shaded leaves, and soil (both if the BRF for soils and vegetation responded like Lambertian surface be constant for all view angles by more than a factor of three band. Detailed models of canopies are scope of this book. Even as quite complicated. However, BRF by small spheres on a flat by the spheres onto the horizon viewed by the sensor. The B a characteristic shape with 1 directly behind the sensor and the sun. Walthall et al. (1985) BRF distributions as a function single sun zenith angle:

$$\text{BRF} = \alpha$$

where \( \alpha \) is the view zenith angle of the sensor when the sun is directly behind the sensor, and \( \beta \) is the change with canopy architecture example, Walthall et al. (1985) with LAF = 2.6 and \( \beta = 5^\circ \) with NIR, \( \alpha = 9.09, b = 7 \) radians). Clearly the BRF is positive and smallest where the sun is directly behind the sensor, and the BRF is negative and largest when the sun is directly in front of the sensor.

In this chapter we are interested in BRF and canopy architecture equations by limiting the d (or within about 10° of d direction used in remote s minimal and interpretation following sections).
Remote Sensing of Canopy Cover and NPAR

For our purposes, the bidirectional reflectance factor (BRF) for a surface can be defined as follows:

\[
\text{BRF} = \frac{\text{flux density leaving the horizontal surface viewed by a sensor}}{\text{flux density incident on the horizontal surface}}
\]

The flux density incident on the surface is usually measured by pointing the sensor at a reference surface (exposed to the same illumination conditions as the target surface) that is close to a perfectly-reflecting, Lambertian surface as possible. Clearly the BRF may be different for various wavelength bands such as the visible (BRF_v) and near-infrared (BRF_i), and the view of the sensor may be occupied by soil, leaves, shaded leaves, and soil (both sunlit and shaded).

If the BRF for soils and vegetation are isomoric that is, the surfaces responded like Lambertian surfaces, then the magnitude of the BRF would be constant for all wavelengths. However the BRF for canopies can vary by more than a factor of three with view angle for a given wavelength band. Detailed models of canopy BRFs are complex and beyond the scope of this book. Even analytical models such as Knapp (1985) are quite complicated. However, from et al. (1992) have proposed the soil BRF by visual sphere on the Lambertian plane, where the shadows case by the sphere onto the horizontal background influence the radiation viewed by the sensor. The BRF distributions for canopies and soils have a characteristic shape with BRF values being highest when the sun is directly behind the sensor and lowest when the sensor view is directed toward the sun. Walkall et al. (1985) present a simple, empirical equation to fit BRF distributions as a function of view zenith and view azimuth for a single sun zenith angle.

\[
\text{BRF} = a \psi_i + b \psi_i \cos(\Delta Z) + c
\]

(15.25)

where \( \psi_i \) is the view zenith angle, \( \Delta Z \) is the difference between the azimuth angle of the sensor and the azimuth angle of the sun \( \Delta Z = \theta \) when the sun is directly behind the viewer so the view is away from the direction of the sun, and \( a, b, \) and \( c \) are empirical coefficients that change with canopy architecture, view wavelength, and sun zenith angle. For example, Walkall et al. (1985) give the coefficients for a soybean canopy with LAI=1.26 and \( \theta = 51^\circ \) as \( \text{VIS} a = 1.49, b = 0.32, \) and \( c = 3.44, \) with \( \text{NIR} a = 9.69, b = 7.62, \) and \( c = 44.8 \) (BRF in % and angles in radians). Clearly the BRF is largest when the true angle from Eq. (15.25) is positive and smallest when the middle term is negative.

In this chapter we are interested in understanding the relation between BRF and canopy architecture. This can be accomplished with simplified equations by limiting the discussion to a sensor viewing from nadir (or within about 10° of directly overhead). This is the most common direction used in remote sensing because atmospheric contamination is minimal and interpretation of radiative data is most straightforward. In the following sections, \( \theta \) refers to the sun zenith angle, and since the sun zenith angle is rarely zero, we use \( \theta \) to refer to the solar view angle so...
that $K_a(\phi)$ refers to the extinction coefficient for beam radiation and $K_a(0)$ refers to the extinction from the nadir view direction. $K_a(0)$ is never used in the following equations to refer to the direction of the sun. Although similar equations are used to describe sun and view effects, the context should always be obvious.

From Eq. (15.22), the fraction of leaves at depth $L$ in a canopy that is sunlit is given by $\exp(-K_a(\phi)L)$. If the sensor is placed at the same zenith and azimuth angles as the sun, and for discussion purposes assume the sensor is so small that it casts a negligibly small shadow, then the sensor would view exactly these sunlit leaves. Thus the same exponential expression can be used to estimate sunlit-leaf-area fraction as to estimate the fraction of leaves in a layer that can be viewed from the same direction.

The special case of identical sun and view directions is referred to as the canopy hot spot, because the canopy appears brighter from this direction than any other direction; all the leaves being sunlit from this direction. For this special case, sunlit leaves and shaded leaves cannot be assumed to be independent because both sun and view directions share the same path through the canopy. As the sensor is moved off the direction of the sun, shaded leaves occupy an increasing fraction of the sensor FOV until the view path and the path of the sun rays are independent. The decrease in radiance as a function of the increasing angle between the direction of the sensor and the sun direction depends on LAI, leaf angle distribution, leaf size, sun zenith angle, and canopy height (Kuniskis, 1995). Typically the hot spot varies from a few degrees wide to a few tens of degrees wide depending on conditions. The equations that follow, which pertain to nadir viewing only, do not consider the hot spot. Hot spot considerations generally would represent a minor refinement for latitudes where the sun zenith angle is rarely less than 20°.

