Hint for Question 3

Let $\epsilon > 0$ be arbitrary. Since $f_n$ is uniformly integrable, we can find a $L$ such that $\sup_n \int f_n 1_{f_n > L} d\mu < \epsilon$. Write

\[ \int f_n d\mu = \int f_n 1_{f_n > L} d\mu + \int f_n 1_{f_n \leq L} d\mu \leq \epsilon + \int (f_n 1_{f_n \leq L} - L) d\mu + L\mu(\Omega). \]

Now take lim sup both sides and try to apply Fatou's lemma.