Sequences and Series Unit Lesson Plan

For this unit lesson plan, I am teaching a Precalculus class about Sequences and Series. This is a topic that is generally not covered very thoroughly, if at all, in Precalc, instead being reserved for Calculus 2 students. However, it is part of the NC DPI Objectives for a Precalculus course, falling under objective 2.07, which states students must find the sum of finite and infinite sequences (beginning with day 3 of the unit, where the concept of a series is introduced), determine whether a given series converges or diverges (beginning also with day 3, where series are introduced), and translate between recursive and explicit representation of both series and sequences (this will be done throughout the entire course, and will play a very strong role on the final assessment). The first two days of the unit are an overview of sequences themselves, since many students will have never seen one before, and the few that have will not have seen it in a while. It also falls strongly into the NCTM Standards of Number and Operations, Algebra, Problem Solving, Reasoning and Proof, Communication, Connections, Representation. I aim to hit the Number and Operations standard through understanding of the nature of sequences and series, which are inherently numerical; students will be given actual data points to turn into sequences, as opposed to just being told that the sequence is a formula. This should strengthen the understanding that a sequence is more than a formula— it’s a representation of a string of real world data, organized into a way that makes mathematical operations easier. Algebra will be met through manipulation of derived sequence functions to find series convergence, as well as algebraic manipulation to attempt to find a sequence for a set of real numbers. I will attempt to encourage problem solving through making sure each student has a complete toolbox of tests for convergence/divergence, but when assigning a problem, no indication of the proper method will be given. Students may choose whichever method is most necessary to solve for the answer. Since in math there are often many ways to get the correct answer, one must gain proficiency at identifying the most appropriate method for solving, lest they waste time. Reasoning and proofs will be important in that half their final assessment grade is based on their ability to show clearly and logically the process by which a set of numbers becomes a sequence, becomes a series, and equals a value. For all other problems, a certain portion of their grade will also depend upon their ability to explain their rationale in a clear and logically sound manner. Communication is incorporated in myriad ways, most prominently in the math journals written at the beginning of every class and the final assessment project, where the students will be writing out their understanding of the material, explaining the development of the material they are being tested on. I hope they will draw connections both through my chosen progression through the material, where it is shown that data becomes sequence, becomes series, becomes a single number (or doesn’t depending on the case). Possibly more important than in many other sections, representation will play a very large role due to the fact that there are a few new methods of notation they are required to learn to be effective at sequences and series (sigma notation, set notation). They should also be able to recognize by the end of the unit that sequences’ primary purpose, and thereby series’ as well, is to model real-world data for ease of manipulation, understanding, and quantifying. While it is not impossible that students have seen something similar to this in the past, it is however very unlikely. As such, the prerequisites are slightly different than for most precalc sections— they simply need to have a strong understanding of functions and the number system. With both of those, solving most sequences becomes possible.
More complicated problems may require more of a grab-bag of topics, including solid understanding of trig functions and the like. As this lesson will likely come very near, if not at, the end of the course, this serves as a good way to pull everything the students have learned in the course together one more time. More than anything else, I think the process of finding whether a given series converges or diverges, and if it converges, to what value, will be the most difficult. This is partly exacerbated by the educational level—many types of series are nearly impossible to evaluate without calculus. As such, the problems assigned to the students need to be carefully selected so as to make sure they are possible at their current learning level. Having read through many textbooks for Precalculus, I have come to the decision that for this section, I would be writing my own problems and lessons, as well as notes. The method that the textbooks took seemed very confusing, and did not foster solid growth and understanding in the subject area—they often seemed tacked in for completeness. The teaching method that I have chosen to follow is a combination of inquiry-based and scaffolding, where, as often as possible, I am playing a minimal role in front of the class, and the students are leading the class through discussion and questioning. I have chosen this method because it seems to promote personal responsibility for learning best, as well as cause a greater understanding of the material through critically thinking about each section of the unit, instead of just taking it for granted. The classroom as a whole will be set up in such a manner that each student will have a “partner” at their table with them. The tables of two will sit in rows in the classroom, facing away from the door and the clock, and toward the overhead projector screen. Students, during times where classwork in being done, will be encouraged to work closely with their table partner, as their understandings individually will be strengthened by the sharing of ideas, and one student may have a way of explaining the material to another that is more effective for some students.

