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Thanks
A Monopolistic Market for Information

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We analyze a model where traders buy information from a monopolistic seller, which is subsequently used in a speculative market. In order to overcome the dilution in the value of information due to its leakage through informative prices, the seller of information may prefer to sell noisier versions of the information he actually has. Moreover, to obtain higher profits, it is desirable for the seller to sell different signals to different traders, so that the added noise realizations do not affect equilibrium prices. One way of doing so, which does not require discrimination, is to sell identically distributed personalized signals to each of a large number of traders. Journal of Economic Literature Classification Numbers: 022, 026, 522, 611. © 1986 Academic Press, Inc.

1. INTRODUCTION

Analyses of markets under uncertainty have been increasingly concerned with the role of information, both public and private. Recently, attempts have been made to model information collection and production activities. A natural complement to the ownership of information by individual agents is the possibility of trade and exchange of information among traders; i.e., if individual agents are endowed or can produce information, then it may also be possible to sell it to others. As an example, we consider markets for information that is to be used for trading in securities markets. Casual observation suggests that investors in securities markets purchase information in the form of newsletters, advising services, etc., so that markets in information do indeed operate in this context. Our purpose is to understand how these markets may behave and what allocations of information are likely to emerge.

It is most convenient to envision information as a signal, a random

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variable that is jointly distributed with the state of the world. When information is used by agents in a market (or in a game), an externality in its use emerges: the valuation of information by each agent depends on the information possessed by others. Moreover, in many situations some of the information purchased may "leak," though its use, to those who did not buy it. Thus, agents may free-ride on the information possessed by others. In our context this occurs because traders in the speculative market use the equilibrium prices of assets to extract information embedded in them. This has important implications for markets in information.

This paper is one step in an investigation of markets for information. We analyze a model of a speculative market where private information about the payoff of a risky asset is sold to traders by a monopolistic seller, and asset prices are determined in a noisy rational expectations equilibrium. The seller's profit maximization problem is not a trivial one since the value buyers place upon the information being sold depends upon the informativeness of equilibrium prices, and this depends in part on the allocation of information. In general, when more traders hold a piece of information, its value decreases, since it is reflected more precisely in the price. If very precise information is sold, it is used very aggressively. This causes its value to deteriorate faster as more traders acquire it than would the value of less precise information. For this reason a monopolistic seller of information may not wish to sell the information he possesses "as is." He or she may prefer instead to add noise to it before selling, if this can be done costlessly. He may also prefer to sell information to only a fraction of the traders in order to increase its value to the buyers.

We provide an analysis of the model for the case in which all traders are identical. We also make, at the start, two strong assumptions. First, the monopolist completely controls the distribution of his products; i.e., he can perfectly discriminate, and he can prevent resale of the information. As is shown later, this assumption can be relaxed. Second, we do not deal with incentive issues: it is assumed that the statistical characteristics of the information sold are common knowledge between the buyers and the seller, and that the seller provides the information truthfully.

First we treat the case where the seller is restricted to selling the same signal to all his clients; i.e., photocopies of the same information are sold to all buyers. The seller chooses the price of the information, or, equivalently, the number of informed traders, as well as a level of noise to be added to his signal, to maximize profits. We show that when the seller's information is precise, he generally chooses to add noise to the information before selling it. The optimal level of added noise and the corresponding optimal

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1 Others have analyzed markets in information under the assumption that agents do not use the information in equilibrium prices. See, for example, B. Allen [3].
fraction of the market that he allows to become informed depend on the precision of prices relative to the information sold. In particular, when the price is a relatively precise signal of the information sold, so that the effects of the externality on the evaluation of information are intense, the seller does not sell information to the entire market. In this situation some traders remain privately uninformed.

Next we allow the monopolist to sell "personalized" signals; i.e., to draw a fresh independent error term of any desired precision (possibly different for different traders) and to add this to what he knows in order to create each of the signals sold. In doing so the seller releases more information in the aggregate (the average of all the signals sold in this case is, in a large market, all the information that he has) than when he sells a noisy photocopied signal. Since this may allow more free-riding, it may seem that personalization would lower his profits.

This intuition is incorrect. In general, the seller wishes to maximize the amount of information (for which he can charge) held by informed traders and, simultaneously, minimize the amount of information in the price. These desires are clearly in conflict, since the more information the informed have, the more informative are the prices. However, the seller is not concerned with the amount of information in the price per se, but with making it more desirable for each trader to buy the private information he offers rather than use the price alone. It turns out that this is achieved when any noise added is added in such a way that its realization does not affect the equilibrium price. For this reason, when the number of traders is large, selling personalized signals is generally better than selling one photocopied signal. In particular, we show that any allocation where photocopied noise has been added is strictly dominated by an allocation where signals are personalized. In contrast to the optimal photocopied allocation, where for some parameters the seller sells to only a fraction of the traders, we show that when signals are personalized the seller always chooses to sell identically distributed signals to the entire market. Thus, in the resulting equilibrium all agents are identical in terms of the amount of information (private and public) they have. For alleviating the effects of the leakage of information to uninformed buyers, adding personalized noise is not only better than adding photocopied noise, it is also better than restricting the number of informed traders.

As mentioned above, the key property of allocations where signals are personalized is that the realizations of any added noise do not affect the asset's equilibrium price. This property can also be achieved by simple allocations involving only a finite number of signals whose added noise terms cancel out in the equilibrium price of the risky asset. While such allocations yield the same profits as those with personalized signals, they may be much more difficult for the monopolist to implement, since he may
need to restrict the number of signals each trader is allowed to buy. Restrictions of this sort are not necessary in the personalized allocations—although many distinct signals are sold, at the profit maximizing prices no agent wishes to purchase more than one. Thus, the seller need not monitor how many signals each trader buys, and the allocation that is optimal for him, at least within a large class of allocations, can be obtained whether or not he can perfectly discriminate among traders.\textsuperscript{2}

The speculative market equilibrium model we use is a general version of those in Grossman and Stiglitz [8], Hellwig [9], and others, where the basic assumptions are joint normality and constant absolute risk aversion. These models have been applied (in [8] and [14]) to analyze information acquisition by individual traders.\textsuperscript{3} In these analyses, however, the cost function of the information signals and their joint distribution with other signals are exogenously determined. In our analysis both the price and (within bounds) the distribution of the signals are determined by the optimizing behavior of the information seller.

While we focus here on the speculative market model, the general theme of our analysis is applicable to other markets in information. In such markets the value of the information to any one individual depends upon the number of other agents possessing it. For example, Kamien and Tauman [11] analyze a monopolistic market for cost-reducing (patentable) innovations that is somewhat similar in spirit to ours. However, in their model there is no leakage of the information to uninformed, so the possibility of selling only part of the patent, or a "noisy" version of it, does not arise.

