## Sports Betting in US (Table 1)

<table>
<thead>
<tr>
<th>Type</th>
<th>Annual Bet Volume</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Legal</td>
<td>$2b</td>
<td>Nevada</td>
</tr>
<tr>
<td>Internet</td>
<td>$20-$30b</td>
<td>Internet/offshore</td>
</tr>
<tr>
<td>Illegal</td>
<td>$80-$380b</td>
<td>Ubiquitous</td>
</tr>
</tbody>
</table>
Legal Sports Wagering in Nevada

<table>
<thead>
<tr>
<th>Year</th>
<th>Bet Volume</th>
<th>Winnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>$2,500,000,000</td>
<td>$220,000,000</td>
</tr>
<tr>
<td>1982</td>
<td>$15,628,000,000</td>
<td>$1,329,300,000</td>
</tr>
<tr>
<td>1985</td>
<td>$20,087,000,000</td>
<td>$1,714,102,000</td>
</tr>
<tr>
<td>1989</td>
<td>$29,506,901,093</td>
<td>$2,517,938,875</td>
</tr>
<tr>
<td>1995</td>
<td>$84,000,000,000</td>
<td>NA</td>
</tr>
<tr>
<td>1999</td>
<td>$80-$380,000,000,000</td>
<td>NA</td>
</tr>
</tbody>
</table>

Illegal Sports Wagering Estimates

<table>
<thead>
<tr>
<th>Year</th>
<th>Bet Volume</th>
<th>Winnings</th>
<th>% Hold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>0</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>1996</td>
<td>2</td>
<td>$60,000,000</td>
<td>NA</td>
</tr>
<tr>
<td>1997</td>
<td>NA</td>
<td>$600,000,000</td>
<td>NA</td>
</tr>
<tr>
<td>1998</td>
<td>53</td>
<td>NA</td>
<td>$260,000,000</td>
</tr>
<tr>
<td>1999</td>
<td>139</td>
<td>$14,300,000,000</td>
<td>$640,000,000</td>
</tr>
<tr>
<td>2000</td>
<td>204</td>
<td>NA</td>
<td>$1,000,000,000</td>
</tr>
</tbody>
</table>

Internet Sports Wagering

Table 1: Sports Wagering Trends (excludes horse racing and greyhounds)
Motivation

- Interesting environment
  - large illegal with legal fringe
  - similar: music (P2P), prostitution

- Key features
  - all segments provide same product (bets, not amenities)
  - yet behave/organized in different ways

- Goal is to explain source of these differences
  → focus on illegal vs internet
  → both unfettered markets
  → but different legal status in home jurisdiction

- Legal environment
  - internets not concerned with arrest
  - illegals can be arrested (pr=5%/yr) → asset confiscation but no jail
  - NB: bettors not arrested but limited access to illegals
This paper

1. Explain Industrial Organization illegal market
   - e.g. firm size, behavior, contracts, . . .
   - illegal status ↑ info issues and ↓ competitive pressures
   - *qualitative evidence in the spirit of the law & econ lit*

2. Economic incentives are central
   - not obvious: little formal education (no college let alone MBA)
   - not brainwashed into using standard structure and conduct
   - inconsistent CW for many practices
     *quantitative evidence here (prices, fin. diversification)*

3. Provides new support for economic theory of crime (Becker, 1968)
   - lit focused on labor issues
     (entry decision: wages/UE, deterrence)
   - new labor evidence here
     (low eqbm income → consistent with RL prefs)
   - additional evidence on IO
     incentives shape criminal enterprise
Data Used in Analysis – Illegal Bookies

- 6 New York city bookies arrested 1995-2000
  [videos of arrests: www.unc.edu/~cigar]
  - wide range sizes:
    * $5-$150 million/yr bet volume
    * 15-300 bettors
    * 0-80 hired employees
  - long-lived (min=2 yrs; average=5 yrs)

- Very detailed internal records (Table 2):
  - observation period: 1 week - 3 months
  - individual bets
  - individual bettors
  - employee records
  - other:
    PC backup, court docs, audio tapes,
    address book, bank statements, ...
<table>
<thead>
<tr>
<th>Location</th>
<th>Observation Period</th>
<th>Number of Bettors</th>
<th>Number of Bets</th>
<th>Bet Volume</th>
<th>Available Records</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bookie 0</td>
<td>Brooklyn, Manhattan, Queens, Staten Island, NY</td>
<td>7/1/95-8/4/95 (22 days with bets)</td>
<td>280 active</td>
<td>10,252</td>
<td>$11,459,310</td>
</tr>
<tr>
<td>Bookie 1</td>
<td>Staten Island, NY</td>
<td>12/16/96-1/12/97 (17 days with bets)</td>
<td>54</td>
<td>1,332</td>
<td>$314,810</td>
</tr>
<tr>
<td>Bookie 2</td>
<td>Newark, NJ</td>
<td>8/31/92-9/30/92 (31 days with bets)</td>
<td>≈ 500</td>
<td>8,400</td>
<td>≈ $9,700,000</td>
</tr>
<tr>
<td>Bookie 3</td>
<td>Brooklyn, NY</td>
<td>1/17/00-1/23/00 (7 days with bets)</td>
<td>112</td>
<td>1,308</td>
<td>$479,433</td>
</tr>
<tr>
<td>Bookie 4</td>
<td>Brooklyn, NY</td>
<td>1/17/00-1/23/00 (7 days with bets)</td>
<td>15</td>
<td>134</td>
<td>$220,880</td>
</tr>
<tr>
<td>Bookie 5</td>
<td>Secaucus, NJ</td>
<td>12/30/96-3/14/97 (77 days with bets)</td>
<td>249</td>
<td>—</td>
<td>$4,732,879</td>
</tr>
<tr>
<td>Bookie 6</td>
<td>Philadelphia, PA</td>
<td>11/1/97</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 2: Data Overview
Data Used in Analysis – Internet Bookies

• Internal docs + discussions w/CEOs of two books (public, private)
  – large: $1^+$ billion/yr bet volume
  – mainly US bettors
  – started in 1995; 1998
  – financial records for lifetime of firm

• Financial reports from ten public books
  – Sportingbet, CanBet, BetInternet, ...
  – larger ones operate > 1 internet site
  – 1-5 years of annual reports
  – $0.1-$1 billion/yr bet volume
Data Used in Analysis – Internet Bookies (CONT)

- Press releases from three large books
  - 2 private (WSEX, Intertops)
  - 1 public (WWTS-Betcorp)
  - $1+ billion/yr bet volume

- Real-time price/odds data (DBC):
  - Feb 1999 - Feb 2000
  - all price changes for 6600 major sport games
  - data for 15 books (11 Nevada; 4 internet)
Sample Selection with Illegal Books

- Non-random data generation.
  - arrests by Kings County DA
  - cases subsequently resolved

- Might bias sample:
  - less successful bookies?
    counter: bettors “dropping a dime”
  - more risk-loving?
  - geographic concentration
    counter: consistency with non-NYC bookies (lit)

- Helps case for rationality: over-sample mistake prone bookies!

- **Argument against SS**
  consistency with non-arrested bookies (lit)
1. Background and Qualitative Results
### STYLIZED FACTS – Firm Structure I (details in paper/next)

<table>
<thead>
<tr>
<th>Issue</th>
<th>Legal</th>
<th>Internet</th>
<th>Illegal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation of Owner/Executive</td>
<td>Y</td>
<td>Y/N</td>
<td>N</td>
</tr>
<tr>
<td>Closely Held/Partnership</td>
<td>N</td>
<td>Y/N</td>
<td>Y</td>
</tr>
<tr>
<td>Customer Interaction</td>
<td>Direct</td>
<td>Direct + Indirect(?)</td>
<td>Direct + Indirect</td>
</tr>
<tr>
<td>Customer Recruiters</td>
<td>N</td>
<td>Y/N</td>
<td>Y</td>
</tr>
<tr>
<td>Incentive Contracts</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Bet on Credit</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
</tbody>
</table>

*Bookie4b, Nov 2004*
<table>
<thead>
<tr>
<th>Issue</th>
<th>Legal</th>
<th>Illegal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bet Size</td>
<td>Large</td>
<td>Y</td>
</tr>
<tr>
<td>Market Power</td>
<td>Medium</td>
<td>N</td>
</tr>
<tr>
<td>Firm Size</td>
<td>Medium</td>
<td>N</td>
</tr>
<tr>
<td>High Growth (Mergers)</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Horizontal Integration</td>
<td>Y/N</td>
<td></td>
</tr>
<tr>
<td>Innovative</td>
<td>N (exchange)</td>
<td>N</td>
</tr>
</tbody>
</table>

Key to above:
- info asymmetry $\rightarrow$ agency (monitor/trust workers and bettors)
- enforceability of contracts
Descriptive Stats: Table 3

- Large bookie: 0
  - $500K/day bets, 280 bettors, 80 employees
  - same volume as large Vegas bookie: Caesars

- Moderate-sized bookie: 3, 5
  - $60K/day bets, 100+ bettors, 10-20 employees

- Small bookie: 1, 4
  - $25K/day bets, 0-10 employees. Few bettors exit.