A remote sensor that is directed toward a canopy-soil system may view both vegetation and soil. For a canopy of randomly positioned leaves, the fraction of the sensor view that is occupied by soil is $\exp(-K_a(0)L_s)$ so that the fraction of view occupied by vegetation is $1 - \exp(-K_a(0)L_s)$. The vegetation portion of the view consists of both sunlit and shaded leaves. The fraction of leaves at a depth $L$ that is sunlit with the sun at zenith angle $\phi$ is given by Eq. (15.22). Therefore the fraction of leaves at a depth $L$ that is sunlit and can be viewed from nadir is given by the product: $\exp(-K_a(0)L_s) \exp(-K_a(\phi)L)$. If the product of these two exponentials is integrated over the depth of the canopy $L_s$ the sunlit leaf area index is obtained that is in the view of the sensor, $L_s^\prime$:

$$L_s^\prime = \frac{1 - \exp(-[K_a(0) + K_a(\phi)L_s]/K_a(0) + K_a(\phi))}{[K_a(0) + K_a(\phi)]}$$

The fraction of the sensor view occupied by sunlit leaves is given by the projection of $L_s^\prime$ in the direction of the sensor or $f_{\phi, L_s^\prime} = L_s^\prime K_a(0)$. The fraction of sensor view occupied by shaded leaves is the difference between the view fraction occupied by vegetation $(1 - \exp(-K_a(0)L_s))$ and the fraction occupied $f_{\phi, L_s^\prime}$.
Remote Sensing of Chaopy Cover and fPAR

and the fraction occupied by sunlit leaves:

\[ f_{v, S} = 1 - \exp(-K_{SL}(0) L) - L^2 L_{b}^2 K_{Sb}(0). \]  

(15.27)

The flux density detected by the sensor, \( Q_{\text{sun}}(\lambda) \), is the sum of the contributions of sunlit leaves, shaded leaves, and soil weighted by the view fractions each occupies:

\[ Q_{\text{sun}}(\lambda) = \rho L_{b} L \left[ 1 - \exp(-K_{SL}(0) L) - L^2 L_{b}^2 K_{Sb}(0) \right] + \rho_1 L_{b} L_1 \exp(-K_{SL}(0) L_1). \]  

(15.28)

where \( \rho \) and \( \rho_1 \) are the leaf and soil reflectivity in the wavelength band of interest. The BRF \( F_{\lambda} \) for a particular wavelength band, for example, the visible, is given by

\[ BRF_{\lambda} = \left( \frac{Q_{\text{sun}}(\lambda)}{Q_{\text{soil}} + Q_{\text{air}}} \right). \]  

(15.29)

The unique feature of leaves that permits remote sensing of canopy bidirectional reflectance to be useful for estimating canopy biophysical characteristics is the strong contrast between absorption in the visible and scattering in the near-infrared with a sharp transition near 700 nm (Fig. 11.3). Usually soils have higher reflectivity in the visible than dense canopies, lower reflectivity in the near-infrared than dense canopies, and only slightly higher reflectivity in the near-infrared than visible; therefore as canopy cover increases, the visible reflectance decreases, near-infrared reflectance increases, and the ratio, given by

\[ SR = \frac{BRF_{\lambda}}{BRF_{\lambda}} \]  

(15.30)

increased (SR is called the simple ratio vegetation index). Another form of the ratio is the normalized difference vegetation index (NDVI) given by

\[ NDVI = \frac{BRF_{\lambda} - BRF_{\lambda}}{BRF_{\lambda} + BRF_{\lambda}} \]  

(15.31)

where \(-1 \leq NDVI \leq 1\). These vegetation indices in the form of ratios are widely used in remote sensing because biophysical characteristics that affect both bands similarly tend to cancel out. Numerous other indices have been developed to optimize the influence of atmospheric or soil contamination and the advantage gained from these variations over SR and NDVI appears to be minor but consistent. NDVI may not be zero for zero vegetation cover because soil reflectances in the two bands may not be equal or because of atmospheric effects. VARI is scattered more than NIR so NDVI can be negative from satellite observations if no atmospheric corrections are done; therefore, an adjusted NDVI \( \text{NDVI}' \) has been proposed by Carlson et al. (1995):

\[ \text{NDVI}' = \left( \frac{\text{NDVI} - \text{NDVI}_{\text{min}}}{\text{NDVI}_{\text{max}} - \text{NDVI}_{\text{min}}} \right)^2 \]  

(15.32)
where \( NDVI_{\text{max}} \) is the NDVI with no vegetation and \( NDVI_{\text{min}} \) is the NDVI with dense vegetation. 

Cihlar et al. (1995) set \( NDVI^* \) equal to the fraction of vegetative cover and this is a reasonable approximation, especially when solar zenith angles are small. Clearly \( NDVI^* \) varies from zero to one over the range of vegetation cover and accounts for the observation that NDVI increases three to five times as rapidly as the fraction of vegetation cover as vegetation density increases.

Remote sensing from satellites has the possibility of sampling the entire land surface of the earth daily at a 1 km spatial resolution on the ground and a spatial resolution of 10 m or less with less frequent temporal sampling. Because of this phenomenal spatial sampling, much effort has been expended to determine what biophysical quantities are most closely related to the remote sensing observations. As examination of Eq. (15.28) provides useful insights here. Remember that optical remote sensing from satellites is possible only under relatively clear- sky conditions when atmospheric transparency is high, because satellites need to view the surface with minimal contamination from the atmosphere. From Eq. (15.28), when \( L_e \) is small, reflection from the soil dominates (third term on the right of Eq. (15.28)). As \( L_e \) increases, the dominant term in Eq. (15.28) becomes the scattering of intercepted near-infrared beam radiation (first term on the right of Eq. (15.28)), which also happens to be closely related to the intercepted PAR radiation. Table 15.2 contains values of the three terms in Eq. (15.28), NDVI, \( NDVI^* \), and IPAR and the fraction of canopy cover \( f_c \) assuming

\[
f_c = \exp(-\alpha_0(0.4L_e))
\]  
(15.32)

Clearly \( NDVI^* \) is most closely related to IPAR and fraction vegetative cover \( f_c \) when \( \alpha \) is small (30°). The relation between \( NDVI^* \) and IPAR is likely to be better at other solar zenith angles because both \( NDVI^* \) and IPAR change but \( f_c \) is fixed with \( \alpha \). The close relation between \( NDVI^* \) and IPAR occurs because intercepted solar radiation dominates both variables; interception in the visible portion of the solar spectrum dominates IPAR and interception in the NIR portion of the solar spectrum dominates NDVI.