The paper is organized as follows. Section 2 describes the basic model of the speculative market and its equilibrium. Section 3 introduces the seller’s problem, and discusses its main ingredients. In Section 4 we analyze the case where there is only one piece of information sold (the case of photocopied signals). We characterize the resulting equilibrium for any level of the seller’s initial information, the degree of risk tolerance of the buyers, and noisiness of the risky asset’s supply. Section 5 compares the seller’s profits when he adds photocopied noise to those obtained when noise is personalized. It is shown that the latter leads to strictly higher profits even if the seller is restricted to selling to a fixed fraction of traders. We first derive the result and discuss the basic intuition for the case where the seller is perfectly informed, and then we prove it for the general case. In Section 6 we find the optimal personalized allocations and conclude the

\textsuperscript{2} We call allocations that do not require active discrimination viable. See Admati and Pfleiderer [2] for a more complete discussion of viability.
\textsuperscript{3} See Sunder [13] for experimental evidence on these issues, which is also relevant to our analysis in this paper.
comparison between photocopying and personalization. Section 7 discusses more general allocations and the issue of discrimination, while Section 8 provides concluding remarks.

2. The Speculative Market

The general structure of our model is as follows. There are three time periods and a continuum of traders indexed by \( v \in [0, 1] \). In period 0, information is sold to a fraction of the traders by a monopolistic seller, as is discussed in the next section. Trading in the speculative market takes place in period 1, and involves one risky asset and a riskless asset. The riskless asset is the numeraire. The price of a share of the risky asset is determined in a noisy rational expectations equilibrium. In period 2, the riskless asset pays off one unit of a single consumption good, a share of the risky asset pays off a random amount \( \tilde{P} \), and consumption takes place.

We make the following assumptions about the speculative market:

(A1) Rational Expectations Equilibrium. The market clears (in a per capita sense) while each trader maximizes the expected utility of final consumption, conditional on all the information available to him including current equilibrium prices.\(^5\)

(A2) Noisy Supplies. The per capita supply of the risky asset is a random variable \( \tilde{Z} \), which is independent of any information the speculators have.

Assumption (A2) is made to ensure that private information has positive value in the rational expectations equilibrium. The randomness in \( \tilde{Z} \) determines in part the informativeness of the price and thereby the intensity of the externality effects on the valuation of information.\(^6\) In general, the noisier the supply, the less informative is the equilibrium price (relative to the information in the market as a whole), and the less is the value of private information signals affected by the information other traders have.

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4 The assumption of a single risky asset is made to keep the analysis simple. Extensions to markets with more than one risky asset are straightforward for the case of a single seller.
5 For a precise definition of equilibrium in the context of an economy with a continuum of traders, see Admati [1].
6 The randomness in supplies can be interpreted in a variety of ways, e.g., through randomness in endowments or the existence of liquidity traders. To prevent fully revealing prices it is only important that supplies are not perfectly correlated with private information. However, the independence assumption greatly simplifies the model and facilitates explicit analysis. See Diamond and Verrecchia [7] for a model of a noisy rational expectations equilibrium where agents have some information on the supply.
To make the model tractable we also assume that:

(A3) Traders' preferences exhibit constant absolute risk aversion; i.e., their utility functions are exponential.

(A4) The random variables $\bar{F}$, $\bar{Z}$ and all the private information signals are jointly normally distributed.

(A5) All traders have the same level of risk tolerance.

The homogeneity assumption enables us to isolate the phenomena in the information market that are due to externality effects on the valuation of (and thereby the demand for) information from those that are due solely to differences in traders' preferences. (See the concluding remarks.)

We define an allocation $A$ of private information to be an assignment of a signal $\bar{Y} = \bar{F} + \bar{u}$ to each trader $v$, $v \in [0, 1]$, where $\bar{u}$ are normally distributed and independent of $\bar{F}$. An allocation is admissible if for this allocation there exists a rational expectations equilibrium price that is linear in the (average of the) private signals and the per capita supply; i.e., that is of the form

$$\bar{P}(A) = \alpha_0(A) + \int_0^1 \alpha_1'(A) \bar{Y} dv + \alpha_2(A)\bar{Z}. \quad (2.1)$$

Note that we have written the equilibrium coefficients as functions of $A$ to emphasize their dependence on the allocation of information. These coefficients, together with some of the parameters of the multivariate distribution of $(\bar{F}, \bar{P}(A), \bar{Z}, \bar{u}, v \in [0, 1])$, e.g., the variance of $\bar{P}(A)$, are determined in equilibrium, as described below. For notational convenience, we allow the variance of $\bar{u}$ to be infinite. (Traders for whom $\text{Var}(\bar{u}) = \infty$ are uninformed.)

Let $\mathcal{A}$ denote the set of admissible allocations. While $\mathcal{A}$ is a very large set, for most of the paper we consider classes of allocations that have the following simple forms: (i) a positive fraction of the market is informed and all informed traders observe the same signal, as in Grossman and Stiglitz [8], or (ii) each informed trader has a signal that is conditionally independent of other signals given some sufficient statistic. The latter type of allocation is a general version of the model found in Hellwig [9] and others.

To understand the effect of equilibrium price information on the valuation of information and thereby on the profits of an information seller, it is important to see the interaction between a given allocation of information and the asset equilibrium price. For a given admissible

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\footnote{Admissibility implies, in particular, that all the integrals that determine equilibrium are well defined.}
allocation \( A \), let \((\text{Var}(\bar{P} \mid \bar{P}^v, \bar{P}(A)))^{-1}\) be denoted by \(r'(A)\). From joint normality, \(r'(A)\) is independent of the realizations of the price and the private signal. Moreover, trader \( v \)'s forecast of the payoff of the risky asset is linearly related to his private signal, \( \bar{P}^v \), and the equilibrium price:

\[
\delta(\bar{P} \mid \bar{P}^v, \bar{P}(A)) = \beta_0^v(A) + \beta_1^v(A) \bar{P}^v + \beta_2^v(A) \bar{P}(A). \tag{2.2}
\]

It can be shown (see [1] or [9]) that the coefficients in the equilibrium price function (2.1) are related to \( r'(A) \) and the regression coefficients in (2.2) (which are themselves functions of the equilibrium parameters) by

\[
\alpha_1^v(A) = \frac{\beta_1^v(A) r'(A)}{\int_0^1 (1 - \beta_2^v(A)) r'(A) \, dv}, \tag{2.3}
\]

and

\[
\alpha_2(A) = -\frac{1}{\int_0^1 \rho(1 - \beta_2^v(A)) r'(A) \, dv}. \tag{2.4}
\]