- General characteristics:
  - average bet $250 - $1000
  - bettors wager 1+/day
  - majority bettors lose $
  - volatile daily net revs (more later)
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Bookmaker 0</th>
<th>Bookmaker 1</th>
<th>Bookmaker 3</th>
<th>Bookmaker 4</th>
<th>Bookmaker 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bet Size</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>$1,117.8</td>
<td>$236.3</td>
<td>$366.5</td>
<td>$1,648.4</td>
<td>—</td>
</tr>
<tr>
<td>Maximum</td>
<td>$23,820</td>
<td>$3,000</td>
<td>$5,000</td>
<td>$4,000</td>
<td>—</td>
</tr>
<tr>
<td>Bets per Bettor-Day</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>3.58</td>
<td>1.45</td>
<td>4.44</td>
<td>1.28</td>
<td>—</td>
</tr>
<tr>
<td>Maximum</td>
<td>43</td>
<td>18</td>
<td>36</td>
<td>25</td>
<td>—</td>
</tr>
<tr>
<td>Bettors (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bet every day</td>
<td>7.1%</td>
<td>3.7%</td>
<td>9.9%</td>
<td>0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>Exit</td>
<td>23.9%</td>
<td>11.1%</td>
<td>5.4%</td>
<td>6.7%</td>
<td>39.0%</td>
</tr>
<tr>
<td>Winnings ≥ 0</td>
<td>49.3%</td>
<td>38.9%</td>
<td>22.5%</td>
<td>53.3%</td>
<td>48.4%</td>
</tr>
<tr>
<td>Bet Frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bets/Minute</td>
<td>1.23</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Bet Types (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exotic Wagers</td>
<td>16.5%</td>
<td>26.3%</td>
<td>19.3%</td>
<td>0%</td>
<td>—</td>
</tr>
<tr>
<td>Football</td>
<td>2.5%</td>
<td>81.7%</td>
<td>21.5%</td>
<td>0%</td>
<td>—</td>
</tr>
<tr>
<td>Basketball</td>
<td>0%</td>
<td>16.2%</td>
<td>71.2%</td>
<td>98.5%</td>
<td>—</td>
</tr>
<tr>
<td>Baseball</td>
<td>97.5%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>—</td>
</tr>
<tr>
<td>Daily Bet Volume</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>$520,877.7</td>
<td>$18,518.2</td>
<td>$68,490.4</td>
<td>$31,554.3</td>
<td>$61,466.0</td>
</tr>
<tr>
<td>Daily Net Revenue</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>$5,892.4</td>
<td>$1,554.9</td>
<td>$9,228.4</td>
<td>-$1,310.0</td>
<td>$9,055.8</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>$87,418.8</td>
<td>$3,501.0</td>
<td>$13,913.5</td>
<td>$6,205.0</td>
<td>$17,916.3</td>
</tr>
<tr>
<td>Maximum</td>
<td>$163,232</td>
<td>$8,461</td>
<td>$37,429</td>
<td>$7,480</td>
<td>$47,960</td>
</tr>
<tr>
<td>Minimum</td>
<td>-$103,891</td>
<td>-$4,700</td>
<td>-$3,190</td>
<td>-$12,400</td>
<td>-$61,205</td>
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<tr>
<td>Employees (number)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salaried pay</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Commission pay</td>
<td>75</td>
<td>7</td>
<td>10</td>
<td>0</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 3: Descriptive Statistics
Basic Facts – Contrast to Internet Bookies

- Internet bet size smaller ($100-$300)
  - reflects adverse selection
  - illegals screen bettors

- Internet book size is much larger
  - annual bet volume = $0.1b-$2b
  - number bettors = 20k-1000k
  - organic growth + mergers
  - illegals require trust (below) → limits expansion/mergers

- Horizontal Integration
  - many internet books provide casino, lottery games
    high margins; leverage customer base (spread out FC)
  - illegals only sports → avoid bettors interacting (↑ arrest prob)
Market Power

• Approach 1: firm-level data (Appendix B).
  – Lerner index ≡ markup prices to MC
  – elasticity-adjusted Lerner index = 0.35
  – e.g. 3 firms under Cournot

• Approach 2: market concentration.
  – big bookie 0 is < 5% NYC market
  – C4 < 35% for NYC
  – suggests perfect competition
Reconciliation:
- bet market is neighborhood
- local oligopoly, e.g. neighborhood bookie
- most bookies only serve one market

Barriers to entry:
- $ for $K$ reserves
- outsider’s difficulty in gaining trust
- access financial diversification services

Internet bookies (contrast)
- no local markets
- less concentrated (no book > 5% US market)
Organization and Operation

- Textbook firm structure:
  
  min transaction costs

- Bookmaker (one or two guys):
  
  - owner: residual claimant on profits/losses
  - executive/policy-setter: prices, new bettors, employees, layoffs
  - multiple prior arrests: rarely served prison sentence
  - but asset forfeiture common

- Wireroom (regular hours):
  
  - writers/clerks
  - collector/bag-man/runner
  - flat wages: $200-$500/week $\approx 2 \times \text{Min Wage}$
  - mainly close friends/relatives of bookie
Organization and Operation (cont)

- Sheetholders:
  - recruit and service customers
  - wages based on bettor losses (next)
  - hi salary: $1500/week for bookie 0
  - most are prior acquaintance of bookie
  - redundancy:
    - management deal with bettors and with sheetholders
  - return to sheetholders below
Organization and Operation (CONT)

• Common practices:
  – low tech (paper records)
  – bet commissions: identical to Vegas

• Betting on credit:
  – each bettor given credit limit
  – weekly settle-up
  – ST credit involves a lot $:
    $3400/week for bettor with bookie 0
  – some allow LT extensions: up to 3 months
  – bettors unable to pay debt allowed to work it off
  – NB: credit requires trust
Organization and Operation – Internet Bookies

- Difference 1: impersonal relationship with bettors
  - implications: no credit, no/few sheetholders
  - geographic separation hinders establishing personal relationship

- Difference 2: innovative (bet exchanges)
  - exchanges require thick market with diverse opinions
    illegals have small, homogeneous mkts (neighborhood)
  - exchanges require continuous transmission of info
    illegals have costly info transmission
  - innovations due to competitive forces
    illegals have barriers to entry and less need to innovate

- Difference 3: competition on commissions
  - easy entry
  - illegals – trust limits entry into neighborhood
Organization and Operation – Internet Bookies (CONT)

- Similarity 1: little separation ownership/management
  - {less true for public internet books}
  - lack of separation for different reasons in each sector
  - internet: economize on costs
  - illegal: info asymmetry (risk of manager stealing), no enforceable contracts (promise to pay manager) discourage separation

- Similarity 2: closely held ownership
  - {less true for public internet books}
  - both overcome classic problems in partnerships
  - free-riding 1: effort is private cost; $\pi$ is PG
  - free-riding 2: monitor other partners is PG
  - mitigated – few partners, daily interaction, long-time relationship
  - still limits ability of illegals to grow
    - how outside investors monitor; adv selection in seeking financing
Revenues, Costs, Profits: **Table 4**

