Swaptions

Professor Eric Ghysels
Nov 30th?

The existing schedule
- Nov. 30: Treasury Bond Futures
- Dec. 7: CDO Pricing
- Dec. 14: Review
- Note: Project 1 due Nov. 30. No Project 2. HW 3 due Dec 14.

Alternative
- Nov. 30: Continuous time affine term structure models (optional)
- Dec. 7: Treasury Bond Futures and/or CDO Pricing
- Dec. 14: Complete Dec 7 lecture and Review
CoCo’s in the news
CoCo’s in the news

• **Contingent Convertible Bonds** are suggested as a possible solution for bank insolvency during financial crises.

• Convert automatically into equity when a bank’s finances deteriorate below a predetermined level – such as capital ratio level.
CoCo’s in the news

The pricing of CoCo’s shares something with typical single name CDS contract – a fixed leg and a contingent leg.

– On the fixed side, coupon until maturity or until capital ratio trigger.

– On the contingent side, switch to equity. The value of the contingent leg is the share value of distressed bank.
0 – Plan

1. Review of Interest Rate Swaps
2. What is a swaption?
3. Why swaption?
4. The uses of swaption
1 – Review of Interest Rate Swaps

A receives a 2-yr bond paying 6%, face $1000 issued by B

B receives a 2-yr FRN Paying 6-m LIBOR, face $1000, issued by A
Review of Interest Rate Swaps

• Within a binomial tree:

\[
\begin{align*}
  &0 & 0.5 & 1 & 1.5 \\
  0 & 0\% & 9\% & 976.30 \\
  0.5 & 7\% & 969.71 & 8\% \\
  1 & 6\% & 990.38 \\
  1.5 & 5\% & \\
  2 & 3\% & 1004.88 \\
  2.5 & 2\% & 1026.94 \\
  3 & 11\% & 1019.80 \\
\end{align*}
\]

\[
\begin{align*}
  &0 & 0.5 & 1 & 1.5 \\
  0 & -23.70 \\
  0.5 & -30.29 \\
  1 & -20.58 & -9.62 \\
  1.5 & 0 & -2.30 \\
  2 & 21.88 & 4.88 \\
  2.5 & 26.94 & \\
  3 & 19.80 & \\
\end{align*}
\]

Swap values
Swaps

- A swap is an exchange of fixed interest rate payments for floating interest rate payments over some period of time.
- Fixed-rate payer (1) long position in a floating-rate bond and (2) short position in a fixed rate bond
1 – Context

• At the beginning of 2004, speculating that the Fed will raise their target Fed funds rate, Sadi Ozelge, manager of Risk-neutral Ltd, decided to buy $100 million face value of risk-free FRN paying semi-annual coupons.

• Coupons of the FRN are tied to the 6-month Treasury rates.
1-Context

Fed Funds Target Rate vs. U.S. Prime Rate vs. 1-Month LIBOR vs. 3-Month LIBOR, December 1, 1999 - October 30, 2009

Copyright © 2009 www.FedPrimeRate.com
1 – Context

• 18 Sep 2007: the Fed reduced their Target by 50 basis points with possibility of further cuts

• This worried Sadi and he was looking for ways to hedge the risk of rates declining for at least the next 2 years

• What can he do?
Swaptions ("swap options")

- Option to enter into a swap at an "off-market" rate
- Types:
  - "payer": right to enter into a pay fixed, receive floating swap
  - "receiver" right to enter into a receive fixed, pay floating swap
  - European, American, Bermudan (modified American) exercise style (European style exercise most common)
2 – Valuing a floor contract

• Given the following 6-month interest rate tree:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13.35%</td>
<td>9.82%</td>
<td>7.10%</td>
<td>5.00%</td>
</tr>
<tr>
<td>0.5</td>
<td>5.00%</td>
<td>6.74%</td>
<td>4.72%</td>
<td>4.45%</td>
</tr>
<tr>
<td>1</td>
<td>7.10%</td>
<td>9.35%</td>
<td>6.39%</td>
<td>4.19%</td>
</tr>
<tr>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

• How could we value the floorlets, assuming a strike rate of 4.75%?
First and Second Floorlets

• **The first floorlet:** The first floorlet protects the first coupon payment, 6 months from today. Note that 5.0% is higher than the floor rate of 4.75%. This means the value of the first floorlet is simply zero.

• **The second floorlet:** The second floorlet protects the second coupon payment, 1 year from today which is function of the 6-month interest rate 6 months from today.
Second Floorlet (cont)

• From the interest rate tree, we know that the interest rate could be either 7.10% in which case there will be no payment from the floor seller; or 4.72% which would trigger the floor contract since 4.72% is smaller than the floor rate of 4.75%.