The assumption that \( A \) is admissible implies that all the above integrals are well defined and that the equations that define the rational expectations equilibrium have a solution. In the cases examined below, the linear equilibrium is also unique. It is convenient to normalize the price (using a simple linear transformation), so that it has the same form as other signals. First, substituting \( \bar{P} + \bar{u} \) for \( \bar{P}^v \) in (2.1) gives

\[
\bar{P}(A) = \alpha_0(A) + \left( \int_0^1 \alpha_1^v(A) \, dv \right) \bar{P} + \int_0^1 \alpha_1^v(A) \bar{u} \, dv + \alpha_2(A) \bar{Z}. \tag{2.5}
\]

When a positive measure of agents are informed, so that \( \int_0^1 \alpha_1^v(A) \, dv \neq 0 \), the price can be normalized as

\[
\bar{P}_n(A) = \bar{P} + \int_0^1 \frac{\alpha_1^v(A) \bar{u}}{\int_0^1 \alpha_1^v(A) \, dv} \, dv + g(A) \bar{Z}, \tag{2.6}
\]

where

\[
g(A) = \frac{\alpha_2(A)}{\int_0^1 \alpha_1^v(A) \, dv}. \tag{2.7}
\]

Clearly, \( \text{Var}(\bar{P} \mid \bar{P}^v, \bar{P}(A)) = \text{Var}(\bar{P} \mid \bar{P}^v, \bar{P}_n(A)) \) for every allocation \( A \) and signal \( \bar{P}^v \), so observing the normalized price \( \bar{P}_n(A) \) is statistically equivalent to observing the price \( \bar{P}(A) \). The advantage of using \( \bar{P}_n(A) \) rather than \( \bar{P}(A) \) is that the covariance of \( \bar{P}_n(A) \) with \( \bar{P} \) is always 1, so that its informativeness (which is the same as that of \( \bar{P}(A) \)) is a simple function
of its variance. Substitution of the equilibrium equations (2.3) and (2.4) into (2.6) gives

$$\mathcal{P}_A(A) = \mathcal{P} + \frac{\beta'(A) r(A) \bar{a}^2}{\int_0^1 \beta'(A) r(A) \, dv} - \left( \rho \int_0^1 \beta'(A) r(A) \, dv \right)^{-1} \bar{Z}. \quad (2.8)$$

These general equilibrium relations are specialized in the particular examples we consider in the following sections.

3. The Seller's Problem

We assume that, initially, all the traders in the speculative markets have homogeneous beliefs, which are summarized by a common prior distribution on $\mathcal{F}$ with mean $\bar{F}$ and variance 1.\(^8\) Traders cannot obtain private information, except by buying it from a monopolistic seller, as described below.\(^9\)

Suppose that a single monopolistic seller possesses some information about the payoff of the risky asset. Our analysis has implications for the way in which information is collected, as is discussed in the concluding remarks. However, the production of information is not directly modeled here and we assume that the seller is endowed with the information at the start of the model. Let $\bar{F} + \bar{\theta}$ be a sufficient statistic for $\mathcal{F}$ given all the information the seller observes, where $\bar{\theta}$ is normally distributed and independent of $\bar{F}$. While the seller cannot sell more precise information than his signal $\bar{F} + \bar{\theta}$, we do not force him to sell his signal "as is." Instead, he can sell signals having lower precision, which can be costlessly generated by adding to $\bar{F} + \bar{\theta}$ normally distributed deviates. We are concerned, in part, with the circumstances under which the seller would like to add noise to his information before selling it, and with the optimal way for him to do so.

We assume that the statistical properties of the information signals sold are common knowledge between the seller and the traders.\(^{10}\) For this

\(^8\)This normalization is without loss of generality, since the units or shares of the risky asset can always be defined so that it is satisfied. In general, if the variance of $\mathcal{F}$ is $\sigma^2$, then we redefine units so that a normalized unit is worth $1/\sigma$ units of the original asset. The variance of the per capita supply is then $1/\sigma^2$ times the supply variance of the original units.

\(^9\)The assumption of initial homogeneity of information, together with that of identical risk attitudes, guarantees that traders are identical in their valuation of and demand for information. Relaxing either assumption would create complex issues relating to discrimination, which are beyond the scope of this paper. See the concluding remarks.

\(^{10}\)Several papers have considered incentive contracts to induce a provider of information to reveal truthfully both the quality of his signal and its realization. See, for example, F. Allen [4] and Bhattacharya and Pfleiderer [5].
assumption to make more sense, and to enable us to focus on the market for information, it is also assumed that the seller cannot (or simply does not) trade in the speculative market. The moral hazard considerations behind the assumptions should be clear. If the seller traded, his incentives to reveal truthfully the information he promised would be severely distorted. Moreover, assuming the monopolist is competitive as a trader, his potential trading profits in our model are negligible relative to profits from selling information. This is because the number of traders is infinite, and, because, due to the random supplies, there are non-trivial per capita gains from trading. Thus (if incentive problems do not arise), letting the monopolist trade would not change our results.\(^{11}\)

We defer to later in the paper a discussion of the mechanism by which the information market operates. Our analysis begins, therefore, by assuming that the seller can discriminate perfectly in selling information. (It is shown below that the ability to discriminate is not used by the monopolist in a homogeneous market.) For now, imagine that the seller approaches each potential buyer \(v \in [0, 1]\) and offers to sell him or her a signal, \(\hat{\hat{Y}} = \hat{F} + \hat{\theta} + \hat{\eta}\) at some price. The buyer can either accept or reject the seller’s offer. If he accepts, he pays the price in units of the riskless asset, and obtains the signal. He can still use the equilibrium price if it provides additional information given his signal. If he rejects, he will be privately informed in the subsequent trading, but will use the price to infer some information freely. Buyers of information can use the information only for making their trading decisions. They cannot resell it to other traders.\(^{12}\)

The fact that the seller may wish to add noise to his information before selling can be seen most clearly when considering an extreme case where the distribution of \(\hat{\theta}\) is degenerate; i.e., the seller has perfect information about \(\hat{F}\). If the perfect information is sold as is to a positive fraction of the market, the rational expectations price is equal to \(\hat{F}\). Thus, the information is completely revealed to traders who do not purchase and the seller cannot

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\(^{11}\) In general, models in which both trading and selling information are possible introduce tradeoffs between trading profits and profits from selling information. Even if an owner of information must choose between trading on the information and selling it (but cannot do both), he may choose to trade rather then sell the information under some conditions. What he chooses to do depends, of course, on the speculative market structure and on the seller’s preferences. These issues are interesting, but are not addressed here. A simple way to justify our assumption is to assume that the seller is extremely risk averse and is not interested in trading in the risky asset.

\(^{12}\) Reselling might be excluded by a contract, or it might be infeasible for various reasons. For example, the information may be sold immediately before trading takes place, and may become useless soon thereafter. Traders would not have time to complete a secondary round of transactions in the information. See our concluding remarks for more discussion.
charge a positive price for the signal. It is therefore never optimal to sell $\bar{P}$.