- **Net revenues:**
  - -4% to 15% hold rate
  - no clear relation to bookie size
  - no clear relation legal mkt hold rate

- **Costs:**
  mainly sheetholder commissions

- **Profits:**
  - $250,000 - $1m per year (<1% bet volume)
  - \( \pi \) rate \( \approx \) non-arrested bookie
  - \( \pi > \) leader’s wages for drug-selling gang (Levitt and Venkatesh, 2000)
  - \( \pi \) attractive?
    * bookie has limited outside opportunities
    * but must invest substantial \( K \) (next!)
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Bookmaker 0</th>
<th>Bookmaker 1</th>
<th>Bookmaker 3</th>
<th>Bookmaker 4</th>
<th>Bookmaker 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs. Period (days)</td>
<td>22</td>
<td>17</td>
<td>7</td>
<td>7</td>
<td>77</td>
</tr>
<tr>
<td><strong>REVENUES</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bet Volume</td>
<td>$11,459,310</td>
<td>$314,810</td>
<td>$479,433</td>
<td>$220,880</td>
<td>$4,732,879</td>
</tr>
<tr>
<td>(Net) Revenue</td>
<td>$129,632</td>
<td>$26,434</td>
<td>$64,599</td>
<td>-$9170</td>
<td>$697,294</td>
</tr>
<tr>
<td>% Hold</td>
<td>1.31%</td>
<td>8.40%</td>
<td>13.47%</td>
<td>-4.15%</td>
<td>14.73%</td>
</tr>
<tr>
<td><strong>COSTS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sheetholder</td>
<td>$99,189</td>
<td>$8,610</td>
<td>$36,840</td>
<td>$0</td>
<td>$453,350</td>
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<tr>
<td>Commissions</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Unpaid/Delayed</td>
<td>$11,486</td>
<td>$720</td>
<td>NA</td>
<td>$50</td>
<td>$40,462</td>
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<tr>
<td>Debt Payment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salaries to Writers and Collectors</td>
<td>$2,520</td>
<td>$1,350</td>
<td>$500</td>
<td>$0</td>
<td>$19,800</td>
</tr>
<tr>
<td>Rent</td>
<td>$3,100</td>
<td>$1,900</td>
<td>$600</td>
<td>$250</td>
<td>$11,000</td>
</tr>
<tr>
<td>Line Service</td>
<td>$200</td>
<td>$550</td>
<td>$200</td>
<td>$100</td>
<td>$1,350</td>
</tr>
<tr>
<td>Subscription</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utilities (phone, electric, pagers)</td>
<td>$497</td>
<td>$611</td>
<td>$701</td>
<td>$478</td>
<td>$3,567</td>
</tr>
<tr>
<td><strong>Total Costs</strong></td>
<td>$116,992</td>
<td>$13,741</td>
<td>$38,841</td>
<td>$878</td>
<td>$529,529</td>
</tr>
<tr>
<td><strong>PROFIT</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td>$12,640</td>
<td>$12,693</td>
<td>$25,758</td>
<td>-$9,648</td>
<td>$167,765</td>
</tr>
<tr>
<td>Profit_{annualized}</td>
<td>$210,000</td>
<td>$220,000</td>
<td>$1,300,000</td>
<td>-$400,000</td>
<td>$800,000</td>
</tr>
</tbody>
</table>

Table 4: Bookmaker Accounting Statement
Revenues, Costs, Profits – Internet Bookies

- **Costs:**
  - lower (no sheetholders)
  - higher (marketing, customer service)
  - costs are 3-7% bet volume (vs 1-11% for illegals)

- **$\pi$ rate:**
  - -20% - 2% bet volume
  - ↑ with age
  - illegals: -4% - 5% bet volume

- higher opportunity cost $K$?
Qualitative Result: Role of Trust

- Bookie must trust bettor (e.g. not to inform police)
- Legally binding contracts not possible

1. Small bookmaker (< 100 bettors) – Appendix C
   - Bookie knows/personally interacts with bettors
   - Stable client base (Table 3)
   - Map listings in address book (Figure 5)
   - Same neighborhood + similar ethnic background (match Fig. 5 address to census tract)

   \[\therefore\] trust develops

2. Big bookmaker (> 100 bettors)
   - more significant problem: little personal ∩
   - solution: sheetholding arrangement (below)
Qualitative Result: Role of Trust (CONT)

**Importance:** allow betting on credit

- Bookie trusts bettors to repay him
- *Adv 1:* ↓ $ exchanges → ↓ theft, arrest
- *Adv 2:* ↑ number bettors
  - illiquid, TI indiv. start addictive behavior
- *Adv 3:* induce more betting | bettor
  - end of week “catch up”
Staten Island Bookie

Figure 5: Bookmaker 1’s Place of Business and His Bettors’ Place of Residence
Sheetholding Arrangement

- Sheetholder recruit customers (average: 8.5)
  - acquaintances
  - exclusive relationship
  - bettors from same neighborhood
  - similar betting patterns

- Multiple layers possible (Figure 1)

- Will see like limited liability franchise
Figure 1: Sheetholding Arrangement for Large Bookmaking Operation
Sheetholding Arrangement (CONT)

- **Red figure system**
  - weekly salary = sheet’s rate \times net weekly position
  - net weekly position = \max(bettors’ current losses − red fig,0)
  - red fig = cumulative bettor wins . . .
  - example from text . . .
  - NB: only paid if bettors repay debts

- Advantage to sheetholder:
  - cannot lose $
  - not need access to capital
Sheetholding Arrangement (CONT)

• Why bookie use red figure?
  1. sheetholders credit constrained (above)
  2. sheetholders superior info on bettors:
     – high power incentives along with insurance
       sheetholder shares $\pi$ but limited liability
     – incentive to bring “high quality” bettors . . .
     – aligns more informed agents’ prefs with principal

     – more on aligning sheetholder preferences.
       * tenure premium in wages (below)
       * like implicit investment in firm
       * gives incentive to ensure firm longevity

     – not have classical problems w/ high power incentives
       * output/effort (bettor losses, payments) accurately measured
       * risk neutral agent $\rightarrow$ not need insurance
Sheetholding Arrangement (CONT)

• DAs for bookie:
  1. high red fig $\rightarrow$ “sheetholder flight”
     solution: reputation + tenure premium (empirics below)
  2. $\downarrow\downarrow$ E(\text{net inc}) AND little effect on Var(\text{net inc})
  3. implicit sheetholder debt repayed with lag.
     bookie 0: hi within-sheet payoff correlation
     $\rightarrow$ pay commissions even if net rev = 0.

• Still nash among feasible incentive contracts
  (if info asymmetry, sheeth. credit constrained)
Sheetholders: Evidence on Link to Information Asymmetry

- Small bookies have no sheetholders → no info problems
- Higher growth rate for big bookies
- Internet books typically not use → trust less important
  no bet on credit; not need to worry about arrest
- Redundancy in structure (Radner, 1993)
  - larger organization has more elaborate arrangement
  - management deals with both bettors and sheetholders (Fig 1)
  - hierarchy shapes management ability to process information
  - bettor behavior provides real time check on sheetholder behavior
  - not true in “balanced tree”
Empirics of Sheetholding: Bookie 5 (Table 5)

1. Wage rates: cols (1) - (2)

Sheetholding Rate\(_{it} = \alpha \times X_{i,t-1} + \nu_i + \epsilon_{it}

- (1): productivity premium
- (2): reputation/tenure wage premium

2. Sheetholder entry/exit: col (3)

\[ \text{Pr}(Employed_{it}) = \lambda_0(t) \exp(X_{i,t-1} \beta) \]

- \{(instrumented) wage rate ↓ exits\}
- no red figure “sheetholder flight”

3. Sheetholder “effort” intensity: cols (4) - (5)

Sheetholder Intensity\(_{it} = \gamma \times X_{i,t-1} + \mu_i + \delta_t + u_{it}