• In such a scenario, the floor seller would have to make a payment equal to 0.5 x $100 million x (4.75% - 4.72%) = $15,000.
• Note that the $15,000 payment is due at the end of time 1. The PV of payment at the end of time 0.5: $15,000/(1+4.72\%\times0.5) = 14,654.16$

• Now take a step back to time 0. With 50% chance (risk-neutral) that floor will be worth nothing (interest 7.10%) and 50% chance that the floor contract will be worth $14,654.16. PV is $[.5*(0 + 14,654.16)/(1+5\%/2)] = 7184$
2 – Valuing a floor contract

• The total cost of the floor contract = sum of the costs of all the floorlets = $74,370

• Sadi thinks that this is too expensive – how could he reduce the cost?
  1. ?
  2. ?
  3. ?
2 – Valuing a floor contract

• The total cost of the floor contract = sum of the costs of all the floorlets =
  0 + 7,184 + 35,156 + 32,030 = $74,370

• Sadi thinks that this is too expensive – how could he reduce the cost?
  1. Reduce maturity of floor contract from 2 years to 1.5
  2. Partial coverage of face value
  3. Reduce strike rate
  4. Sell cap contract to pay for floor contract
2 – Valuing a cap contract

• What is a cap contract?
  – By selling a cap contract, Sadi agrees to give the part of the coupon that is above a strike rate (or a cap rate, or a ceiling rate) to the buyer of the cap contract
  – The buyer of the cap contract can use it as an insurance against rising interest rates if he/she borrows floating
2 – Valuing a cap contract

Coupon payments from a FRN
2 – Valuing a cap contract

• Valuing a cap contract is very similar to valuing a floor contract
• Again, a 2-year semi-annual cap consists of 4 caplets – which can be valued separately
• What does the cost of the cap depend on intuitively?
  1. ?
  2. ?
  3. ?
2 – Valuing a cap contract

• Valuing a cap contract is very similar to valuing a floor contract
• Again, a 2-year semi-annual cap consists of 4 caplets – which can be valued separately
• What does the cost of the cap depend on intuitively?
  1. The maturity of the cap
  2. The strike rate
  3. Interest rate volatility
2 – Valuing a cap contract

• The fourth caplet of a 2-year semi-annual cap with a cap rate of 10%
2 – Collar and reverse collar

• If we buy a floor and sell a cap like Sadi did, we enter into a reverse collar
  – A reverse collar limits the floating coupons we earn within certain bands
  – Depending on how we design the bands, the cost of the reverse collar can be 0 or even negative

• If we buy a cap and sell a floor, we enter into a collar
  – If we are borrowers, a collar limits the interest we pay within certain bands
2 – What is a swaption?

- Swaption = option to enter into a swap with a pre-specified fixed rate $K$
  - Receiver swaption: option to enter into a swap to receive the fixed rate $K$ (and pay the floating)
  - Payer swaption: option to enter into a swap to pay the fixed rate $K$ (end receive the floating)
- You pay a premium for the swaption at time 0 and are not required to pay anything when entering the swap
The value of a swap = the value of the fixed rate bond - $100.

A receives a 2-yr bond paying 8%, face $100 issued by B

B receives a 2-yr FRN paying 6-m LIBOR, face $100, issued by A
• Within a binomial tree:

\[
\begin{array}{cccc}
0 & 0.5 & 1 & 1.5 \\
0 & 10.01\% & 97.40 & 13.54\% \\
7.29\% & 97.45 & 9.54\% & 13.54\% \\
5.19\% & 99.35 & 6.93\% & 99.26 \\
102.58 & 4.91\% & 100.50 & 6.58\% \\
103.14 & 4.64\% & 100.69 & 101.77 \\
102.85 & 4.38\% & 101.77 & \\
\end{array}
\]

Swap values:

\[
\begin{array}{cccc}
0 & 0.5 & 1 & 1.5 \\
0 & 2.58 & 0.50 & -2.60 \\
-0.65 & -0.74 & -2.60 & \\
2.58 & 3.14 & 0.69 & \\
2.85 & 1.77 & 1.77 & \\
\end{array}
\]

\[
\frac{0.5(100.69 + 101.77) + 4}{1 + \frac{4.64\%}{2}} = 104 \\
\frac{102.58-100}{1 + \frac{4.38\%}{2}} = 2.85
\]
2 – Why swaption?

The swap values here are for fixed-receiver or fixed payer?

- Receiver swaption participates in the positive and stay away from negative values
3 – Swaption Valuation

- Valuing a 2-year receiver swaption $100 face

\[
\frac{0.5(0 + 0.69)}{1 + \frac{6.93\%}{2}} = 0.33
\]

\[
\frac{0.5(0.69 + 1.77)}{1 + \frac{4.64\%}{2}} = 1.63
\]
3 – Swaption Valuation

• For simplicity, I have assumed that the maturity of the swaption is the same as the maturity of the swap (both 2 years)
• In general, the maturity of the swaption can be smaller than the maturity of the underlying swap
• How can we redo our previous valuation if the maturity of the swaption is 1 year?
3 – Swaption Valuation

• We’ve been dealing with American swaption ➔ the option to enter the swap can be exercised anytime before the maturity of the swaption

• For European swaption ➔ the option can only be exercised on the maturity date

• Bermudan swaption ➔ the option can only be exercised on certain dates prior to maturity

• What are the effects on valuation?
4 – Uses of swaptions

Can you home-make:

• a callable bond from a non-callable bond and a swaption?
• A cancelable swap?
4 – Uses of swaptions

Can you home-make:

• a callable bond from a non-callable bond and a swaption?
• A cancelable swap?