Technology will play a noticeable, but minor role in this section of the course. Each student will be allowed to use their calculators, but will be encouraged for many sections to try it without them, especially considering this unit does not lend itself strongly to standard TI calculator use. Overheads of problems, as well as the internet, will be used to further incorporate technology into the classroom. Common misconceptions for this material will occur primarily in the convergence and divergence section, the most common one likely being misinterpretation of the strength of the Nth Term Test for Divergence. Other likely trouble spots will be the transition from sequences to series, and recalling the differences in nature between the two. Care will be taken to avoid these misconceptions and correct them if they do arise—they will ideally be noticed quickly thanks to the daily journals, where the students will write a short paragraph at the beginning of each class, explaining what we learned in the previous lesson. I will then evaluate the explanation for mathematical correctness, and use the most common errors in those to guide my corrections to their understanding. In regard to special education, the two broader categories, academically gifted and behaviorally/learning disabled will be handled with care. As Precalculus is generally considered to be an honors level class at most high schools, the problems will be made with the aim of being challenging to the whole class, academically gifted students as well. If a student needs additional attention or care, I will make sure to have additional time during lunch and before and after school to meet with them and work with it. Further, if I notice a student is struggling but not asking to come in for additional help, I will suggest to them that perhaps they could benefit from some extra tutorial. All students will be held to the same behavioral standard, but as with any situation, exceptions can be made. They will need to be respectful of one another, myself, and the classroom and follow instructions when asked, and that’s about it. As the students will largely be leading the discourse in the classroom, with
guidance from me, a slightly more open, discussion-based course structure is being used here. If a student desperately needs something explained, I will, of course, oblige them, but not without exhausting available options to lead them to the answer on their own, first. Students will be assessed through daily math journals, nightly homework, class participation, and a two-part final assessment testing both rationale and ability to perform requested mathematical operations. The journals are a short, 5 minute assignment at the beginning of every class, designed to get them to remember what we last did in class, so it is fresh in their minds when we begin to build on it. Further, it allows me to see how their understanding of the material is evolving, and insure that it does so in a correct manner. The nightly homework will only be a few problems, between 4-5, and will be challenging, or in some cases will be a real world application they need to search for. The homework will be graded based on effort, but whether an answer is correct will be marked on the paper. Further, at the beginning of each lesson, the students will work out, on the board, student-selected problems from the homework, and be in charge of explaining to the class their method and answer. This, as well as active participation in the class discussions, will create the class participation grade. The final assessment consists of a 10 question formal test where a string of numbers is given, and the students must find a sequence to represent it, find a series to represent that sequence, and explain whether the series converges or diverges. On the test, the value of convergence will not be tested, though it will be discussed in class. Points will be awarded based on both methodology and on correct answer. The other half of the assessment will be a writing-based project, where the students will have to explain what sequences and series are, how they are used, and how to tell if a sequence converges or diverges. They will also have to write about how sequences and series can be used in real life, a topic which will be emphasized during class discussions. Both halves of the assessment will be weighted evenly, and the project will be due on the day of the test. The final goal of the unit is, obviously, for the students to have learned sequences and series, but also to gain an appreciation for the power that they hold, and to encourage self-exploration of mathematics material and to become lifelong math learners.
Sequences Day 1: 90 minutes

-Daily Writing Warm-Up: 5-10 minutes at beginning of class writing a short paragraph about what we learned the previous day. In this case, the previous day would have been the last unit’s test post-meet, so we are doing something different, which I will explain at the start of class. I will tell them that we are starting sequences and series, and I’ll ask them to write about what they think a sequence is, and what they think a series is, both in general, and in math terms. During this time, I will look at the previous day’s homework (in this case, the last test’s corrections).