- effort = num bettors, bet volume
- red figure has no effect
- e.g. not channel portion bettors away
<table>
<thead>
<tr>
<th>Dependent Var.</th>
<th>Sheet. Rate</th>
<th>Pr(Employed)</th>
<th>Bet Volume ($\times 10^{-4}$)</th>
<th>Num. Bettors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (lagged)</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>constant</td>
<td>0.220</td>
<td>0.333</td>
<td>0.214</td>
<td>6.204</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.109)</td>
<td>(0.021)</td>
<td>(0.456)</td>
</tr>
<tr>
<td>Net Revenue from Bettors ($\times 10^{-3}$)</td>
<td>0.090</td>
<td>0.071</td>
<td>-0.030</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.045)</td>
<td>(0.056)</td>
<td></td>
</tr>
<tr>
<td>Bet Volume ($\times 10^{-4}$)</td>
<td>0.012</td>
<td>0.009</td>
<td>0.911</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.007)</td>
<td>(0.292)</td>
<td></td>
</tr>
<tr>
<td>Number Bettors</td>
<td>0.003</td>
<td>0.004</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Previous Years of Service</td>
<td>0.031</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.016)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Previous Year’s Net Rev. ($\times 10^{-4}$)</td>
<td>0.061</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commission Rate</td>
<td>0.886</td>
<td>0.446</td>
<td>1.511</td>
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<tr>
<td></td>
<td>(0.422)</td>
<td>(0.084)</td>
<td>(0.599)</td>
<td></td>
</tr>
<tr>
<td>Red Figure ($\times 10^{-3}$)</td>
<td>0.006</td>
<td>0.011</td>
<td>-0.137</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.078)</td>
<td>(0.561)</td>
<td></td>
</tr>
<tr>
<td>Sheetholder FE?</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Time FE?</td>
<td>No</td>
<td>No</td>
<td>baseline hazard</td>
<td>Yes</td>
</tr>
<tr>
<td>Frequency</td>
<td>Week</td>
<td>Week</td>
<td>Week</td>
<td>Day</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.41</td>
<td>0.19</td>
<td>0.63</td>
<td>0.69</td>
</tr>
<tr>
<td>log $L$</td>
<td>-159.23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>121</td>
<td>121</td>
<td>121</td>
<td>825</td>
</tr>
</tbody>
</table>

Table 5: Sheetholder Incentive System Evaluation: Bookmaker 5.
Trust/Sheetholders – Internet Bookies

- Hard to develop trust
  - no personal/physical interaction
  - in different countries
  - no property rights

- Trust less important anyway
  not need to worry about arrest

- Implications:
  - (typically) not use sheetholders
  - no bet on credit (cash accounts only)
  - faster growth (easier to add bettors)
Trust/Sheetholders – Internet Bookies (CONT)

• WWTS uses “agents” (bettor recruiters)
  – likely illegal bookies/sheetholders in US
  – pay proportionate to losses of referred bettors
  – huge expense: 9% net revs in 2003
    www.betwwts.com: “You can earn up to 40% back on your referred customers’ losses”
  – only a few recruiters
    top agent=40% agent revs; top eight agents=80% agent revs
  – bring in over a quarter of bet volume

• Formal alliances with illegal bookies:
  – used to extend credit
  – overcomes inability to enforce contracts (illegal bookies collect)
  – source: NYC arrests; CEO discussions
2. Prices and Financial Risk/Profits
### STYLIZED FACTS – Firm Behavior

<table>
<thead>
<tr>
<th>Issue</th>
<th>Legal</th>
<th>Internet</th>
<th>Illegal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pricing</td>
<td>Vegas</td>
<td>Vegas</td>
<td>Vegas + Home Team + Discrimination</td>
</tr>
<tr>
<td>Competition on Commissions</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Financial Risk Management</td>
<td>Balanced Book</td>
<td>little hedging + small risk</td>
<td>little hedging + large risk</td>
</tr>
</tbody>
</table>
2a. Prices
Prices – Overview

- “Price” fixed at time of wager
  - F, BB, H: price = −spread
  - BS: price = odds (from (8))

- {No competition on commissions (discuss later)}
Prices – Objectives

1. Compare with legal price:
   • easy for bookies to offer legal price if want
   • real-time Vegas prices readily available
   • bets only on games that day

2. Check for price discrimination:
   • harder for bookie to do
   • difficulty 1: competition from other bookies (not too important)
   • difficulty 2: bettor picks side he wagers on
     – bad price on $A \rightarrow$ good price on $\sim A$
     e.g. spread
     – {NB: never set separate lines for each side of game in data}
     – BUT bookie has betting history of each bettor:
       can use this to infer bettor prefs and price discriminate

Theory: Appendix D.2
Large Bookmaker (Bookie 0) Prices: Preliminaries

- Records exact minute wager placed (mainly BS):
  \( price_t \)

- Match with real time Las Vegas price:
  \( price_{vegas,t} \)
Bookie 0 Price Empirics: Regressions (Table 7)

\[ price_{i,t} = \alpha + \beta \times price_{vegas,t} + \gamma_1 X_{i,t} + \gamma_2 X_i + \nu_i + \nu_t + \epsilon_{i,t} \]  \hspace{1cm} (4)

- (1): prices largely follow legal line
  - \( \alpha \approx 0, \beta \approx 1 \rightarrow \text{BUT can reject} \)
  - \( \alpha > 0 \rightarrow \text{unfavorable bettor price.} \)
    matches raw data (Table 6)
  - NB: not linear shift legal price (game×team FE)

- (2): deviations due to bettor-specific factors
  - \( R^2 \rightarrow 1 \)
  - 10\% bettors FE \( \gg 0 \rightarrow \text{unfavorable price} \)
  - < 1\% bettors FE \( \ll 0 \)
Bookie 0 Price Regressions: Table 7 (cont)

\[ price_{i,t} = \alpha + \beta \times price_{vegas,t} + \gamma_1 X_{i,t} + \gamma_2 X_i + \nu_i + \nu_t + \epsilon_{i,t} \]  \hspace{1cm} (4)

- (3): bettor-specific factors help explain prices
  - small effect:
    - Bookie Position, Time from Game, Debt
  - big effect:
    - Herfindahl = bet pattern persistence
    - ex: perfect team loyalty
      \[ Herf_{FA}=1 \rightarrow price \uparrow 0.024 \text{ (allow bookie arbitrage)} \]
    - interaction:
      - loyalty effect \uparrow bet frequency
Bookie 0 Price Regressions: Table 7 (cont)

NY Yankees games

- sentimental favorite in much of NYC

- (4): $R^2$ only 0.8

- (5): pro-NYY bets have constant $\gg 0$
  - anti-NYY bets have constant $\approx 0$ (omitted)
  - $\therefore$ not linear price shift

- Bettor FE: (omitted)
  $R^2 \rightarrow 1$. Herf key to explain bettor FE.
Bookie 0 Price Regressions: **NY Yankees games (CONT)**

- **Implication:**
  - Yankees loyalists (\(\frac{1}{4}\) sample)
  - \(WTP = -(\Pr(Y) \times \frac{1-price}{price} - (1 - \Pr(Y)) \times 1)\)
  - Yankees loyalists \(WTP = 4.2\)¢ / dollar bet
    \(\rightarrow 2x\) BS commission

- **State-dependent utility function:** \(U(Y, SOW)\)
  - VN prefs:
    - insure against bad SOW \(\rightarrow\) bet against Yankees
  - SD utility:
    - ↑ MU income in happier SOW \(\rightarrow\) bet on Yankees
  - prior evidence – extreme SOW (death) or health outcomes
  - here – may apply in financial context as well
    \(\rightarrow\) not insure against loss by “sentimental favorite”
  - ex: election outcome
Bookie 0 Price Regressions: NY Yankees games (CONT)