The monopolist's profits are generally a complex function of the allocation of the information. Since the asset market is assumed to be in a rational expectations equilibrium, traders have correct expectations with regard to the allocation of information. For a given admissible allocation of information, the value of a private signal to a trader, which is the certainty equivalent of the information, depends on the joint distribution of the asset payoff, the signal, and the equilibrium price at the allocation. It is determined by comparing the level of ex ante expected utility attained when the private signal is observed to the level that would be attained if only the equilibrium price was observed. The profit the monopolist obtains from a particular allocation depends, therefore, on the value each trader places on the signal assigned to him, when the allocation (and therefore the characteristics of the equilibrium price it produces) are taken as given. Recall that $r^v(A)$ is the total precision of the forecasts based on signal $\bar{P}$ and the equilibrium price $\bar{P}(A)$, and let $r^{\sigma^2}(A) = (\text{Var}(\bar{P} \mid \bar{P}(A)))^{-1}$ be the precision of the forecast based only on the price. We show in [2] that the monetary value of a signal $\bar{P}^v$ to a trader with a risk tolerance coefficient $\rho$, who can observe the equilibrium price $\bar{P}(A)$, is a simple function of the increment in the precision of the prediction brought about by observing $\bar{P}^v$. Letting this value be denoted by $\phi^v(A)$, we have

$$\phi^v(A) = \left(\frac{\rho}{2}\right) \log \left(\frac{r^v(A)}{r^{\sigma^2}(A)}\right). \quad (3.1)$$

This is, for a given allocation, the maximum price the seller can charge trader $v$ for the signal $\bar{P}^v$. Since the monopolist extracts all surplus, profits are

$$\Pi(A) = \int_0^1 \phi^v(A) dv. \quad (3.2)$$

An allocation $A$ is feasible if it is admissible as defined in the previous sec-

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13 Note that the fact that infinitely precise information is worthless is a consequence of the price taking assumptions we have made in describing traders' behavior. This extreme situation would not occur in models in which individual traders' private signals have positive weight in prices and traders account for the effect of their demands on price. Nevertheless, our qualitative conclusions in this section and in the rest of the paper would not change. It can be shown that even in models that are not perfectly competitive (e.g., as in Kyle [12]), a monopolistic seller of information wishes to add noise to a very precise signal he sells when the noise level in prices is sufficiently small.

14 Of course, traders are, in the end, indifferent between being informed and uninformed, since the monopolist extracts all surplus. See Section 7 for some discussion of the selling mechanism.
tion, and if, in addition, the signals of each trader have the form $\bar{F} + \bar{G} + \bar{E}$. (The variables $\bar{E}$ may have infinite variance, corresponding to an uninformed trader.) The seller's problem is to choose a feasible allocation that maximizes profits.

For each feasible allocation of information $A$, the normalized equilibrium price is of the form

$$P_n(A) = \bar{F} + \bar{G} + \bar{h}(A) + g(A)\bar{Z},$$  \hspace{1cm} (3.3)

where

$$\bar{h}(A) = \frac{\int_0^1 \alpha^2_n(A) \bar{E} \, dv}{\int_0^1 \alpha^2_n(A) \, dv}. \hspace{1cm} (3.4)$$

The informativeness of the normalized price is determined by the variance of the sum $\bar{G} + \bar{h}(A) + g(A)\bar{Z}$, which, by independence, is the sum of the variances. The first term $\bar{G}$ is a parameter of the model, and is exogenous. By varying the amount of correlation between the added noise in the signals sold to traders, the seller has complete control over the variance of the average error term $\bar{h}(A)$ in the normalized price. (For example, if photocopied versions of $\bar{F} + \bar{G} + \bar{h}$ are sold to all buyers, then $\bar{h}(A) = \bar{h}$; if the error terms in individual signals are mutually independent across traders, then $\bar{h}(A) = 0$.\footnote{This follows from an extension to the law of large numbers. See footnote 17, below.}) The parameter $g(A)$, which is the coefficient of $\bar{Z}$ in the normalized price, affects the amount of exogenous supply noise in the price, and therefore measures, for a given $\sigma^2_2$, how precise the price is relative to the aggregated information of the informed traders in the market. Thus, it determines how well the price transmits the aggregated information. When $g(A)$ is large in absolute value, the effects of the externality are relatively weak. However, if the seller sells more or better information, $g(A)$ is smaller in absolute value, and the effects of the externality are more intense. In general, $g(A)$ is a complex equilibrium function of the allocation.

The main elements of the seller's problem can be seen in the simple case where identically distributed signals are sold. In this case the allocation of information is defined by (i) the fraction $\lambda$ of the market that buys information, (ii) a parameter $s$ that represents the quality of each signal sold; e.g., the variance of the signal given $\bar{F}$, and (iii) the correlation among signals sold to different traders. Among such allocations the seller wishes to maximize the value to each trader of being informed relative to being uninformed times the fraction of informed traders $\lambda$.

In general, the precision of the price increases when the seller sells higher
quality signals or sells to more traders. Either type of change allows the
uninformed traders to learn more through the price and this affects the
value of being informed relative to being uninformed. Increasing $s$ (selling
noisier information) and decreasing $\lambda$ (selling to fewer traders) are two
distinct, but interrelated, ways for the seller to alleviate the adverse effects of
the leakage of information through prices. Both are costly to the seller in
the sense that they would clearly not be used absent externalities. Similarly,
if the equilibrium price did not transmit information, the value of each
signal (and therefore the seller's profits) would be independent of the con-
ditional correlation between the signals sold to different traders. In the
presence of the externalities, the effect of this correlation is somewhat sub-
tle, and is discussed in Section 5 and 6.

4. THE CASE OF PHOTOCOPIED INFORMATION

This section analyzes the case where the seller is restricted to selling the
same (i.e., photocopied) signals to all buyers but where either directly or by
pricing the information appropriately, he can control the fraction of the
market that becomes informed. The seller can determine (i) the precision of
the signal, as long as it is not more precise than the information he obser-
ves, and (ii) the price of the information, or, equivalently, the fraction of
the market that is privately informed.\footnote{The equivalence of setting prices and quantities follows here since for a given signal, each
price is consistent with a unique $\lambda \in [0, 1]$ of informed traders. Conversely, the fraction $\lambda \in (0, 1)$ of informed traders determines a unique price for the information. At $\lambda = 0$ many
prices are possible, all of which lead to zero profits, while for $\lambda = 1$ the seller clearly sells at the
highest price consistent with everyone buying information.} That is, the seller can choose
parameters $\lambda_s$ and $s_s$, where $\lambda_s$ is the fraction of the total market that is
informed and $s_s$ is the variance of the error added to $\hat{F} + \hat{b}$ to create the
private signal. (The subscript $s$ stands for a well known trade name in the
photocopying industry.) Note that $\lambda_s$ and $s_s$ determine the allocation
completely. After the sale of information, a fraction $\lambda_s$ of the traders know
both the equilibrium price and $\hat{P} = \hat{F} + \hat{b} + \hat{\eta}$, where $\text{Var}(\hat{\eta}) = s_s$. Note that
these traders only use $\hat{P}$ to predict $\hat{\eta}$, since this is the best information
available to the market. The remaining traders, fraction $1 - \lambda_s$, use only
the equilibrium price to forecast the risky asset payoff.