-Introduction of Web-based textbook and project: 5 minutes; I will explain to the students that for this section, instead of the textbook, we’ll be using my website as our “reference source.” The website will be, in essence, an improved version of the Weebly site I made in class: http://seriessequencessp2010.weebly.com/index.html This site contains a majority of the information that we will cover during the unit. After that, I’ll tell the students about their unit project, the paper about sequences and series. The rubric will be handed out to them, and the due date of “the day of the test” will be announced. Any questions about the project will be answered.

- Class Discussion of Sequence First Impressions: 5-10 minutes; after the students write their journals, I’ll ask for volunteers to share their first impressions. I will emphasize at this stage that there is no wrong answer to what they think a sequence is. Once many students have gotten to share their opinions, I’ll move on.

-Numerical representation of sequences: 15 minutes; I will call on a student and ask them to write 10 random whole numbers on the board, in a straight line. After they have done so, I’ll ask them to assign each number they wrote a natural number, starting with 1 for the first, and increasing by one for each subsequent number. I’ll then ask the class if they think that what the student wrote was a sequence and why. I’ll collect answers from many students, until I have a solid consensus amongst the students. This process will repeat with 2 more students, 1 of which will write fractions, one of which will write decimals. The discussion will occur each time a sequence is written on the board. After the third sequence, I’ll inform the class that a sequence in math is any list of numbers that you can assign the natural numbers to.

-Recursive representation of sequences:15 minutes; I will write up my own sequence on the board, 1/n, explicitly, and ask the students if they think that it’s possible to come up with an equation that would give us the set of numbers. After some discussion, I will suggest that we use the index of the numbers, the natural number that we assign each one, to try to do it. I’ll then ask the students if they can see any sort of pattern between one number to the next. When they identify that the denominator is increasing by 1 each time, I’ll encourage them and ask them to tell me the equation that will work for it, if the denominator is just increasing by one each time. We’ll continue discussing it until either the 20 minutes have been used or someone suggests 1/n.
-Rules about sequences: 15 minutes; I’ll ask the class if they can think of any circumstances where it wouldn’t be possible to create a recursively represented sequence. If necessary, I’ll have them come to the board and write the numbers themselves, and then try to work it out. Emphasis will be put at the end of the discussion upon the necessity of a predictable pattern behind the sequence being necessary.

-Applications of sequences: 15 minutes; Using the topic of patterns, I’ll ask the students to talk about real life patterns that they might want to find an equation, and thus a sequence, for. This should allow them a chance to talk about their interests, as patterns are common in a vast majority of hobbies and the like.

-Daily Assessment: 5 minutes to explain homework and dismiss class. HW will be to find a real life example of a sequence and come in to class ready to talk about it.
Sequences Day 2: 90 minutes

-Daily Writing Warm-Up: 5-10 minutes at beginning of class writing a short paragraph about what we learned the previous day.

-Homework Discussion: 10 minutes; I’ll ask for volunteers who want to share the applications they came up with. Once again, emphasis in my comments will be placed on the concept of a pattern, and wanting to, in real terms, find the pattern.

-Finite and Infinite Sequences: 20 minutes; I’ll ask the students to think about what would happen if n never stopped increasing. After some discussion, I’ll indicate that such a sequence is “infinite”, and the only way to be able to accurately write an infinite series is to have it in recursive form. As we’ll have just finished limits, I will ask the students to review limits by telling me what the limit of a few functions I write on the board are as n goes to infinity. We’ll then discuss finite sequences, where I’ll ask the students what would be different if n stopped at some point. Once someone chimes in that this is an endpoint, or if it’s taking too long I will say it, and we’ll move on. To motivate them, I will point out that understanding how a pattern behaves at its endpoints (even if that endpoint is infinite). This is important because what a pattern will do long-term is generally the point of a pattern.