• Small effect on $\pi$:
  – unfavorable wagers involve small stakes
  – $\uparrow$ net revs per bet by 0.3%
<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sample Size</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N )</td>
<td>number of wagers</td>
<td>11797</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( N_i )</td>
<td>number of bettors</td>
<td>263</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td><strong>Wager-Level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_{\text{bookmaker}} )</td>
<td>bookmaker price on wager, see (8)</td>
<td>0.522</td>
<td>0.080</td>
<td>0.278</td>
<td>0.737</td>
</tr>
<tr>
<td>( p_{\text{vegas}} )</td>
<td>Las Vegas price at the exact minute of the bookmaker wager</td>
<td>0.519</td>
<td>0.008</td>
<td>0.286</td>
<td>0.737</td>
</tr>
<tr>
<td><strong>Bookie Position</strong></td>
<td>$ bookmaker must pay if wagered team wins game</td>
<td>11734.78</td>
<td>13490.36</td>
<td>-59566</td>
<td>61847</td>
</tr>
<tr>
<td><strong>Time from Game</strong></td>
<td>time between bet and games (minutes)</td>
<td>140.90</td>
<td>160.35</td>
<td>0</td>
<td>1465</td>
</tr>
<tr>
<td><strong>Bet Amount</strong></td>
<td>$ on each part of wager presuming the favorite team wins (this assumption is needed since under a money line the size of the payoff depends on which side wins)</td>
<td>920.73</td>
<td>1293.95</td>
<td>0</td>
<td>10400</td>
</tr>
<tr>
<td><strong>Bettor-Level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Prior Debt</strong></td>
<td>$ which bettor owes the bookmaker at the start of the betting day (it is reset each week and equals zero if the bettor has cumulative winnings)</td>
<td>2410.56</td>
<td>5488.34</td>
<td>0</td>
<td>45750</td>
</tr>
<tr>
<td><strong>Sunday Debt</strong></td>
<td>( \text{PriorDebt} \times I_{\text{Sunday}} )</td>
<td>550.058</td>
<td>3120.01</td>
<td>0</td>
<td>39040</td>
</tr>
<tr>
<td><strong>Number Bets</strong></td>
<td>number of bets bettor places over the sample period</td>
<td>37.91</td>
<td>55.90</td>
<td>1</td>
<td>533</td>
</tr>
<tr>
<td>( \sum_{t \in \text{teams}} \frac{Number Bets \times p_t^2 - 1}{Number Bets - 1} )</td>
<td>where ( p_t ) is the proportion of the bettor’s wagers on team ( t )</td>
<td>0.086</td>
<td>0.166</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \text{Herfindahl}_{T} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sum_{t \in \text{teams}} 2p_t(p_{tF}^2 + p_{tA}^2) - 1 )</td>
<td>where ( p_{tF} ) (( p_{tA} )) is the proportion of wagers involving team ( t ) which are for (against) team ( t )</td>
<td>0.624</td>
<td>0.273</td>
<td>0.036</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6: Definitions and Descriptive Statistics: Bookmaker 0 Prices and Betting Patterns
<table>
<thead>
<tr>
<th>Covariate</th>
<th>All Games</th>
<th>NYY</th>
<th>pro-NYY</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.013</td>
<td>0.006</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>price&lt;sub&gt;vegas&lt;/sub&gt;,&lt;sub&gt;t&lt;/sub&gt;</td>
<td>0.979</td>
<td>0.989</td>
<td>0.970</td>
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<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Bookie Position (&lt;span&gt;10&lt;/span&gt;&lt;sup&gt;-6&lt;/span&gt;)</td>
<td>0.167</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time from Game (&lt;span&gt;10&lt;/span&gt;&lt;sup&gt;-5&lt;/span&gt;)</td>
<td>-0.194</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.077)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bet Amount (&lt;span&gt;10&lt;/span&gt;&lt;sup&gt;-5&lt;/span&gt;)</td>
<td>-0.199</td>
<td></td>
<td></td>
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<td></td>
<td>(0.010)</td>
<td></td>
<td></td>
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<tr>
<td>Prior Debt (&lt;span&gt;10&lt;/span&gt;&lt;sup&gt;-7&lt;/span&gt;)</td>
<td>0.547</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.272)</td>
<td></td>
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<tr>
<td>Sunday Debt (&lt;span&gt;10&lt;/span&gt;&lt;sup&gt;-6&lt;/span&gt;)</td>
<td>0.180</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.072)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number Bets (&lt;span&gt;10&lt;/span&gt;&lt;sup&gt;-4&lt;/span&gt;)</td>
<td>0.135</td>
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<tr>
<td></td>
<td>(0.018)</td>
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<tr>
<td>Herfindahl&lt;sub&gt;T&lt;/sub&gt; (&lt;span&gt;10&lt;/span&gt;&lt;sup&gt;-2&lt;/span&gt;)</td>
<td>0.104</td>
<td></td>
<td></td>
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<td></td>
<td>(0.077)</td>
<td></td>
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<tr>
<td>Herfindahl&lt;sub&gt;F,A&lt;/sub&gt;</td>
<td>0.013</td>
<td></td>
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<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Herfindahl&lt;sub&gt;F,A&lt;/sub&gt; × Number Bets (&lt;span&gt;10&lt;/span&gt;&lt;sup&gt;-3&lt;/span&gt;)</td>
<td>0.230</td>
<td></td>
<td></td>
</tr>
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<td></td>
<td>(0.061)</td>
<td></td>
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</tr>
<tr>
<td>Day of the week indicators?</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Better FE?</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>&lt;span&gt;R&lt;/span&gt;&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.91</td>
<td>0.98</td>
<td>0.98</td>
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<td>&lt;i&gt;N&lt;/i&gt;</td>
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<td>811</td>
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</tbody>
</table>

Table 7: Bookmaker 0 Prices. OLS Regression Analysis
Price Discrimination Interp

1. Most bookie 0 bettors wager at Vegas prices

2. Price discriminate against bettors with team loyalty
   - e.g. always bet *for* some set of teams
   - $\frac{1}{10}$ bettors
   - consistently wager at unfavorable price
   - NB: bookie must know bettor will pick unfavorably priced side (use betting history)

3. Bettors wagering at favorable price
   - few bettors $\rightarrow$ heavy, infrequent gambles
   - arbitageurs?

Yankees games: patterns accentuated
Supplemental Evidence for Price Discrimination

1. Simultaneous wagers on same game
   - > 50% at different price
   - only bettors with strong team loyalty consistently wager at hi price
   - evidence of bettor-specific prices

2. Convert to price for favorite team
   - controls for bettor’s choice of side
   - persistence vars no longer significant
   - consistent with price discrim:
     sentimental team not always favorite

3. Only wagers with big price diff from Vegas
   - i.e. allow arbitrage
   - 8% allow bookie arbitrage
     → mainly with persistent bettors
   - < 1% allow bettor arbitrage
Price Discrimination: Qualitative Evidence

voice 1: Was that Norm40? Did he play the Knicks again?

voice 2: Yeh. Picked them for a nickel. [a $500 bet]

voice 1: What a *****! Don’t he realize the boss is charging him four points extra? We might as well just pick his pocket...Andy50 never gets a soft line, he always gets Vegas. A real wise guy [a well informed bettor]. [Re]member when he busted us on Jack[sonville]? He can always get his boys in Vegas to lay off the other way.

excerpted conversation between wire room operators employed by Bookmaker 5
Old evidence: Price Crosstabs (Bookie 0)

- **Old Table 7:**
  - most bettors receive price near Las Vegas
  - few get mainly good lines ($\Delta price < 0$)
    very few get (and bet on) Arbitrage
  - $\approx 10\%$ get mainly bad lines.
    $\approx 10\%$ get $\sim$Arbitrage
  - bettor characteristics
    Herfindahl: bet pattern persistence
  - consistent with price discrimination ... 