To analyze the seller's problem, we first find profits as a function of $\lambda_s$
and $s_s$. The special case of a perfectly informed seller ($\sigma^2_\eta = 0$) is then con-
sidered and the optimal $\lambda_s$ and $s_s$ for this seller is derived. We then state
the general solution, which applies when the seller may be imperfectly informed.

The profits earned by a seller of photocopied information depend upon
the precision levels of the informed and the uninformed. For a given \( \lambda_x \) and \( s_x \), it can be shown that the normalized rational expectations equilibrium price \( \overline{P}_N(\lambda_x, s_x) \) is given by

\[
\overline{P}_N(\lambda_x, s_x) = \overline{P} + \overline{\theta} + \overline{\eta} - \frac{(s_x + \sigma_\theta^2)}{\lambda_x \rho} Z.
\]  

(4.1)

The total precision of the uninformed traders in the equilibrium is therefore (using the normalization \( \sigma_x^2 = 1 \))

\[
r(\lambda_x, s_x) = 1 + \frac{1}{s_x + \sigma_\theta^2 + ((s_x + \sigma_\theta^2)/\lambda_x \rho)^2 \sigma_\zeta^2},
\]

while that of the informed traders (who do not need to use the price as an additional signal) is given by

\[
r(\lambda_x, s_x) = 1 + \frac{1}{s_x + \sigma_\theta^2}.
\]

(4.3)

The monopolist’s problem is

\[
\max_{\lambda_x \in [0, 1], \theta \in [0, \lambda_x]} \lambda_x \rho \log \left( \frac{r(\lambda_x, s_x)}{r(\overline{\lambda}_x, s_x)} \right).
\]

(4.4)

By substituting (4.2) and (4.3) into (4.4) and simplifying we obtain

\[
\max_{\lambda_x \in [0, 1], \theta \in [0, \lambda_x]} \lambda_x \rho \log \left( 1 + \frac{1}{(\lambda_x \rho/\sigma_\zeta^2)(1 + 1/(s_x + \sigma_\theta^2)) + (s_x + \sigma_\theta^2)} \right).
\]

(4.5)

We can now see clearly the way profits depend on \( \lambda_x \) and \( s_x \), and the effects of externalities. First note that if the price did not transmit any information, then \( r^\rho \) would be equal to one (since \( \sigma_x^2 = 1 \) in our normalization). In this case the seller is able to charge for the incremental increase in precision relative to the prior on \( \overline{P} \). The value of the signal \( \overline{P} + \overline{\theta} + \overline{\eta} \) would therefore be

\[
\left( \frac{\rho}{2} \right) \log \left( 1 + \frac{1}{s_x + \sigma_\theta^2} \right).
\]

When prices do transmit information, the “benchmark” for evaluating information is no longer the prior distribution but the conditional distribution of \( \overline{P} \) given \( \overline{P}(\lambda_x) \). The extra term in the denominator in (4.5), \((\lambda_x \rho^2/\sigma_\zeta^2)(1 + 1/(s_x + \sigma_\theta^2))\), is related to the amount the seller must reduce
the price charged due to the fact that the asset price reveals information. As is intuitive, this reduction is increasing in \( \rho \), the risk tolerance of the traders, and in \( \lambda_s \), the fraction of informed traders, and is decreasing in \( \sigma_Z^2 \), the variance of the per capita supply. If the informativeness of the asset price could be held constant, then an increase in \( s_s \) would never be desirable. However, the dilution of the value of information for a fixed \( \lambda_s \) is larger when \( 1/(s_s + \sigma_Z^2) \), the precision of signal sold, is higher. This is why the seller may wish to choose a positive \( s_s \).

Let the optimal allocation for a perfectly informed monopolist, \((\sigma_Z^2 = 0)\), be denoted by \((\lambda^*_x, s^*_x)\). For each fixed fraction \( \lambda_x \) of the market the monopolist might sell to, there is an optimal level of noise to add to \( \hat{p} \). That level, \( s^*_x(\lambda_x) \), maximizes the price the monopolist can charge when selling to the fraction \( \lambda_x \). Equivalently, \( s^*_x(\lambda_x) \) maximizes

\[
\frac{1}{(\lambda_x \rho/\sigma_Z)^2(1 + 1/s_s) + s_s},
\]

or minimizes:

\[
\left(\frac{\lambda_x \rho}{\sigma_Z}\right)^2 \left(1 + \frac{1}{s_s}\right) + s_s.
\]

It is easy to verify that \( s_s = \lambda_x \rho/\sigma_Z \) minimizes (4.7). Substituting \( s^*_x(\lambda_x) = \lambda_x \rho/\sigma_Z \) into the profit function yields profits as a function of \( \lambda_x \):

\[
\Pi(\lambda_x, s^*_x(\lambda_x)) = \left(\frac{\lambda_x \rho}{2}\right) \log \left(1 + \frac{1}{(\lambda_x \rho/\sigma_Z)(2 + \lambda_x \rho/\sigma_Z)}\right).
\]

Let \( m = \lambda_x \rho/\sigma_Z \). Then maximizing \( \Pi(\lambda_x, s^*_x(\lambda_x)) \) is equivalent to choosing \( m \in [0, \rho/\sigma_Z] \) to maximize

\[
m \log \left(1 + \frac{1}{m(2 + m)}\right).
\]

In the Appendix we show that there exists a unique \( m^* > 0 \) that maximizes (4.9) subject only to the constraint that \( m \) be nonnegative, and that the optimal policy for the perfectly informed monopolist is to set \( \lambda^*_x = \min(\sigma_Z m^*/\rho, 1) \). The added noise level \( s^*_x \) that maximizes profits is then \( \lambda^*_x \rho/\sigma_Z \).

The parameter values for which \( \lambda^*_x < 1 \) are those satisfying \( \sigma_Z/\rho < 1/m^* \). This describes situations where the relative informativeness of the price for a given signal is high. Thus, the seller faces strong dilution in the value of the product when selling to more and more traders and prefers both to add noise and to restrict the fraction of informed traders. In the resulting
equilibrium there are two classes of traders, informed and uninformed, even though the traders are ex ante homogeneous. There is, unfortunately, no explicit solution for \( m^* \) and therefore for \( \lambda_o^* \) when it is strictly less than 1. However, it is easy to show that \( \lambda_o^* \) is increasing in \( \sigma_Z/(\rho) \), as is intuitive. The noisier the supply (or the lower the risk tolerance), the larger the fraction of the market that the seller "can afford" to inform, since prices transmit less information.