-Factorials: 20 minutes; I’ll write 5! on the board, and then ask anyone if they know what it means. I’ll make the “Well, it doesn’t mean that 5 is really excited” joke that nearly every math teacher ever has made, and then explain to the class the what it really means is $5\times4\times3\times2\times1$, or 5 times every natural number less than 5. Then, we’ll look at our calculators and see that there is a factorial button on there that functions just like we defined. I’ll then ask the students if they can think of a way this might be helpful to us in sequences. If a student mentions using n!, then we’ll have a discussion about the types of sequences that would pop up using that. If no one suggests that, I’ll ask the class what they think about using n!, and what they think it will mean. Once we have defined n! as $(n)(n-1)(n-2)\ldots(1)$, I’ll move onto asking what happens if you divide factorials. I will write $(n+1)!/n!$ on the board, and ask for a volunteer to come and show me what it equals. I’ll take volunteers until we have shown it equals $n+1$ by expanding the terms of both out and seeing they will cancel. I will ask if there are any general rules that we can derive from the relation we just noticed. Once $(n+1)n!=(n+1)!$ is suggested, we’ll move on to the next part of the lesson.

-Cooperative classwork: 25 minutes; The students will have a sheet of 5 sets of numbers, and will have to find the sequence for them. They may work with their tablemate. I will circulate, and ask students how they reached their answers, and be available for questions.

-Daily Assessment: 5 minutes to explain homework and dismiss class to begin work on homework if there is time. HW will be to complete the classwork, if it wasn’t completed in class.
Series Day 3: 90 minutes

-Daily Writing Warm-Up: 5-10 minutes at beginning of class writing a short paragraph about what we learned the previous day. During this time, I will check for the previous night’s homework.

-Homework Questions: 20 minutes; Upon walking in the classroom, students write the number of HW problems they would like worked out on the top left corner of the chalkboard. I then ask students who completed the problems to come to the front of the classroom and work the problems on the board (all at the same time). Once that’s done, they will walk me through what they did, and if their answer or method was wrong, I’ll help them along until the correct answer and method to that answer to all requested problems is on the board.

-Series: 15 minutes; I’ll ask the class what they think we should do if we want to find the sum of every value of the sequence. I’ll ask the students to brainstorm situations where they’d want to know the total value of a sequence (the sum of a pattern)- seeking their own motivation. When we’ve come to the conclusion that all we have to do is add them together, I’ll point out “But what do we do if it’s an infinite sequence? Won’t we have to add infinitely?” I’ll let the students discuss it for a while, and when everyone has had a say on what to try, I’ll move on.

-Sigma Notation: 5 minutes; To answer the question of what to do with infinite sequences, I’ll show the class sigma notation, explaining how it works and so on, and defining it as a sum. I’ll also show that it doesn’t have to be infinite.

-Series Properties: 15 minutes; After a brief review of the commutative and distributive properties, I’ll ask how we can apply these to the sigma sum. We’ll move on when it is pointed out that we can pull constants outside of the sigma, as well as splitting a sum of the form sigma(a_n+b_n) into sigma a_n + sigma b_n.

-Partial Fractions: 15 minutes; In the hopes of prepping them for the next day where they will learn telescoping series (which frequently need partial fractions), I will quickly run through partial fractions. After writing out the initial condition, I will have the students try to work through the example of it- guiding them where I need to.

-Daily Assignment: 5 minutes before the bell, I’ll stop teaching and hand out their homework worksheet. 5 problems.
Series Day 4: 90 minutes

-Daily Writing Warm-Up: 5-10 minutes at beginning of class writing a short paragraph about what we learned the previous day. During this time, I will check for the previous night’s homework.

-Homework Questions: 20 minutes; Upon walking in the classroom, students write the number of HW problems they would like worked out on the top left corner of the chalkboard. I then ask students who completed the problems to come to the front of the classroom and work the problems on the board (all at the same time). Once that’s done, they will walk me through what they did, and if their answer or method was wrong, I’ll help them along until the correct answer and method to that answer to all requested problems is on the board.

-Convergence: 10 minutes; Opening conversation will be about convergence, which means that the sum is a finite value. We’ll talk about what kind of sequences are converge briefly, and create together a few rules, like the fact that as n goes to infinity, a_n must decrease. We’ll use these to create our convergence tests. I will remind them of the reasons they stated the previous day for why finding the value of convergence to help motivate them.