- **Old Table 8:** Just Yankees games
  - earlier patterns accentuated
  - loyalists (“mainly bad”).
    many; $\frac{3}{4}$th prices satisfy $\sim$Arbitrage
  - not just uniform line shift ...
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Overall</th>
<th>Mainly Good</th>
<th>Many Good Arb.</th>
<th>Mainly Bad</th>
<th>Many Bad Arb.</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>0.530</td>
<td>0.525</td>
<td>0.534</td>
<td>0.556</td>
<td>0.584</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.035)</td>
<td>(-)</td>
<td>(0.037)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Δprice</td>
<td>0.0021</td>
<td>-0.0093</td>
<td>-0.0206</td>
<td>0.0131</td>
<td>0.0145</td>
</tr>
<tr>
<td></td>
<td>(0.0078)</td>
<td>(0.0041)</td>
<td>(-)</td>
<td>(0.0045)</td>
<td>(0.0034)</td>
</tr>
<tr>
<td>Arbitrage</td>
<td>0.007</td>
<td>0.009</td>
<td>0.417</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.015)</td>
<td>(-)</td>
<td>(0.003)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>~Arbitrage</td>
<td>0.084</td>
<td>0</td>
<td>0</td>
<td>0.214</td>
<td>0.349</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(-)</td>
<td>(-)</td>
<td>(0.161)</td>
<td>(0.082)</td>
</tr>
<tr>
<td>Bet Amount</td>
<td>1052.66</td>
<td>1513.81</td>
<td>2837.08</td>
<td>264.89</td>
<td>472.95</td>
</tr>
<tr>
<td>per wager</td>
<td>(1290.84)</td>
<td>(1330.56)</td>
<td>(-)</td>
<td>(372.94)</td>
<td>(772.83)</td>
</tr>
<tr>
<td>Prior Debt</td>
<td>1321.90</td>
<td>865.79</td>
<td>1514.58</td>
<td>1483.54</td>
<td>1373.5038</td>
</tr>
<tr>
<td>at start of</td>
<td>(2897.96)</td>
<td>(1628.67)</td>
<td>(-)</td>
<td>(664.38)</td>
<td>(1542.35)</td>
</tr>
<tr>
<td>day</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number Bets</td>
<td>37.91</td>
<td>7.75</td>
<td>4.00</td>
<td>33.08</td>
<td>31.73</td>
</tr>
<tr>
<td>in sample</td>
<td>(55.90)</td>
<td>(8.01)</td>
<td>(-)</td>
<td>(38.22)</td>
<td>(35.13)</td>
</tr>
<tr>
<td>Herfindahl_T</td>
<td>0.086</td>
<td>0.140</td>
<td>0</td>
<td>0.258</td>
<td>0.179</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td>(0.435)</td>
<td>(-)</td>
<td>(0.355)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>Herfindahl_{FA}</td>
<td>0.624</td>
<td>0.678</td>
<td>0.500</td>
<td>0.878</td>
<td>0.877</td>
</tr>
<tr>
<td></td>
<td>(0.273)</td>
<td>(0.234)</td>
<td>(-)</td>
<td>(0.234)</td>
<td>(0.229)</td>
</tr>
<tr>
<td>Herfindahl_{BT}</td>
<td>0.878</td>
<td>0.977</td>
<td>1</td>
<td>0.878</td>
<td>0.880</td>
</tr>
<tr>
<td></td>
<td>(0.213)</td>
<td>(0.078)</td>
<td>(-)</td>
<td>(0.203)</td>
<td>(0.228)</td>
</tr>
</tbody>
</table>

| N            | 263     | 12          | 1              | 39         | 26            |

OLD Table 7: Cross Tabulations of Bookmaker 0 Prices and Betting Patterns
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>For and Against Yankees Wagers</th>
<th>For Yankees Wagers only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overall</td>
<td>Mainly Good</td>
</tr>
<tr>
<td><strong>price</strong></td>
<td>0.532</td>
<td>0.516</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.055)</td>
</tr>
<tr>
<td><strong>Δprice</strong></td>
<td>0.0079</td>
<td>-0.0049</td>
</tr>
<tr>
<td></td>
<td>(0.0112)</td>
<td>(0.0046)</td>
</tr>
<tr>
<td>Arbitrage</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>~Arbitrage</td>
<td>0.300</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>(0.257)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>Bet Amount per wager</td>
<td>1137.01</td>
<td>1839.20</td>
</tr>
<tr>
<td></td>
<td>(1457.59)</td>
<td>(1920.27)</td>
</tr>
<tr>
<td>Prior Debt at start of day</td>
<td>1465.213</td>
<td>1167.96</td>
</tr>
<tr>
<td></td>
<td>(3402.66)</td>
<td>(1887.23)</td>
</tr>
<tr>
<td>Number Bets in sample</td>
<td>5.91</td>
<td>2.81</td>
</tr>
<tr>
<td></td>
<td>(7.74)</td>
<td>(2.03)</td>
</tr>
<tr>
<td>Herfindahl_{FA}</td>
<td>0.774</td>
<td>0.605</td>
</tr>
<tr>
<td></td>
<td>(0.323)</td>
<td>(0.323)</td>
</tr>
<tr>
<td>Herfindahl_{BT}</td>
<td>0.917</td>
<td>0.966</td>
</tr>
<tr>
<td></td>
<td>(0.193)</td>
<td>(0.127)</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>207</td>
<td>29</td>
</tr>
</tbody>
</table>

OLD Table 8: Bookmaker 0 Prices and Betting Patterns: Bets on New York Yankees games

Bookie4b, Nov 2004
Smaller Bookmakers Prices (Table 8)

- Bookie 1 (very small)
  - (1): legal prices explain 99.9% variation.
  - (2)-(3): 1 pt uniform line shift against NY teams
  - (1)-(3): bettor FE, other covariates insign.

- Bookie 3 (moderate size):
  - (4): mainly legal prices
  - (5): but some bettor-specific prices
  - (6): high price linked to persistent betting
  - e.g. price discrimination.
    BUT only $\frac{1}{16}$ th bettors ($\ll$ bookie 0)

- Bookie 4:
  - contemporaneous with bookie 3
  - Corr(prices) = 0.98
  - e.g. little illegal mkt. price dispersion
  - contrast: 1970s had dispersion (role tech $\Delta$)
<table>
<thead>
<tr>
<th>Sample</th>
<th>Bookmaker 1</th>
<th>Bookmaker 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep. Var</td>
<td>full</td>
<td>pro-NY bets</td>
</tr>
<tr>
<td>Covariate</td>
<td>$price_{i,t}$</td>
<td>$price_{i,t}$</td>
</tr>
<tr>
<td>constant</td>
<td>0.018 (0.029)</td>
<td>1.212 (0.087)</td>
</tr>
<tr>
<td>$price_{vegas,t}$</td>
<td>1.001 (0.001)</td>
<td>1.007 (0.003)</td>
</tr>
<tr>
<td>Herfindahl$_T$</td>
<td>0.530 (0.135)</td>
<td></td>
</tr>
<tr>
<td>Herfindahl$_{FA}$</td>
<td>1.172 (0.070)</td>
<td></td>
</tr>
<tr>
<td>Bettor FE?</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>$N$</td>
<td>1893</td>
<td>58</td>
</tr>
</tbody>
</table>

Table 8: Bookmakers 1, 3 Prices. OLS Regression Analysis
Prices: Conclusion

- Bookie price almost lockstep with legal

- ... except home-town price unfavorable

- ... except price discrimination by moderate, big books
  → more evidence of mkt power
Prices – Internet Books Differences

1. Competition on commissions
   - illegal/legal: standardized commissions (4.5% for F,BB)
   - Tradesports:
     - 8¢ per lot → 2% commission
   - BOS:
     - loyalty rewards; initial deposit bonus
   - WWTS:
     - Friday night reduced commissions; initial deposit bonus
   - explanation:
     - role of competition (price shopping difficult in illegal sector)
Prices – Internet Books Differences (cont)

2. Legal prices offered

- initial lines from Vegas / Don Best
- only small price shifts when bet imbalance
- little price dispersion (across internet book and with Vegas)
  - 0.001% have > 1 pt differential in football
- evidence:
  - real-time DBC data (6600 games; \( N = 10^6 \) prices for day of event); www.oddschecker.com; CEO discussion
- explanation:
  - role of competition (easy to arbitrage on internet)
- note:
  - illegal books have less “representative” bettors (local mkt)
  - need to shift prices
Prices – Internet Books Differences (CONT)

3. Little price discrimination
   • explanation:
     greater churning of bettors; role of competition
   • NB: technically feasible to price discriminate
     → individ-specific prices when log-in
   • evidence:
     CEO discussion + internal docs

Overall
   • legal status is crucial
   • influences info about bettor, need for price competition
   • but constrained by bettors’ ability to choose
2b. Financial Risk/Profits
Financial Risk Taking