Now consider the imperfectly informed seller \( (\sigma_0^2 > 0) \). If \( \sigma_0^2 \leq s_o^2 = \lambda_o^*/\sigma_Z \), then this seller can achieve the same allocation and same profits as the perfectly informed seller, by setting \( s^*_o = s_o^2 - \sigma_0^2 \) and \( \lambda^*_o = \lambda_o^* \). If, however, \( \sigma_0^2 > s_o^2 \), then it follows from the uniqueness of the solution \( (\lambda_o^*, s_o^*) \) of the perfectly informed seller’s problem, that the imperfectly informed seller achieves lower profits.

It turns out that when \( \sigma_0^2 > s_o^2 \) the seller prefers to sell the signal as is, without adding noise. That is, if the seller’s best information is less precise than the signal he would have chosen to sell if he was perfectly informed, then he does not wish to decrease its precision before selling it. What may change in the optimal solution is the choice of \( \lambda_o^* \), the fraction of the market that becomes informed. Intuitively, since the signal sold is now less precise, the price is less informative for a given fraction of informed traders. This means that the externality effects are less severe, so that the seller can sell to more traders. We confirm this intuition: if \( \lambda_o^* < 1 \) then a seller with \( \sigma_0^2 > s_o^2 \) sells to a fraction \( \lambda_o^* \) that is strictly greater than \( \lambda_o^* \).

The following proposition summarizes the analysis for a seller restricted to selling one piece of photocopied information. Additional details of the proof are in the Appendix.

**Proposition 1.** There is a unique pair \( \lambda_o^*, s_o^* \) which solves (4.5), and it is given by

\[
\lambda^*_o = \min \left( 1, \frac{\sigma_Z}{\rho} m^* \right), \quad \text{if} \quad s^*_o > 0; \\
\lambda^*_o = \min \left( 1, \frac{\sigma_Z}{\rho} m(\sigma_0^2) \right), \quad \text{if} \quad s^*_o = 0,
\]

where

\[
m^* = \arg \max_m m \log \left( 1 + \frac{1}{m(2 + m)} \right),
\]

and

\[
m(x) = \arg \max_m m \log \left( 1 + \frac{1}{m(2 + m) + (m - x)^2/x} \right).
\]
The optimal solution to the seller's problem is depicted in Fig. 1, which describes the different regimes of the solution as a function of the model's parameters, $\sigma_Z/\rho$ and $\sigma_0^2$. The space of parameters is divided according to whether noise is added and whether the seller sells to the entire market or not. For example, in region B noise is added and information is sold only to a fraction of the market. In region A no noise is added but still only a fraction of the market is informed. Note that when $\sigma_Z/\rho < 1/m^*$, the decision to add noise depends only on the precision of the seller's information and not on the amount of supply noise. In this range of parameters the only response to an increase or decrease in $\sigma_Z/\rho$ is a change in $\lambda_x$. Similarly, when $\sigma_0^2 < m^*$, whether information is sold to all traders or not is independent of $\sigma_0^2$. It only depends on the value of $\sigma_Z/\rho$. Loosely speaking, the instruments of adding xeroxed noise and restricting the fraction of informed traders have distinct roles in the optimal solution. Adding noise is primarily used when the seller's information is too precise, and restricting the fraction of informed traders is used primarily when the supply noise is small or risk tolerance is high. These results are discussed in more detail in the following sections.

5. Photocopied vs Personalized Noise

There are many ways for the seller to add noise to his information other than selling the same signal to all buyers. We now consider allocations
obtained when the seller adds to his information a fresh realization of mutually independent noise terms to create each of the signals sold. We refer to the signals sold and to the resulting allocations as "personalized." (As discussed later, our analysis of this case covers the more general allocations where the seller may add both photocopied noise and personalized noise, as well as some allocations where a finite number of different signals are sold.) We also allow the seller to add personalized error terms with different variances; i.e., sell different commodities to different traders. A special case of this is to sell to some traders signals with infinite variance (at zero price).

Three important features distinguish the current model from that of the previous section. First, when all informed traders get an identical signal, informed traders do not learn any additional information from the price. However, when private information is diverse, as when personalized noise is added, all traders use the equilibrium prices as information. Second, since the seller adds independent errors to his information, and since there is a large number of traders, traders in the aggregate have as much information as the seller himself, even though each individual trader's signal is noisier. Third, because each trader is small, none of the individual error terms affects the price. Only the realization of what is common to all the signals, namely the seller's information, affects equilibrium. Thus, each trader's information is conditionally independent of the equilibrium price given the information the seller has. These facts are crucial to the results of the rest of the paper.

Can the seller obtain higher profits by personalization than by photocopying? It is clear that if the parameters of the problem are such that the seller chooses not to add photocopied noise (i.e., in regions A or D of Fig. 1), then he cannot do worse by personalizing. This follows because a special case of the allocations we termed "personalized" is an allocation where no noise is added to the signals of some of the traders, while an infinite amount of personalized noise is added to those of the rest (i.e., $s_p = 0$ for $v \in [0, \lambda]$ and $s_p = \infty$ for $v \in (\lambda, 1]$), where $s_p$ is the variance of the personalized noise in the signal sold to trader $v$). Thus, personalization at least weakly dominates photocopying in these regions.

We show below that the seller can generally obtain higher profits by personalization than by photocopying. Although this is proved for the general case, where the seller may be imperfectly informed, our results are most easily illustrated for the special case where the seller knows $\bar{F}$ perfectly ($\sigma_2^2 = 0$). In the context of personalized signals this is the only value of $\sigma_2^2$ for which the solution for the rational expectations equilibrium is tractable—see Admati [1] and Hellwig [9].

In this section we consider only allocations in which the seller sells identically distributed signals to a fraction of the market. This facilitates the
comparison between the personalized and the photocopied case. When the fraction of informed traders is fixed and each informed trader purchases an identical commodity, the seller’s profits are proportional to the value of the identically distributed signals sold to each trader. The only difference between the photocopied and personalized allocations is then the level of statistical dependence (perfect correlation vs. independence) between the noise terms added to create the signals. The optimal allocation for the general case where personalization is possible is discussed in the next section.

If $\sigma_p^2 = 0$, then for any given values of $\lambda_x, \lambda_p, s_x$ and $s_p$ the normalized prices in the photocopied and personalized allocations are given by

$$ P_{Nx} = \bar{P} + \bar{\eta} - \left( \frac{s_x}{\lambda_x \rho} \right) Z,$$

(5.1)

and

$$ P_{Np} = \bar{P} - \left( \frac{s_p}{\lambda_p \rho} \right) Z,$$

(5.2)

respectively.\(^{17}\)

Now, let us assume that $\lambda_x = \lambda_p = \lambda$ and $s_x = s_p = s$; i.e., the same fraction of the market is informed and the intrinsic quality of the signals sold is the same for each case. Then the coefficient of supply in the normalized price is the same in both cases, so that the only difference between (5.1) and (5.2) is the inclusion of the photocopied noise $\bar{\eta}$ in $P_{Nx}$.