The effects of leaf angle and sea zenith angle can be seen from Table 15.3. Clearly \( NDVI^* \) is a reasonable predictor of fraction of IPAR for a modest range of conditions. Since IPAR is closely related to vegetation productivity potential (Eq. (14.13)) with \( L_e \) replaced by IPAR and conversion efficiency \( \mathcal{E} \) adjusted accordingly [as shown doubled], remote sensing has something significant to contribute to global vegetation stud- ies. The robustness of the relation between NDVI and fraction of IPAR is further established by studies that have shown \( NDVI^* \) to be related to the fraction of IPAR associated with the green vegetation in canopies that have both green and dead foliage.

**Example 15.3.** Compare the nadir, near-infrared BRF (BRF_\( \text{nir} \)) for a canopy with a spherical leaf angle distribution (\( x = \pi \)) with the hemi-

<table>
<thead>
<tr>
<th>( L_e )</th>
<th>NIR Term 1 (W m(^{-2}))</th>
<th>NIR Term 2 (W m(^{-2}))</th>
<th>NIR Term 3 (W m(^{-2}))</th>
<th>NIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>9</td>
<td>17</td>
<td>26</td>
<td>0.2</td>
</tr>
<tr>
<td>0.3</td>
<td>25</td>
<td>61</td>
<td>92</td>
<td>0.2</td>
</tr>
<tr>
<td>0.5</td>
<td>47</td>
<td>103</td>
<td>206</td>
<td>0.2</td>
</tr>
<tr>
<td>0.9</td>
<td>68</td>
<td>135</td>
<td>513</td>
<td>0.2</td>
</tr>
<tr>
<td>1.5</td>
<td>86</td>
<td>165</td>
<td>205</td>
<td>2.8</td>
</tr>
<tr>
<td>2.0</td>
<td>98</td>
<td>250</td>
<td>206</td>
<td>20</td>
</tr>
<tr>
<td>4.0</td>
<td>113</td>
<td>50</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6.0</td>
<td>113</td>
<td>57</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

**Conclusion.** The canopy BDF terms is that equation is

\[
\psi = 30
\]

<table>
<thead>
<tr>
<th>( L_e )</th>
<th>( \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.06</td>
</tr>
<tr>
<td>0.3</td>
<td>0.06</td>
</tr>
<tr>
<td>0.6</td>
<td>0.20</td>
</tr>
<tr>
<td>1.3</td>
<td>0.41</td>
</tr>
<tr>
<td>1.5</td>
<td>0.59</td>
</tr>
<tr>
<td>2.0</td>
<td>0.72</td>
</tr>
<tr>
<td>4.0</td>
<td>0.90</td>
</tr>
<tr>
<td>6.0</td>
<td>0.92</td>
</tr>
</tbody>
</table>
Remote Sensing of Canopy Cover and IPAR

Table 15.2. Variation of some quantities related to remote sensing as a function of several canopy biophysical characteristics. The wavelengths used for remote sensing calculations are about 350 nm and 750 nm. The three terms from Eq. (15.28) are for the NIR wavelength band. The canopy is assumed to have a spherical leaf angle distribution and $\psi = 30^\circ$. All fluxes are in units of W m$^{-2}$.

<table>
<thead>
<tr>
<th>$L_s$</th>
<th>NIR</th>
<th>NIR$^*$</th>
<th>NDIR</th>
<th>NDVI</th>
<th>NDVI$^*$</th>
<th>IPAR</th>
<th>IPAR</th>
<th>Fraction</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>27</td>
<td>0.20</td>
<td>0</td>
<td>0.06</td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>9</td>
<td>0</td>
<td>70</td>
<td>0.27</td>
<td>0.01</td>
<td>28</td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>23</td>
<td>1</td>
<td>61</td>
<td>0.39</td>
<td>0.06</td>
<td>78</td>
<td>0.16</td>
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<td></td>
</tr>
<tr>
<td>0.6</td>
<td>47</td>
<td>3</td>
<td>50</td>
<td>0.54</td>
<td>0.20</td>
<td>143</td>
<td>0.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>68</td>
<td>9</td>
<td>38</td>
<td>0.64</td>
<td>0.41</td>
<td>212</td>
<td>0.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>86</td>
<td>17</td>
<td>28</td>
<td>0.78</td>
<td>0.79</td>
<td>280</td>
<td>0.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>98</td>
<td>25</td>
<td>20</td>
<td>0.84</td>
<td>0.57</td>
<td>311</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>113</td>
<td>50</td>
<td>6</td>
<td>0.91</td>
<td>0.90</td>
<td>441</td>
<td>0.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>113</td>
<td>57</td>
<td>2</td>
<td>0.92</td>
<td>0.92</td>
<td>479</td>
<td>0.96</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solidation. The canopy BRF$_\psi$ is estimated from Eq. (15.28) so the three terms in that equation need to be evaluated. The following quantities are

Table 15.3. Relation between NDVI$^*$ and IPAR fraction ($f_{IPAR}$) for two sun zenith angles and two leaf angle distributions.

<table>
<thead>
<tr>
<th>$L_s$</th>
<th>$\psi = 30^\circ$</th>
<th>$\psi = 60^\circ$</th>
<th>$\psi = 30^\circ$</th>
<th>$\psi = 60^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>0.3</td>
<td>0.06</td>
<td>0.16</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>0.6</td>
<td>0.20</td>
<td>0.29</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>1.0</td>
<td>0.41</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
</tr>
<tr>
<td>1.5</td>
<td>0.56</td>
<td>0.71</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>2.0</td>
<td>0.72</td>
<td>0.86</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>4.0</td>
<td>0.90</td>
<td>0.92</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>6.0</td>
<td>0.92</td>
<td>0.96</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Note: The NDVI$^*$ and IPAR$^*$ are calculated from the leaf angle distribution (LAD) and the leaf area index (LAI) model.
needed:

\[ K_{se} = \sqrt{\frac{1.0 + \tan^2(0)}{1.0 + 1.774(1.0 + 1.82)^{-0.73}}} = 0.50 \]

\[ K_{se}(60) = \frac{1.0 + \tan^2(60)}{1.0 + 1.774(1.0 + 1.82)^{-0.73}} = 1.00 \]

\[ K_D = 0.76 \quad \text{(from Fig. 15.4)} \]

\[ L^e_1 = \frac{1 - \exp[-(0.5 + 1.02a)]}{0.5 + 1.0} = 0.633 \]

\[ Q_{sun}(\phi) = \text{Term 1 + Term 2 + Term 3} \]