-Convergence Tests: 35 minutes; On the board, I’ll write down the nth term test for divergence, the ratio test, the root test, basic comparison test, limit comparison test, geometric series, telescoping series, and the alternating series test. We’ll then have a class discussion on why each of them works- I’ll ask what property of series that we’ve discussed makes that specific test work.

-Finding the sum: 10 minutes; For the telescoping series, I’ll show quickly how we can use partial sums to make telescoping series in many cases, and that we can find the sum of a telescoping series by writing out the first few terms, and noticing the pattern of cancellation. We then take the non-cancelling terms, and that’s the sum. For a geometric series, I will show them that for a geometric series a*(r)^n it converges to a/(1-r).

-Daily Assignment: 5 minutes before the bell, I’ll stop teaching and hand out their homework worksheet. 5 problems.
Series Day 5: 90 minutes

-Daily Writing Warm-Up: 5-10 minutes at beginning of class writing a short paragraph about what we learned the previous day. During this time, I will check for the previous night’s homework.

-Homework Questions: 20 minutes; Upon walking in the classroom, students write the number of HW problems they would like worked out on the top left corner of the chalkboard. I then ask students who completed the problems to come to the front of the classroom and work the problems on the board (all at the same time). Once that’s done, they will walk me through what they did, and if their answer or method was wrong, I’ll help them along until the correct answer and method to that answer to all requested problems is on the board.

-Group activity: 5 minutes to explain the situation and have them form groups. Constructing series based on a given sequence, and stating whether the series converges. If geometric or telescoping, state what it converges to. Work in teams of three. They will receive sheet of 20 very difficult problems. You have 50 minutes to get as many as possible. Teams that get the most questions right win a homework pass for that night’s homework.

-Inquiry: I will not be lecturing at all that day- they can ask me questions on how to do the work, and if they state something, I’ll ask them a question back.

-Motivation: Free homework pass (I feel confident handing this out, because if they win the competition, they understand the homework for that night anyway.). Further, the motivation from the previous day of knowing how much a pattern will occur still exists here.

--If a team happens to complete the sheet before the 50 minutes are up, and has them all correct, then I will have them begin work on their HW.

-Daily Assessment: 5 minutes to explain homework and dismiss class to begin work on homework if there is time. HW will be about 5 problems I assign, and to finish/correct any problems you got wrong or didn’t complete in the assignment during class that day.
Review Day 6: 90 minutes

-Daily Writing Warm-Up: 5-10 minutes at beginning of class writing a short paragraph about what we learned the previous day. During this time, I will check for the previous night’s homework.

-Student-Question Work Day: 75 minutes; Review day is run entirely by the students. If I am asked to work a problem on the board, I will do so. Their only assignments are to work on their projects, to study over their notes and review, and ask me any questions they don’t understand. If the students request it, we will have a more formal review, where I will ask the students to supply me with definitions of the various concepts taught, what the different tests are, etc.

-Daily Assessment: 5 minutes before the bell, I’ll suggest that the students read over the website once more. I will also remind them that their project is due the next day, and that working on it and reading it over should be a good way to review the fundamentals of sequences and series for the next day’s test.
**Test Day 7: 90 minutes**

- Daily Writing Warm-Up: 5-10 minutes at beginning of class writing a short paragraph about what we learned the previous day.

- Project Turn-In and Hand Out Tests:<1 minute; I will ask the students to have their projects on their table. I will give them their test, face down, and pick up their project. As I do this, I’ll briefly explain the format of the test and what they need to do. Once all students have a test, they can begin.

- Test: 75 minutes; I will sit at my desk and be ready to answer any questions about the test the students have.

- Daily Assessment: 5 minutes to explain homework and dismiss class to begin work on homework if there is time. HW will be 5 problems I assign.
Post-Test Discussion Day 8: 90 minutes

-Daily Writing Warm-Up: 5-10 minutes at beginning of class writing a short paragraph about how they think the test went.

-Discussion of the Projects: 20 minutes; Write project grade distribution of projects on the board. Tell students what mistakes were commonly made, which sections people scored best and worst on. Hand back projects graded.