It is simple, given (5.1) and (5.2), to derive the precisions obtained by informed and uninform ed traders in each allocation. In the photocopied allocation the precisions of each informed and uniformed are

$$ r_x(\lambda, s) = 1 + \frac{1}{s},$$

(5.3)

and

$$ r_{x'}(\lambda, s) = 1 + \frac{1}{s + s^2 \sigma_x^2 / \lambda^2 \rho^2},$$

(5.4)

\(^{17}\) As already noted, the fact that individual error terms, $\xi$, do not appear in (5.2) relies on the heuristic definition that if $\xi''$ are mutually independent for $v \in [0, 1]$, $E \xi'' = 0$, and $\text{Var}(\xi'')$ is bounded then $\int_0^1 \xi'' dv = 0$ almost surely. (For a more complete discussion see [1, 5, 10].) In our context, it can be shown from the equilibrium relations that $\text{Var}(s_x, \xi')$ is bounded, where $s_x$ is the coefficient of $P$ in the price, which justifies writing $\int_0^1 x, \xi' dv = 0$. 
respectively. In the personalized allocation the precisions are

\[ r_p(\lambda, s) = 1 + \frac{1}{s} \frac{\lambda^2 \rho^2}{\sigma_Z^2}, \tag{5.5} \]

and

\[ r_p^s(\lambda, s) = 1 + \frac{\lambda^2 \rho^2}{s \sigma_Z^2}. \tag{5.6} \]

Note that both informed and uninformed traders have higher precisions in the personalized allocation than in the photocopied allocation. That is, both the benchmark for evaluating information and the contribution of each signal to the total precision of an informed trader are higher when signals are personalized.

The monetary value of a private signal in each allocation is determined by the (logarithm of the) ratio of the precisions of informed and uninformed traders in equilibrium. From (5.3) and (5.4), the value of the signal in the photocopied allocation is

\[ \frac{\rho}{2} \log \left( 1 + \frac{1}{(\lambda^2 \rho^2 / \sigma_Z^2)(1 + 1/s) + s} \right). \tag{5.7} \]

Similarly, using (5.5) and (5.6), the value of each personalized signal is

\[ \frac{\rho}{2} \log \left( 1 + \frac{1}{(\lambda^2 \rho^2 / \sigma_Z^2)(1/s) + s} \right). \tag{5.8} \]

To interpret these expressions recall that if the price did not transmit any information then the value of each signal sold (photocopied or personalized) would be \((\rho/2) \log(1 + 1/s)\). The extra terms in (5.7) and (5.8), \((\lambda^2 \rho^2 / \sigma_Z^2)(1 + 1/s)\) in the photocopied case and \((\lambda^2 \rho^2 / \sigma_Z^2)(1/s)\) in the personalized, are related to the dilution in the value of the signals sold that is due to the leakage of some of the aggregated private information through the asset price. Note that for any fixed values for \(\lambda\) and \(s\) the term determining the discount is larger for the photocopied allocation than it is for the personalized allocation. The term in the denominator of (5.7) that does not appear in (5.8), \((\lambda^2 \rho^2 / \sigma_Z^2)\), indicates that there is an additional discount in the price of the photocopied signal over and above the discount in the price of a personalized signal. It follows that for each \(\lambda\) and \(s\), although the equilibrium price is more precise in the personalized allocation than it is in the photocopied allocation, the price the seller can charge for each personalized signal is strictly higher than the price he can charge for the photocopied signal.

Intuitively, the value of a personalized signal does not deteriorate as fast as the value of a photocopied signal when more traders are informed. In
both cases dilution occurs because the price is becoming more informative, but in the photocopied case the dilution is more pronounced. This is because in the photocopied case the (normalized) equilibrium price is simply a noisier version of the photocopied signal; it is equal to the photocopied signal plus some noise. An individual trader who acquires the photocopied signal gains because he can eliminate the additional noise in the price, but when this additional noise is small, the gain is small. The gain would be larger if the trader received a signal with an error that is independent of the price, as in the personalized allocation; in this case both the private signal and the price provide useful information to an informed trader. Personalized signals held by other traders affect the informativeness of the price, but have no other impact on the information content of a fresh personalized signal.\footnote{It would be misleading to conclude from our discussion that the informational content of a signal is generally lower the more conditionally correlated it is with information that is otherwise available to a trader. In general, suppose a trader observes $\bar{F} + \hat{\varnothing}_1$ and a signal $\bar{F} + \hat{\varnothing}_2$ is available. If the variances of $\bar{F}, \hat{\varnothing}_1$, and $\hat{\varnothing}_2$ are $1, s_1$ and $s_2$ respectively, and $c$ is the correlation between $\hat{\varnothing}_1$ and $\hat{\varnothing}_2$, then the increment in total precision brought about by using the second signal in addition to the first to forecast $\bar{F}$ is \((1/s_2)(1 - c\sqrt{s_2/s_1})(1 - c^2)\). The effect of conditional correlation (positive or negative) on this value is ambiguous. For example, if $\hat{\varnothing}_2 = \hat{\varnothing}_1$ the increment is zero, while if $\hat{\varnothing}_2 = 2\hat{\varnothing}_1$ it is infinite. The personalized allocation (with $\sigma^2_F = 0$) corresponds to $c = 0$, while in the photocopied allocation $c = \sqrt{s_2/s_1}$. In these cases the private signal increases total precision (relative to the price) by \((1/s_2)(1 - c^2)\). This increment is decreasing in the absolute value of $c$, and is highest when $c = 0$.}

We have shown that if the seller is perfectly informed, then for any $\lambda$ and $\gamma$ selling personalized signals with conditional variance $s$ to a fraction $\lambda$ of the traders yields strictly higher profits than selling a photocopied signal with conditional variance $s$ to the same traders.\footnote{In passing, it is interesting to note that when the seller is perfectly informed (but not otherwise), the choice of $s$ which maximizes the price he can charge (and therefore profits) when selling to a given fraction $\lambda$ is the same in both the photocopied and the personalized allocations. In each case the optimal $s$ for a given $\lambda$ is $\lambda \rho \sigma_D$. Thus, a perfectly informed seller who is forced to sell (identically distributed signals) to fraction $\lambda$ of the market sells signal(s) having the same intrinsic quality whether he photocopies or personalizes.} The next proposition shows that adding personalized noise strictly dominates adding photocopied noise also when the seller is imperfectly informed and sells to the same fraction of the traders.