**Term 1**

\[ \rho \Omega_D(\phi) L^1_0 K_{se}(0) \]

**Term 2**

\[ Q_{sun}(60) = K_{sun}(60) \rho_{se} + \bar{Q}_a + \bar{Q}_i \]

\[ Q_a = 20 \exp(-0.04 \times 0.76 \times 2.0) = 200(0.738) \]

\[ = 15 \text{ W m}^{-2} \]

\[ Q_i = \left( \frac{20 + 15}{2} \right) = 18 \text{ W m}^{-2} \]

\[ Q_{se}(60) = 230 \left[ \exp(-0.04 \times 1.0 \times 2.9) - \exp(-1.0 \times 2.0) \right] \]

\[ = 123 \text{ W m}^{-2} \]

\[ Q_{se} = \frac{0 + 123}{2} = 62 \text{ W m}^{-2} \]

\[ Q_{se}(60) = 1.0(230) + 18 = 62 = 310 \text{ W m}^{-2} \]

**Term 1**

\[ 0.48 \times 310 \times 0.633 \times 0.5 = 47 \text{ W m}^{-2} \]

**Term 2**

\[ \frac{1}{2} \left[ \exp(-K_{sun}(0) L^1_0) - L^1_0 K_{se}(0) \right] \]

\[ Q_{sm} = \frac{Q_a + Q_i}{2} + 18 = 62 = 80 \text{ W m}^{-2} \]

**Term 2**

\[ 0.48 \times 80 \left[ 1 - \exp(-0.5 \times 2.0) \right] \times 0.633 \times 0.5 \]

\[ \approx 12 \text{ W m}^{-2} \]

**Term 3**

\[ \frac{1}{2} \left[ \exp(-K_{se}(0) L^1_0) \right] \]

\[ Q_{sm}(60) = 230 \times \left[ \exp(-0.04 \times 1.0 \times 2.0) \right] = 230 \times 0.67 \]

\[ = 154 \text{ W m}^{-2} \]

**Term 3**

\[ 0.15 \times (154 + 15) \exp(-0.5 \times 2.0) = 9 \text{ W m}^{-2} \]

Therefore \[ Q_{sun}(60) = 47 + 12 + 9 = 66 \text{ W m}^{-2} \] so that BRF \[ = \frac{66}{310} = 0.27. \]

If we had used the more precise Eq. (15.11) instead of Eq. (15.6), then BRF \[ = 0.32 \text{ instead of 0.27}. \]

**Remote Sensing and Canopy Temperature**

The hemispherical reflectance of beam and diffuse correlation

\[ \rho_{se}(60) \approx \rho^e_{se}(60) - \rho^e_{se}(60) \]

\[ = \frac{1 - \sqrt{0.04}}{1 + \sqrt{0.04}} \]

\[ = 2.0 \times 0.7 \]

\[ = 0.76 \]

\[ = 0.576 - 0.36 \]

Therefore the hemispherical correlation

\[ \rho_{se}(60) = \frac{Q}{Q} \]

If we had used the more \[ \rho_{se}(60) = 0.48 \text{ instead of 0.27}. \]

The reason BRF \[ = 2 \]

is that the radiometer sensor erases the nadir and the undesirability of using about remote sensing.

**15.11 Remote Sensing an**

Aerodynamic surface flow of net radiation into seas particularly in Eq. (14.6) that can be measure spatial scales, numerous sensed radiometric temp and latent heat fluxes. To examining Eq. (14.8) of surface temperature, an examination between radionuclides included in Eq. (14.8). Able globally, most of th are not.
The hemispherical reflectance can be estimated from Eq. (15.10) for both beam and diffuse components. For the beam component:

\[ \rho_{\text{bp}}(60) = \rho_{\text{bp}}^{\text{app}}(60) - \rho_{\text{bp}}^{\text{app}}(60) \exp(-2\sqrt{\alpha} L_{\text{a}}(L_{\text{b}})) \]

\[ \rho_{\text{bp}}^{\text{app}}(60) = 1 - \frac{\text{0.04}}{1 + \text{0.04}} \times \frac{\text{1.0}}{1.0} = 0.667 \quad \text{(Eq. (15.7))} \]

\[ \rho_{\text{bp}}^{\text{app}}(60) = 0.667 \times 0.667 = 0.667 \quad \text{(Eq. (15.8))} \]

Therefore, the hemispherical reflectance of the canopy is

\[ \rho_{\text{app}}(60) = \frac{Q_{\text{soil}}R_{\text{exp}}(60) + Q_{\text{soil}}F_{\text{tree}}(60)}{230 \times 0.435 + 20 \times 0.344} = 0.43. \]

If we had used the more precise Eq. (15.9) instead of Eq. (15.10), then \( \rho_{\text{soil}}(60) = 0.48 \) instead of 0.43.

The reason \( \text{BRF}_{\text{soil}} \) is lower than \( \rho_{\text{soil}}(60) \) in the near-infrared is that the radian-viewing sensor views deeper into the canopy than the sun penetrates and thus the radian \( \text{BRF}_{\text{soil}} \) is lower by 37 percent. This indicates the undesirability of using hemispherical reflectances to make inferences about remote sensing with narrow FOV sensors.