-Discussion of Tests: 55 minutes; Write test grade distribution on the board. Hand back tests to students. Tell them which problems were most commonly missed. Take volunteers to work those problems on the board.

-Daily Assessment: 5 minutes to explain homework and dismiss class. HW will be to correct your test.
Final Assessment:

2 part- half in-class assessment (test), half descriptive project.

*Assessment- Full class period, find the sequence for 10 sets of numbers, and state whether the series of the sequence converges (and if possible using the tools we currently have, what it converges to) and why. Different types of test will have to be applied. Each problem is worth 5 points- 1 for the sequence, 1 point for correct syntax for the series, 1 point for the correct answer to the “converge or diverge” question, and 2 points for a correct explanation of why.

*Project- Paper with illustrative examples that they should have been working on the entire unit. They should use the daily journals they’ve been writing as a source of material, as well as the homeworks and the textbook to create their own how-to book on series and sequences, explaining what a sequence is, what a series is, convergence, and anything relevant to learning about them (sigma notation, set brackets, etc). As a conclusion, I want them to write a bit about what sequences and series can be used for in real life (to discover their own motivation for its use). This will also be worth 50 points, combining with test to create a full 100 point assignment that counts as a test. It will be due the day of the formal assessment. The primary grading point in this assignment is the logic behind why each thing was done, why each convergence test works, etc. Papers should be about 5 pages or so in size 12 font, single spaced, and contain large amounts of examples. The purpose of this is, after grading, they will be returned to the students as study material for their final. This is also an attempt to grade their qualitative understanding of the material, as opposed to just their quantitative via a test.
Sequences and Series Paper Rubric:

While there is, technically, no length minimum, if you have done a thorough job, your paper should be at least 4 pages long. The final draft will be due on the same day as your test. If you need any advising on how to handle this paper, or help with using the equation builder in Word, please come talk to me before or after class, and I’ll be happy to help you out.

Sequences 20pts

-What is a sequence? 5 pts
-How to construct a sequence from a string of numbers 10 pts
-Infinite Sequences vs. Finite Sequences addressed 5 pts

Series 25 pts

-What is a series? 5 pts
-How do I construct a series from a sequence? 5 pts
-What does it mean to converge? 5 pts
-How do I test for convergence? 10 pts

Conclusion 5 pts

-Did you pull everything together well, did you answer the question of “how is this useful to me?”

Total 50 pts
CW/HW Day 2

1. $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{42}$, …

2. 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, …, $\frac{1}{100}$,…

3. 3, 5, 7, 9, 11, 13, …

4. 1, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, …, $\frac{1}{100}$, …

5. 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, $\frac{1}{256}$, …
HW Day 3

Write in sigma notation in simplest form.

1. $\frac{1}{n}$, $n$ to infinity

2. $n!$, $n$ to 10

3. $\frac{3}{n^2 - 2n + 1}$, $n$ to infinity

4. $\frac{n}{n^2}$, $n$ to 6

5. $\frac{m}{n^2}$, $n$ to infinity, $m$ constant
HW Day 4

State whether the given series converges or diverges, and why.

1. \(\frac{1}{\ln(n)^n}\)

2. \(\frac{7}{n^2+n}\)

3. \(\frac{1}{2^n}\)

4. \((-1)^n \left(\frac{1}{n}\right)\)

5. \(\frac{1}{((n(n-2)(n+1)))^{\frac{1}{2}}}\)
In-Class Game Questions:

1. \( \{ \frac{1}{n!} \} \)

2. \( \{ (-1)^n \left( \frac{1}{n} \right) \} \)

3. \( \{ \frac{1}{2^n} \} \)

4. \( \{ \frac{7}{n^2+n} \} \)

5. \( \{ \frac{1}{n!n} \} \)

6. \( \{ \frac{2^n}{n} \} \)

7. \( \{ \frac{n}{n^n} \} \)

8. \( \{ \frac{1}{n} \} \)

9. \( \{ \frac{1}{(n(n-2)(n+1))^2} \} \)