• CW: “balanced book” → Std Dev(net rev)=0

• How obtain balanced book:
  – layoff
  – adjust price

• Costs to each form of hedging:
  – layoff: pay commissions
  – adjust price: “middling;” bet cascades

• ∴ CW hard to understand:
  – hedging is costly
  – balanced book requires extreme RA

• Goal:
  Calc extent of financial risk in practice
Calculating Net Revs Distribution

Overview

• distribution $\equiv$ prob-wt. sum all possible rev outcomes
• prob: use prices to infer distribution of game results
• rev outcome: calc bookie net rev for given outcome (vector)
• Figure: unbalanced bets on one event
Calculating Net Revs Distribution

Details (Appendix E.1)

• *game/event*:
  – prob of outcome → use mkts
    * BS (money line): odds via (8)
    * F,BB,H: outcome ∼ \( N(spread_Y, \sigma_{season}^2) \)
  – revenue → net revs given outcome and bets which were placed

• *daily* net rev distribution:
  – BS: calc exact distrib \( 2^N \approx 16K \) outcomes
  – F,BB,H: draw from distribution vector
  – NB: presume game outcomes independent

• caveat:
  – calc compromised if a central authority pools risk → e.g. O.C.
  – evidence against:
    competition, layoffs, observed losses
Distribution Bookmaker 3's [Simulated] Revenues ($): Day 7

Fraction

(\text{sum}) \text{BookiePr}

-50000 -25000 0 25000 50000

0

.02

.04

.06
Risk Taking: Large Bookmaker (Bookie 0)

- Takes very large positions (and never lays off)
  e.g. payout depend on which team wins

- Daily/weekly distribution net revs, $\tilde{R}$.
  weekly → since settle-up period.
  Monte Carlo simulated

- Net revs quite variable (Figs 2,3):
  - **Fig 2**: mean=$20K$, std dev=$70K$
  - **Fig 3**: mean=$60K$, std dev=$240K$
  - Actual: weekly net revs $\in [-400k, 300k]$

  $\tilde{R} = \sum_{g \in G} \tilde{R}_g \overset{CLT}{\to} \text{Std Dev}(\tilde{R}) \propto \sqrt{G}$

  - intuition:
    (i) idiosyncratic risk not wash out → games independent
    (ii) systematic risk too (home-town teams)
Figure 2: Daily Revenue Distribution: Bookmaker 0

Distribution Bookmaker's Revenues ($): Day 4

Fraction

(sum) BookiePr

-200000 -100000 0 100000 200000

0

.02

.04

.06

-200000

-100000

0

100000

200000

-0.06

-0.04

-0.02

0

Fraction
Figure 3: [Simulated] Weekly Revenue Distribution: Bookmaker 0

Distribution Bookmaker’s [Simulated] Weekly Revenues: Week 2
Risk Taking: Bookie 0 (CONT)

- Min $K$: **Table 9** (top panel)
  - negative outcomes necessitate $ reserve
  - deal with worst weekly loss
  
    $$K_T^* \equiv - \min R^* : Pr(\tilde{R} < R^*) = T$$

  - $K_{0.001}^* \approx 750,000$
  - NB: close to available $ when arrested!
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Average Daily Return</th>
<th>Normal Distribution</th>
<th>Monte Carlo Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MINIMUM CAPITAL ($)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_0^*$</td>
<td>1,219,132</td>
<td>1,023,564</td>
<td>934,262</td>
</tr>
<tr>
<td>$K_{0.001}^*$</td>
<td>1,095,576</td>
<td>669,090</td>
<td>766,527</td>
</tr>
<tr>
<td>$K_{0.01}^*$</td>
<td>889,619</td>
<td>539,607</td>
<td>630,615</td>
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</tbody>
</table>

**Bookmaker 0**

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Average Daily Return</th>
<th>Normal Distribution</th>
<th>Monte Carlo Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MINIMUM CAPITAL ($)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_0^*$</td>
<td>47,852</td>
<td>47,639</td>
<td>27,295</td>
</tr>
<tr>
<td>$K_{0.001}^*$</td>
<td>33,768</td>
<td>17,804</td>
<td>17,765</td>
</tr>
<tr>
<td>$K_{0.01}^*$</td>
<td>27,690</td>
<td>10,837</td>
<td>11,474</td>
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</table>

**Bookmaker 1**

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Average Daily Return</th>
<th>Normal Distribution</th>
<th>Monte Carlo Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MINIMUM CAPITAL ($)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_0^*$</td>
<td>143,280</td>
<td>154,503</td>
<td>85,663</td>
</tr>
<tr>
<td>$K_{0.001}^*$</td>
<td>103,542</td>
<td>77,182</td>
<td>73,307</td>
</tr>
<tr>
<td>$K_{0.01}^*$</td>
<td>76,665</td>
<td>59,125</td>
<td>51,813</td>
</tr>
</tbody>
</table>

**Bookmaker 3**

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Average Daily Return</th>
<th>Normal Distribution</th>
<th>Monte Carlo Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MINIMUM CAPITAL ($)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_0^*$</td>
<td>143,280</td>
<td>154,503</td>
<td>85,663</td>
</tr>
<tr>
<td>$K_{0.001}^*$</td>
<td>103,542</td>
<td>77,182</td>
<td>73,307</td>
</tr>
<tr>
<td>$K_{0.01}^*$</td>
<td>76,665</td>
<td>59,125</td>
<td>51,813</td>
</tr>
</tbody>
</table>

Table 9: Minimum Capital Reserve
Annual Returns Bookie 0

- Algorithm:
  - Monte Carlo simul from weekly distribution
  - Instead: calibrated model for realism
  - details: Appendix E.2, below

- \( \text{Net Revs} \sim N(6.7m, 2.0m^2) \)
  \[ \Pr(\text{NetRevs} < 0) \approx 0 \]

- \( \text{Profits} \sim N(0.9m, 2m^2) \)
  - old Fig 5: mean=$500k, std dev=$1.5m
  - \( \Pr(\text{Profits} < 0) = \frac{1}{3} \)
  - NB: \( \pi < 0 \)
    - ... 1 of 5 yrs with data
    - ... 8% S&P 500 firms
    - ... 50% public internet books
OLD Figure 5: [Simulated] Annual Profits Distribution: Bookmaker 0
Annual Returns Bookie 0 (CONT)

- Annual ror on $K$:
  - $E(ror) = 90\%$, Std Dev$(ror) = 200\%$
  - below one std dev confidence band of legal fin mkt efficient frontier: $\text{SEE (5)}$
Annual Distributions (Appendix E.2)

Framework

- bet payoff: \((0.5, -M) \oplus (0.5, (1 + v)M)\)
- \(s\) offsetting wagers, \(N - 2s\) unmatched
- \(G\) independent events

Distributions (via CLT)

\[
Net \ Revs \sim \frac{M}{2} N \left( NG \ v, (N - 2s)^2 G(2 + v)^2 \right)
\]

\[
Profits = Net \ Revs(v = \tilde{v}) - DFC - IFC
\]

Advantage: realism

- bookie commission rate \((v)\)
- sheetholder commission \((v \rightarrow \tilde{v})\)
- costs \((DFC, IFC)\)
- see text for param values
- NB: \(\approx\) if just Monte Carlo from weekly dist
1. Not invest $K$ in legal mkt.
   get higher risk-adjusted ror in legal mkt

2. Not invest $K$ in safe asset.
   - intuition: annual $\pi$ highly uncertain
   - details: Appendix E.3, next sheet

3. Not hedge/layoff
   - ↓ std dev $\$1 \rightarrow ↓$ mean $\$0.8.$
     comparable to legal fin. mkt
   - hedging inexpensive and yet not use
     $\rightarrow$ not strongly dislike risk
   - details: Appendix E.3, next sheet

Aside: bias to find RA (since ignore non-fin risk).