15.11 Remote Sensing and Canopy Temperature

Aerodynamic surface temperature is a key variable in the partitioning of net radiation into sensible and latent heat fluxes, as shown in Ch. 14, particularly in Eq. (14.8). Since radiometric surface temperature is a quantity that can be measured from satellites over the globe on kilometer spatial scales, numerous attempts have been made to use these remotely sensed radiometric temperatures to monitor the partitioning of sensible and latent heat fluxes. The magnitude of this challenge is apparent from examining Eq. (14.8); obviously many variables can affect aerodynamic surface temperature, and the additional variables involved in the relation between radiometric and aerodynamic temperatures are not even included in Eq. (14.8). Although radiometric temperature may be available globally, most of the other variables that affect surface temperature are not.
The sensible heat flux from the vegetation/soil system is closely related to surface aerodynamic temperature by

$$H_{se} = c_p (T_{se} - T_a)$$  (15.34)

where $H_{se}$ is the aerodynamic conductance or canopy boundary-layer conductance given by Eq. (14.9) and $T_a$ is the air temperature. The apparent simplicity of Eq. (15.34) is deceptive. Assuming the information is available on a continental basis to estimate $H_{se}$, and this is no minor task because vegetation height, cover, and wind speed are required (remote sensing of NDVI may help here), the major challenge remains in trying to use aerodynamic temperature to estimate sensible heat flux:

1. The radiometric temperature and aerodynamic temperature are not the same and usually differ by 1 to 5°C.
2. The near-surface air temperature is not known on the same spatial scale as radiometric temperature and can vary by 5°C or more depending on the temperature of the underlying surface.
3. Atmospheric corrections and uncertainties in surface emissivity associated with satellite borne surface radiometric temperatures have uncertainties of 1 to 3°C.

Unfortunately, an uncertainty of 1°C in $T_{se} - T_a$ can result in a 50 W m$^{-2}$ uncertainty in $H_{se}$; a reasonable estimate of a tolerable maximum error. These challenges have not deterred scientists from searching for a solution.

From this discussion a practical method for using satellite surface temperature measurements would have at least three qualifiers:

1. Accommodate the difference between aerodynamic temperature and radiometric temperature.
2. Not require a measurement of non-surface air temperature.
3. Relate more on differences of surface temperature over time or space rather than absolute surface temperatures to minimize the influence of atmospheric corrections and uncertainties in surface emissivity.

Anderson et al. (1997) have proposed such a method based on satellite observations from the Geosynchronous Orbiting Environmental Satellite (GOES), which is used primarily for observations of clouds and weather forecasting, having a ground spatial resolution of 4 km. In addition to the satellite temperature observations, they use ground measurements and balloon measurements from the weather forecasting network, a continental vegetation classification map, and vegetation cover estimated with NDVI as described in the previous section. Uncertainties in sensible and latent heat of 70 to 50 W m$^{-2}$ are achievable by this method. Practical methods for using satellite observations of surface temperature to partition sensible and latent heat fluxes on a continental scale are most challenging.

15.12 Canopy Reflectivity Leaf Reflectivity (E)

Canopy reflectance is in radiance incident on leaf multiple interactions between absorption of the radiant flux from the canopy that is absorbed on soil surface. Either Eq. (15) trapping phenomenon E and PAR transmissivity is the thermal wave along leaf reflectivity $p_L = 0$ canopy, $p_{L,0} = 0.01$. Therefore a deep canopy that makes it up, and hits to have thermal emissivities.

15.13 Heterogeneous Canopies

The simplified radiative balance to vegetation leaves are often inferred finding a leaf anywhere position. When leaves are considered heterogeneous they take many forms. We heterogeneity:

1. Incorporate a clear by replacing L with depending on zenith.
2. Assume leaves to some appropriate envelopes, to reposition.

The clumping factor to extend the previous chapter to heteroclumped foliage has been strongly clump...
15.12 Canopy Reflectivity (Emisivity) versus Leaf Reflectivity (Emisivity)

Canopy reflectance is less than leaf reflectance because some of the radiation incident on leaves is transmitted deeper into the canopy where multiple interactions between the radiation and leaves cause additional absorption of the radiation. In effect, the canopy behaves as a trap for the radiation that is absorbed at the deeper depths in the canopy or at the soil surface. Either Eq. (15.7) or Eq. (15.8) can be used to illustrate this trapping phenomenon. For a deep canopy with PAR reflectivity \( \rho_r = 0.1 \) and PAR transmissivity \( \tau_p = 0.1 \), the canopy reflectance \( \rho_c = 0.056 \). In the thermal wavelength band, if the leaf emissivity \( \varepsilon_L = 0.95 \), then the leaf reflectivity \( \rho_L = 0.03 \) because \( \varepsilon_L = 0.95 \). Using Eq. (15.7) for a deep canopy, \( \rho_{c, opt} = 0.013 \) or the emissivity of this deep canopy is 0.987. Therefore a deep canopy is much closer to a blackbody than the leaves that make it up, and this explains why dense canopies often are assumed to have thermal emissivities of 0.99 even though leaves may have lower emissivities.

15.13 Heterogeneous Canopies

The simplified radiative exchange principles described in this chapter apply to vegetative canopies with leaves that are randomly distributed throughout the canopy space. Such canopies of randomly-positioned leaves are often referred to as homogeneous because the probability of finding a leaf anywhere in the canopy space is independent of horizontal position. When leaves are not randomly distributed in space, the canopy is considered heterogeneous; and the character of the heterogeneity can take many forms. We briefly consider two approaches to characterizing heterogeneity.

1. Incorporate a clumping factor in the exponential extinction equations by replacing \( \Omega \) with \( \Omega(\psi) \) where \( \Omega(\psi) \) is the clumping factor that depends on zenith angle.

2. Assume leaves to be randomly distributed within the confines of some appropriate geometric volumes, which we refer to as canopy envelopes, to represent widely-spaced tree crowns or crop rows.

The clumping-factor approach has the advantage of making it possible to extend the previous equations for random canopies discussed earlier in this chapter to heterogeneous cases. For random canopies \( \Omega(\psi) = 1 \), clumped foliage has \( \Omega(\psi) < 1 \), and if foliage is more nearly uniformly spaced, \( \Omega(\psi) > 1 \). For forest canopies, which tend to be the most strongly clumped, the dependence of clumping factor on \( \psi \) can be
approximated by the following equations:

\[
\Omega(\psi) = \frac{\Omega(0) [1 - \Omega(0)] \exp(-2.2(\psi \delta))}{p = 3.80 - 0.46D \quad 1 \leq p \leq 3.34}
\]

\[
D = \frac{\text{crown depth}}{\text{crown diameter}}
\]

where \( \Omega(0) \) is the clumping factor when the canopy is viewed from nadir or when looking up out of the canopy toward the zenith. Table 15.4 contains some values of \( \Omega(0) \) for mature stands of several species.