10. \( \{ \frac{3n^2}{n^2} \} \)
11. \(\{n^{\frac{3}{4}}\}\)

12. \(\{\frac{1}{n^2}\}\)

13. \(\{\frac{1}{\ln(n)^n}\}\)

14. \(\{\sin(n)\}\)

15. \(\{\frac{n+1}{n^2+2n+1}\}\)

16. \(\{n^2\}\)

17. \(\{\frac{n!}{(n+1)!}\}\)

18. \(\{\frac{n}{x^n}\}\)

19. \(\{((-2)^n) \cdot \frac{n^2}{(n^2)+4}\}\)

20. \(\{\frac{1}{x^n}\}\)
HW Day 5 Questions:

Show that the following infinite sequences converge or diverge, and why.

1. \( \frac{1}{n^2} \)

2. \( \frac{1}{n^4} \)

3. \( \left( \frac{1}{3 \ln(n)^n} \right) \)

4. \( \frac{1}{n^2 + n} \)

5. \( \frac{1}{\left( (n)(n-2)(n+1)(n-4) \right)^2} \)
**Series and Sequences Test:**

For each problem, do the following:

1 point: Find a recursive representation of the sequence if it is explicit, find an explicit representation if it is recursive.

1 point: Correct syntax on a sigma-notated series

1 point: Does the series converge or diverge?

2 points: Show all work and label which method of convergence testing you used, and why.

1. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots, \frac{1}{100}, \ldots$

2. $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \frac{1}{42}, \ldots$

3. $2, -\frac{1}{2}, \frac{2}{9}, -\frac{1}{8}, \ldots$

4. $\frac{1}{n^{n-1}}$
5. $1, 1/4, 1/16, 1/64, 1/256, \ldots$

6. $1/n!$

7. $n^{-\frac{3}{4}}$

8. $1, 1/2, 1/3, 1/4, 1/5, 1/6, \ldots, 1/100,\ldots$

9. $\frac{1}{\ln(n)^n}$

10. $1, 1/2, 1/3, 1/4, 1/5, 1/6, \ldots, 1/100$
In the end, my hope is that this approach to sequences and series, though simplified from what the students would see in a calculus class, will be enough to emphasize the importance of patterns, and analysis of patterns to determine the end result of it. If you have a success rate modeled by sequence a_n, you really should want to know the sum of all a_n, to see how much success you’re going to have. The inquiry/scaffolding based method where my and the students’ questions guide the lesson, then the students help each other learn the material enhances the social structure of school, encourages cooperation, and should increase overall understanding. Assessment was made as varied as possible to encourage multiple learning styles to encounter success- the class participation grade emphasizes hands-on and audio learning, the discussion encourages audio and visual learning (as the discussions entail students writing on the board), the journals encourage the more literarily inclined, and the standard test helps out the more quantitative. The partner work enables the students to practice sharing ideas about math in a non-judgemental environment, and thereby gain confidence in their ability to analyze and think critically about mathematical topics. It is through this that I hope to meet both the DPI and the NCTM standards for this unit. The DPI standards are essentially just to teach the material, but the NCTM standards also require the students to be able to describe the underlying processes in the math, which is what the class discussions are meant to promote. Through intense practice with the material and constant discourse, students will come to be able to describe in various media the nature of patterns, sequences, and series, and will likely gain more technical ability with math in general to boot. As such, I’m confident that at the end of the unit, the students having completed this set of lessons will be able to express more clearly, more intelligently, the material they learned than students who attended a standard lecture alone. This is, I suppose, the point of this exercise- to find a more interesting, more engaging way to teach traditionally dry subjects. Since the students are, within reason, essentially in charge of what they are learning here (with guidance), it should serve to keep their attention. Further, the assignments to find real world uses for what they are learning should prove to them first hand that this material is extremely useful to them, allowing them to self-motivate- an important skill for higher learning in any field. Our focus is, of course, to teach math, but helping kids become lifelong learners in the process is certainly a desirable side-effect. In this way, I hope to have a classroom that fosters each student’s individual growth and interests, as well as also instilling in them a strong love of math and a respect for all that can be done through math.