Explanation for results: Becker (1968) eqbm entry model of crime
Risk Pref Calc Details (Appendix E.3)

Framework

• **A1** CARA utility, \( u(x) = - \exp(-\gamma x) \)
• **A2** Annual profits, \( x \sim \mathcal{N}(\mu, \sigma^2) \)

Results

2. Not invest \( K \) in safe asset

\[
CE \geq \text{safe} \\
CE = \mu - \frac{1}{2}\gamma \sigma^2 \\
\rightarrow \gamma \leq 0
\]

3. Not hedge/layoff

\[
\max_{b \geq 0} \ CE = \mu(b) - \frac{1}{2}\gamma \sigma(b)^2 \\
\text{s.t.} \quad \frac{\mu'}{\sigma'} = 0.8 \\
b^* = 0 \rightarrow \gamma \leq 0
\]

NB: \( CRRA \equiv \gamma \times \text{income} \leq 0 \)
Risk Taking: Smaller Bookmakers (Bookies 1, 3)

- Some hedging (layoffs) but still unbalanced books

- Weekly net rev dist (Fig 4)
  - Bookie 1: $5k
  - Bookie 3: $28k
  - (volume-scaled) dispersion $\uparrow$ bookie size

- Min $K$: Table 9 (bottom panels)
  - $K_{\text{bookie 1}}^* \approx 15K$
  - $K_{\text{bookie 3}}^* \approx 75K$
  - scale by bet volume:
    $K^* \uparrow$ bookie size

- $\Pr(\pi \leq 0) \ll$ bookie 0
Figure 4: [Simulated] Weekly Revenue Distribution: Bookmakers 1, 3

Distribution Bookmaker 1’s [Simulated] Weekly Revenues: Week 3

Distribution Bookmaker 3’s [Simulated] Weekly Revenues: Week 1
Risk Taking: Smaller Bookmakers (CONT)

- Risk prefs:
  - must be RA (since layoff)
  - but $\approx$ RN (since net rev dispersion)
  - risk tolerance ↑ bookie size
Risk Taking: Conclusion

- all bookies take on financial risk

- only small ones even partially hedge. consistent with $\pi$ max

- balanced book is poor approximation
Risk Taking – Internet Books Differences

- Hedging: virtually never layoff (CEO; internal docs)

- Net revs less volatile
  - Annual.
    Internet: StdDev(Hold) = 0.6%-1.0% (firms w/ ≥ 3 yrs data)
    Bookie 0: StdDev(Hold)=1.3%
  - Weekly.
    Internet: Pr(Weekly Net Revs<0)=0.05 → Figure
    Bookie 0: Pr(Weekly Net Revs<0)=0.32

- {Profits
  - annual \( \pi < 0 \) for 50% public internet books (\( N = 51 \))
  - somewhat volatile (above) }

- Data sources: Internal docs; CEO discussion; annual reports
BetOnSports: Weekly gross margin distribution

Source: Company data
Risk Taking – Internet Books Differences (CONT)

Role of legality

1. Illegal
   - mkt size = neighborhood and/or city (trust . . .)
   - means systematic differences in demand (≠ bettors in Vegas)
     - hard to attract bettors on both sides since homogeneous prefs
   - hard to cheaply diversify

   - what about across-city betting?
   - could arbitrage (given home town bias documented here)
   - but hard to do
     - difficult to attract bettors in unfamiliar area
     - layoffs increase chance of arrest (other bookie)
     - layoffs expensive (pay commission)
     - RICO (federal prosecution)
Risk Taking – Internet Books Differences (CONT)

Role of legality (CONT)

2. Internet Books

- not have such imbalanced demand
- since have worldwide pool of bettors (less systematic risk)
- reduces financial risk from charging Vegas prices
Conclusion

- Self-interested behavior powerful
  - explains expensive incentive contract
  - highlights weakness CW on risk
  - why prices track legal mkt

- Outstanding issues:
  - identical commissions as legal mkt
  - bookie’s precise obj function ($\pi$ max?)
  - role legal fringe

- Do results extend to other organized illegal markets with legal quasi-substitutes?
  - ex: file sharing on internet; prostitution
  - how does trust develop?
Conclusion: Policy Implications

1. All sports gambling move to internet/off shore?
   - not completely
   - bettors not have tech sophistication
   - no financial credit:
     - unlikely in future since no trust (below)
     - bettors $ constrained and unlikely to have access to legit credit mkts
   - hard to develop trust
     - no personal/physical interaction
     - in different countries
     - no property rights
   - alternative: specialization
     - off shore contract with on shore to service accounts
     - common in recent NYC arrests
Conclusion: Policy Implications (cont)

2. Can prohibitions against internet bookies work?
   - unlikely
   - internet is a-geographic and porous; overcome trust issue
   - little success at eliminating on-shore version

3. Changes if legalized:
   - ↓ role trust
   - ↓ credit (?): might ↓ gambling addiction
   - no price discrimination
   - bookie size grow (size, geographic area)
   - no sheetholders
   - illegal fringe persist.
     horses currently
EXTRA SLIDES: betting primer
Betting Primer: Straight Bets (Appendix A)

- **Spread:**
  - bet team A for $1
  - spotted $P$ points ($P \leq 0$)
  - win if actual points $>$ $P$
  - if win: get $1$
  - if lose: pay $(1 + vig)$ where $vig = 0.1$

- **Money line:**
  - bet team at money line $l$ for $1$
  - win bet if team wins (no spread)
  - if lose: pay $1$
  - if win and $l \geq 100$ (underdog): get $l/100$
  - if win and $l < -100$ (favorite): get $100/|l|$
  - bookie commission:
    gap in favorite-underdog line

- **Table 10** for more advanced stuff
<table>
<thead>
<tr>
<th>Bet Type</th>
<th>Definition</th>
<th>Payoff</th>
<th>Commission Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>Bet on $A$</td>
<td>$S(A) \equiv (I(\text{win } A) - (1 - I(\text{win } A)(1 + \text{vig})) \text{Bet}_A$</td>
<td>FB, BB: 4.8%</td>
</tr>
<tr>
<td></td>
<td>FB, BB:</td>
<td>$S(A) \equiv (I(\text{win } A)c_A - (1 - I(\text{win } A)) \text{Bet}_A$</td>
<td>B:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\leq 2.3%$</td>
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<tr>
<td>Exotics</td>
<td></td>
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<tr>
<td>If</td>
<td>If $A$ then $B$</td>
<td>$I(A, B) \equiv S(A) + I(\text{win } A)S(B)$</td>
<td>FB, BB: 7.0%</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>B:</td>
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<td></td>
<td></td>
<td></td>
<td>$\leq 2.3%$</td>
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<tr>
<td>Reverse</td>
<td>If $A$ then $B$, AND</td>
<td>$R(A, B) \equiv I(A, B) + I(B, A)$</td>
<td>FB, BB: 7.0%</td>
</tr>
<tr>
<td></td>
<td>If $B$ then $A$</td>
<td></td>
<td>B:</td>
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<td></td>
<td></td>
<td></td>
<td>$\leq 2.3%$</td>
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<tr>
<td>Reverse</td>
<td></td>
<td>$\binom{N}{2}$ Reverses</td>
<td>FB, BB: 7.0%</td>
</tr>
<tr>
<td>Box</td>
<td></td>
<td>$\sum_{i,j \in \text{Bets}} R(i, j)$</td>
<td>B:</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$\leq 2.3%$</td>
</tr>
<tr>
<td>Parlay</td>
<td>Joint bet on multiple events</td>
<td>$P(\text{Bets}) \equiv \left( \prod_{i \in \text{Bets}} I(\text{win } i) \times 2^{#\text{Bets}} - 1 - #\text{Bets} \times \text{vig} \right) \text{Bet}$</td>
<td>FB, BB: 12.5%</td>
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<td></td>
<td></td>
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<td>B:</td>
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<td>$\leq 3.2%$</td>
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<td></td>
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<tr>
<td>N team</td>
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<tr>
<td>Round Robin</td>
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<tr>
<td>$\binom{N}{2}$ two team Parlays</td>
<td>$\sum_{i,j \in \text{Bets}} P(i, j)$</td>
<td>FB, BB: 12.5%</td>
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<td></td>
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<td>B:</td>
</tr>
<tr>
<td></td>
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<td>$\leq 3.2%$</td>
</tr>
</tbody>
</table>

Table 10: Types of Bets