Using Eq. (15.35), sunlit leaf area index can be estimated for a clumped canopy by using Eq. (15.23) and replacing \( L_c \) by \( \Omega(\psi)L_c \), and diffuse penetration estimated from the same substitution into Eq. (15.5). This approach is only approximate because the scattering equations imply a random distribution of leaves.

With conifers, an additional level of clumping occurs because needles are organized into shoots. Typically the hemi-surface area of a conifer shoot is about 1.3 to 2 times greater than the effective light-intercepting area of a shoot. This shoot clumping factor is quite important when canopy architecture is estimated from indirect measurements such as those discussed in the next section. Fassnacht et al. (1994) describe a method for estimating shoot clumping factors, and show that the difference in HSAL of fertilized and unfertilized pine stands is 30 percent; with 23 percent of this difference arising because fertilized shoots contain more needle surface area (more strongly clumped) and only seven percent difference arising from the increased light interception as determined by an indirect measurement of HSAL.

The second approach to characterizing heterogeneous canopies requires knowing the dimensions of geometric canopy envelopes that contain all the foliage. This approach is most useful when the spatial distribution of radiation beneath canopies is needed, such as in agroforestry where crop placement beneath tree crowns may be critical. If canopy envelopes are assumed to be ellipsoids, such as Norman and Wellis (1983) use, then a wide variety of crown shapes can be simulated. Given an ex-

**Table 15.4.** Canopy clumping factors in the zenith direction for mature, healthy stands of several species.

<table>
<thead>
<tr>
<th>Species (Location)</th>
<th>Hemisurface Area Index</th>
<th>( \Omega(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sugar Maple (Northern Wisconsin, U.S.A.)</td>
<td>5.5</td>
<td>~ 1</td>
</tr>
<tr>
<td>Oak (North Carolina, U.S.A.)</td>
<td>4</td>
<td>~ 1</td>
</tr>
<tr>
<td>Aspen (Saskatchewan, Canada)</td>
<td>3.5</td>
<td>1.5-2</td>
</tr>
<tr>
<td>Jack Pine (Saskatchewan, Canada)</td>
<td>2.5</td>
<td>3-4</td>
</tr>
<tr>
<td>Black Spruce (Saskatchewan, Canada)</td>
<td>6.5</td>
<td>5-6</td>
</tr>
</tbody>
</table>

\[ S(\psi, AZ) = \text{eq.} \]

where \( \mu \) is the leaf area (volume) and \( S(\psi, AZ) \) is of canopy envelopes between depth in the canopy or at Models of BRF in both several approaches are \( d \) and \( P \) eff (1991).

### 15.14 Indirect Sensing of Canopy Architecture

A description of canopy architecture involves determination of leaf area distribution and some mechanism needed to describe canopy structure, particularly forests, grasslands, and tundra bits of information. Plants and measuring alternative measurements of gap fraction as a function of \( L_c \) and \( \Omega(\psi) \). The estimates canopy architecture correspond to the order of effect of leaf angle distribution of zenith angle of gap fraction as a flat area can be done from Fig. 15.3 calculated to that best fit the data at measurements original method appears to be the most and must be done carefully. Approach is available.

Heterogeneous (no formation about the heterogeneity can be then additional methods beyond the mean angle. If heterogeneous ci that contain foliage is \( S(\psi, AZ) \) may be de...
Indirect Sensing of Canopy Architecture

When the canopy is viewed from e canopy toward the zenith. Table or mature stands of several species. Index can be estimated for clumped 
where the scattering equations imply a

Where clumping occurs because nearly the hemi-surface area of conifer than the effective light-intercepting 
measurements such as those of et al. (1994) describe a method for 
show that the difference in HSAI 
with 23 percent east 
Sided shoots contain more needles and only seven percent difference 
spire as determined by an indirect

15.14 Indirect Sensing of Canopy Architecture

A description of canopy architecture includes the position and orienta-
tion distributions of leaves, branches, stems, flowers, and fruit. For most 
canopies, leaves dominate the canopy space so leaf area index, leaf angle 
distributions and some measure of clumping provide most of the informa-
tion needed to describe canopy architecture. If we limit our discussion to 
canopies that approximate random positioning (most full-cover deciduous 
forests, grasslands, and crops), then LAI and s are the minimum essen-
tial hints of information. Direct measurements of LAI and s, by cutting 
plants and measuring leaf areas and angles are exceedingly laborious, so 
alternative measurement methods are desirable. Measurements of canopy 
gap fraction as a function of zenith angle can be used to obtain estimates of 
LAI and K_{0}. The strategy for using gap-fraction measurements to 
estimate canopy architecture is illustrated in Fig. 15.3. The gap fraction 
corresponds to the ordinate labeled transmission and the curves show the 
effect of leaf angle distribution (s) on transmission or gap fraction as a 
function of zenith angle for L_{0} = 1. Given a number of measurements of 
gap fraction as s as function of zenith angle, the curve that best fits the 
data can be chosen from numerous families of curves such as shown in 
Fig. 15.3 calculated for a range of LAI values. The values of s and L_{0} 
that best fit the data are assigned to the canopy where the gap-fraction measurements originated (Norman and Campbell, 1989). Although this 
method appears to be simple, the inversion procedure can be error prone 
and must be done carefully. Several commercial instruments that use this 
approach are available and have been discussed by Welles (1990).

Heterogeneous (nerrandom) canopies require some additional in-
formation about the characteristics of the heterogeneity. If canopy 
heterogeneity can be represented by the parameter Ω(ξ) in Eq. (15.35), 
then additional methods must be available for estimating Ω(ξ) (Chen, 1996) 
beyond the measurements of gap fraction as s function of zenith angle.

If heterogeneous canopies are composed of regular geometric shapes 
that contain foliage with large gaps between them, then the path length 
S(φ, ΑΖ) may be determined for the particular geometry (horizontal

<table>
<thead>
<tr>
<th>Hemi-Surface Area Index</th>
<th>D (m)</th>
<th>Ω (ξ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>~ 1</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>~ 1</td>
<td>0.9</td>
</tr>
<tr>
<td>3.5</td>
<td>1.5–2</td>
<td>0.7</td>
</tr>
<tr>
<td>2.5</td>
<td>3–4</td>
<td>0.5</td>
</tr>
<tr>
<td>6.5</td>
<td>5–6</td>
<td>0.4</td>
</tr>
</tbody>
</table>
cylinders for row crops or regularly spaced spheres for an orchard) and added to the inversion process (Welles, 1990).

Indirect methods exist to estimate $L_n$ and $z$ for a wide variety of homogeneous and heterogeneous canopies including prairies, row crops, deciduous, and coniferous forests. Even though direct destructive measurements remain the reference standards for evaluating the accuracy of indirect methods, indirect measurements are faster, easier, and provide better spatial sampling.

The indirect sensing of canopy architecture provides an example of how an improved understanding of the fundament of radiant exchange in vegetation has provided a solution to the practical problem of characterizing plant canopies.

References


Problems

15.1 A canopy with a mean height of 4 m and leaf area index of 2 includes a shaded area where leaf area index is 1. Such area with a zenith angle of 30°.

15.2 Find the daily fraction of incoming radiation by a plant leaf with leaf angle distribution.

15.3 If the ratio of two problems 15.1 is

$$\text{Assume that}$$
Light Environment of Plant Canopies

Problems

15.1. A canopy with a spherical leaf angle distribution has a total leaf area index of three. Find the flux density of PAR on soil and on shaded leaves at the bottom of the canopy, and the fraction of the leaves which are soil-lit and shaded. Assume a clear sky with a solar zenith angle of 30°.

15.2. Find the daily fractional transmission of PAR, NIR, and total solar radiation by a canopy with leaf area index, $L_a = 2$. Assume that the leaf angle distribution is approximated by an elliptical angle distribution with $x = 2$.

15.3. If the ratio of red to far red radiation at the top of the canopy in the problem 15.1 is 1, what is the ratio at the bottom of the canopy. Assume $a_{red} = 0.8$ and $a_{far red} = 0.2$. 


15.4. Using Eq. (15.25) and the coefficient values in the text for VIS and NIR wavelength bands, plot the BRF at and BRF as a function of view zenith angle for the principal plane of the sun between nadir and 60°. The principal plane occurs when ΔAZ = 0 or ΔAZ = π. The horizontal axis of the graph will go from zenith view angles of −60° to +60° with positive view angles corresponding to ΔAZ = 0 and negative view zenith angles corresponding to ΔAZ = π (with LAI = 2.6 and Ψ = 61°). For VIS, a = 1.49, b = 0.32, and c = 3.44, and NIR, a = 9.09, b = 7.62, and c = 46.8 (BRF in % and angles in radians). Considering that ΔAZ = 0 corresponds to having the sun behind the sensor and ΔAZ = π corresponds to the viewer looking toward the sun but downward at the canopy, explain the shape of this curve.

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>θ (°)</th>
<th>ρ (g/cm³)</th>
<th>v (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>44.6</td>
<td>13.3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>43.8</td>
<td>13.7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>43.0</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>42.3</td>
<td>14.6</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>41.6</td>
<td>15.1</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>40.9</td>
<td>15.5</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>40.2</td>
<td>16.0</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>39.5</td>
<td>16.4</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>38.9</td>
<td>16.9</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>38.3</td>
<td>17.4</td>
<td></td>
</tr>
</tbody>
</table>

Specific heat of air: \( c_p = \) 
Molecular mass of air: \( M_a = \) 
Molecular mass of water: \( M_w = \)

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>ρ (g/cm³)</th>
<th>λ (W/m·K)</th>
<th>λ (kJ/kg·°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.99087</td>
<td>45.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.00000</td>
<td>44.8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.99973</td>
<td>44.6</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.99823</td>
<td>44.1</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.99568</td>
<td>43.7</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.99225</td>
<td>43.4</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.98807</td>
<td>42.8</td>
<td></td>
</tr>
</tbody>
</table>

Specific heat of water
Latent heat of freezing
Thermodynamic freezing point
Appendix

Table A1. Temperature dependent properties of gases in air at 101 kPa.

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>ρ (mol m⁻³)</th>
<th>v (m³/m³)</th>
<th>Dp (m/s)</th>
<th>Dr (m/s)</th>
<th>Dr (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>44.6</td>
<td>13.3</td>
<td>18.6</td>
<td>21.2</td>
<td>13.0</td>
</tr>
<tr>
<td>5</td>
<td>43.8</td>
<td>13.7</td>
<td>20.5</td>
<td>21.9</td>
<td>14.3</td>
</tr>
<tr>
<td>10</td>
<td>43.0</td>
<td>14.2</td>
<td>20.1</td>
<td>22.6</td>
<td>14.8</td>
</tr>
<tr>
<td>15</td>
<td>42.3</td>
<td>14.6</td>
<td>20.8</td>
<td>22.3</td>
<td>15.3</td>
</tr>
<tr>
<td>20</td>
<td>41.6</td>
<td>15.1</td>
<td>21.4</td>
<td>24.0</td>
<td>15.7</td>
</tr>
<tr>
<td>25</td>
<td>40.9</td>
<td>15.5</td>
<td>22.0</td>
<td>24.7</td>
<td>16.7</td>
</tr>
<tr>
<td>30</td>
<td>40.2</td>
<td>16.0</td>
<td>22.7</td>
<td>25.4</td>
<td>17.1</td>
</tr>
<tr>
<td>35</td>
<td>39.5</td>
<td>16.4</td>
<td>23.3</td>
<td>26.2</td>
<td>17.2</td>
</tr>
<tr>
<td>40</td>
<td>38.9</td>
<td>16.9</td>
<td>24.0</td>
<td>26.9</td>
<td>17.7</td>
</tr>
<tr>
<td>45</td>
<td>38.3</td>
<td>17.3</td>
<td>24.7</td>
<td>27.7</td>
<td>18.2</td>
</tr>
</tbody>
</table>

Specific heat of air (c_p = 29.3 J mol⁻¹ K⁻¹)
Molecular mass of air: M_A = 28 g/mol.
Molar heat of water: M_W = 18 g/mol.

Table A2. Properties of water.

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>ρ (g/cm³)</th>
<th>k (W/m K)</th>
<th>α (10⁻⁶/m°C)</th>
<th>D_p (m²/s)</th>
<th>D_r (m²/s)</th>
<th>D_s (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.99987</td>
<td>45.0</td>
<td>1.79</td>
<td>0.184</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.00880</td>
<td>44.8</td>
<td>1.57</td>
<td>0.136</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.99973</td>
<td>44.4</td>
<td>1.31</td>
<td>0.140</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.99872</td>
<td>44.1</td>
<td>1.01</td>
<td>0.144</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>30</td>
<td>0.99558</td>
<td>43.7</td>
<td>0.80</td>
<td>0.148</td>
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</tr>
<tr>
<td>40</td>
<td>0.98225</td>
<td>43.4</td>
<td>0.66</td>
<td>0.151</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.98007</td>
<td>42.8</td>
<td>0.56</td>
<td>0.154</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Specific heat of water (c_p = 4.184 J g⁻¹ K⁻¹)
Latent heat of freezing (L) = 334 kJ kg⁻¹
Thermodynamic property constant at 20°C: c_p = 4.184 J g⁻¹ K⁻¹
### Table A3. Temperature dependence of saturation vapor pressure, slope of the vapor pressure function, black body emissance, radiative conductance, and clear sky emissivity.

<table>
<thead>
<tr>
<th>Temp K</th>
<th>Temp C</th>
<th>e(T) kPa</th>
<th>Pa</th>
<th>W m⁻²</th>
<th>m⁻² s⁻¹</th>
<th>eₜ</th>
</tr>
</thead>
<tbody>
<tr>
<td>268.2</td>
<td>-5</td>
<td>0.422</td>
<td>32</td>
<td>293</td>
<td>0.149</td>
<td>0.66</td>
</tr>
<tr>
<td>269.2</td>
<td>-4</td>
<td>0.455</td>
<td>34</td>
<td>298</td>
<td>0.151</td>
<td>0.67</td>
</tr>
<tr>
<td>270.2</td>
<td>-3</td>
<td>0.490</td>
<td>36</td>
<td>302</td>
<td>0.153</td>
<td>0.67</td>
</tr>
<tr>
<td>271.2</td>
<td>-2</td>
<td>0.525</td>
<td>39</td>
<td>307</td>
<td>0.154</td>
<td>0.68</td>
</tr>
<tr>
<td>272.2</td>
<td>-1</td>
<td>0.568</td>
<td>42</td>
<td>311</td>
<td>0.156</td>
<td>0.68</td>
</tr>
<tr>
<td>273.2</td>
<td>0</td>
<td>0.611</td>
<td>44</td>
<td>316</td>
<td>0.158</td>
<td>0.69</td>
</tr>
<tr>
<td>274.2</td>
<td>1</td>
<td>0.657</td>
<td>47</td>
<td>320</td>
<td>0.160</td>
<td>0.69</td>
</tr>
<tr>
<td>275.2</td>
<td>2</td>
<td>0.706</td>
<td>50</td>
<td>325</td>
<td>0.161</td>
<td>0.70</td>
</tr>
<tr>
<td>276.2</td>
<td>3</td>
<td>0.756</td>
<td>54</td>
<td>330</td>
<td>0.163</td>
<td>0.70</td>
</tr>
<tr>
<td>277.2</td>
<td>4</td>
<td>0.813</td>
<td>57</td>
<td>335</td>
<td>0.165</td>
<td>0.71</td>
</tr>
<tr>
<td>278.2</td>
<td>5</td>
<td>0.872</td>
<td>61</td>
<td>339</td>
<td>0.167</td>
<td>0.71</td>
</tr>
<tr>
<td>279.2</td>
<td>6</td>
<td>0.935</td>
<td>65</td>
<td>344</td>
<td>0.168</td>
<td>0.72</td>
</tr>
<tr>
<td>280.2</td>
<td>7</td>
<td>1.001</td>
<td>69</td>
<td>349</td>
<td>0.170</td>
<td>0.72</td>
</tr>
<tr>
<td>281.2</td>
<td>8</td>
<td>1.072</td>
<td>73</td>
<td>354</td>
<td>0.172</td>
<td>0.73</td>
</tr>
<tr>
<td>282.2</td>
<td>9</td>
<td>1.147</td>
<td>77</td>
<td>359</td>
<td>0.174</td>
<td>0.73</td>
</tr>
<tr>
<td>283.2</td>
<td>10</td>
<td>1.227</td>
<td>82</td>
<td>365</td>
<td>0.176</td>
<td>0.74</td>
</tr>
<tr>
<td>284.2</td>
<td>11</td>
<td>1.312</td>
<td>87</td>
<td>371</td>
<td>0.178</td>
<td>0.74</td>
</tr>
<tr>
<td>285.2</td>
<td>12</td>
<td>1.402</td>
<td>92</td>
<td>375</td>
<td>0.179</td>
<td>0.75</td>
</tr>
<tr>
<td>286.2</td>
<td>13</td>
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Appendix

Table A3. (continued)

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Table A4. Conversion factors

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<tr>
<td>Volume</td>
<td>1 m³ = 10⁶ cm³ = 10⁶ mm³</td>
</tr>
<tr>
<td>Density</td>
<td>1 Mg/m³ = 10³ kg/m³ = 1 g/cm³</td>
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<tr>
<td>Pressure</td>
<td>1 kPa = 10³ Pa</td>
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<tr>
<td>Heat</td>
<td>1 Joule = 1.0038 cal</td>
</tr>
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<td>Watt = 3.4121 kcal/h</td>
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<td>Heat flux density</td>
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<tr>
<td></td>
<td>1 W/m² = 1.433 x 10⁻⁷ cal cm⁻² min⁻¹</td>
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<td></td>
<td>1 W/m² = 2.388 x 10⁻⁶ cal cm⁻² s⁻¹</td>
</tr>
</tbody>
</table>

Table A5. Physical constants

| Speed of light in vacuum | 2.997925 x 10⁸ m/s |
| Avogadro constant | 6.02252 x 10²³ mol⁻¹ |
| Planck constant | 6.62607 x 10⁻³⁴ J s |
| Gas constant | 8.314 J mol⁻¹ K⁻¹ |
| Boltzmann constant | 1.38054 x 10⁻²³ J K⁻¹ |
| Stefan-Boltzmann constant | 5.6703 x 10⁻⁸ W m⁻² K⁻⁴ |