**Proposition 5.1.** For any allocation of information where a positive fraction $\lambda$ of the traders are privately informed and each informed trader observes the signal $\bar{Y} = \bar{F} + \hat{\varnothing} + \hat{\varnothing}$ with $\text{Var}(\hat{\varnothing}) > 0$, there exists an allocation where (i) the same fraction $\lambda$ of the traders are privately informed, (ii) the signal observed by informed trader $v$ is $\bar{Y} = \bar{F} + \hat{\varnothing} + \hat{\varnothing}'$ where $\hat{\varnothing}'$ are independent across traders (and identically distributed), and (iii) the value of each private information signal is strictly higher than in the original allocation.
Proof. Let $A_s$ be an allocation in which the monopolist sells $Y = F + \tilde{Y} + \tilde{\eta}$ to fraction $\lambda$ of the market and let $s_x = \text{Var}(\tilde{\eta})$. Then the precisions of the informed and uniformed are

$$r(A_s) = 1 + \frac{1}{\sigma_0^2 + s_x},$$ (5.9)

and

$$r^2(A_s) = 1 + \frac{1}{\sigma_0^2 + s_x + ((\sigma_0^2 + s_x)/\rho \lambda)^2 \sigma_Z^2}.$$ (5.10)

Now consider an alternative allocation in which the monopolist sells to fraction $\lambda$ of the market signals $Y = F + \tilde{Y} + \tilde{\eta}$, where the $\tilde{\eta}$ are i.i.d. with variance $s_p$. For this allocation the precisions of the informed and uniformed are

$$r(A_p) = 1 + \frac{1}{\sigma_0^2 + s_p(g(A_p))^2 \sigma_Z^2/(s_p + (g(A_p))^2 \sigma_Z^2)},$$ (5.11)

and

$$r^2(A_p) = 1 + \frac{1}{\sigma_0^2 + (g(A_p))^2 \sigma_Z^2},$$ (5.12)

where $g(A_p)$ is the coefficient of $\tilde{Z}$ in the normalized price. Assume $s_p$ is chosen so that

$$\frac{s_p(g(A_p))^2 \sigma_Z^2}{s_p + (g(A_p))^2 \sigma_Z^2} = s_x.$$ (5.13)

Since $(g(A_p))^2 \sigma_Z^2 > 0$, $s_p$ is larger than $s_x$. If condition (5.13) is met, $r(A_s) = r(A_p)$; i.e., the informed are equally well informed in each allocation. Hence the desired result will follow if we show that uninformed traders in the photocopied allocation are better informed than they are in the personalized allocation. This requires that we show

$$(g(A_p))^2 \sigma_Z^2 > s_x + \left(\frac{\sigma_0^2 + s_x}{\rho \lambda}\right)^2 \sigma_Z^2.$$ (5.14)

Solving (5.13) for $(g(A_p))^2 \sigma_Z^2$, we obtain

$$(g(A_p))^2 \sigma_Z^2 = \frac{s_p^2 s_x}{s_p - s_x}.$$ (5.15)

The equilibrium relation between $g(A_p)$ and $s_p$ can be shown to be

$$\sigma_0^2 s_p + (g(A_p))^2 \sigma_Z^2 (\sigma_0^2 + s_p) + \rho \lambda (g(A_p))^2 \sigma_Z^2 = 0.$$ (5.16)
Combining (5.16) with (5.15) and simplifying, we obtain

$$\frac{s_p}{s_x} \left( \frac{s_p}{s_x} - 1 \right) = \left( \frac{\rho \lambda}{\sigma_x^2 + s_x} \right)^2 \frac{s_x}{\sigma_2^2}. \quad (5.17)$$

This implies that

$$\frac{s_p s_x}{s_p - s_x} = s_x + \frac{s_p}{s_x} \left( \frac{\sigma_x^2 + s_x}{\rho \lambda} \right)^2 \sigma_2^2$$

$$> s_x + \left( \frac{\sigma_x^2 + s_x}{\rho \lambda} \right)^2 \sigma_2^2. \quad (5.18)$$

Thus, keeping the total precision achieved by informed traders equal in both allocations, we find that the precision of the uninformed traders is lower in the allocation where signals are personalized than where they are photocopied.

Consequently, the seller can always obtain strictly higher profits than those obtained by adding photocopied noise by adding, instead, an appropriate amount of personalized noise. This can be done even if the same fraction of traders are informed and all the personalized signals are identically distributed.

In the analysis above we have restricted the seller to a specific subset of the personalized allocations. In particular, the level of personalized noise was chosen (in the proof of Proposition 5.1) to keep informed traders as well informed as they are in the corresponding photocopied allocation. The seller may do still better by choosing optimally the added noise level in each trader’s personalized signal. The optimal allocation when personalization is possible, found in the next section, takes a particularly simple form and provides a sharp contrast with the optimal photocopied allocation.

### 6. Optimal Personalized Allocations

Suppose the monopolist sells to trader \( v \) the personalized signal \( \tilde{Y}^v = \bar{F} + \bar{\theta} + \tilde{\varepsilon}^v \), where \( \tilde{\varepsilon}^v \) are independent and \( \text{Var}(\tilde{\varepsilon}^v) = s_\pi^2 \), for \( s_\pi \in [0, \infty] \). Because of the independence of the added error terms \( \tilde{\varepsilon}^v \), they disappear in the aggregation so that the equilibrium price for such an allocation \( A \) is of the form

$$\bar{P}(A) = a_0(A) + \left( \int_0^1 a_1(A) \; dv \right) (\bar{F} + \bar{\theta}) + a_2(A) Z. \quad (6.1)$$
The normalized price can be written as
\[ \tilde{P}_N(A) = \bar{P} + \bar{\theta} + g(A) \tilde{Z}, \] (6.2)
where
\[ g(A) = \alpha_2(A) \left( \int_0^1 \alpha'_2(A) \, dv \right)^{-1}. \] (6.3)

Since the seller can add noise with any desired variance, the coefficient \( g(A) \), and therefore profits, are generally complex functions of the allocation. A major simplification of the seller's problem in this context is obtained in the following proposition, where we show that it is optimal for the seller to set \( s^*_p \equiv s_p \) for some constant \( s_p \). That is, when noise is added in a personalized fashion, the seller (strictly) prefers allocations of information that are perfectly symmetric across traders. This implies that, in contrast with the analysis of the case with one photocopied signal, there are no uninformed traders in the resulting equilibrium—information is sold to all the traders, no matter how small is the supply noise or how risk tolerant are the traders.

**Proposition 6.1.** Let \( A \) be an admissible allocation of information where individual signals are conditionally independent given \( \bar{P} + \bar{\theta} \). Let \( \bar{r}(A) = \int_0^1 r'(A) \, dv \) be the average total precision (given both signals and prices) attained by traders in the allocation \( A \). If \( \int_0^1 (r'(A) - \bar{r}(A))^2 \, dv \neq 0 \), then there exists another allocation \( A' \) where signals are conditionally independent and identically distributed given \( \bar{P} + \bar{\theta} \), and such that \( \Pi(A') > \Pi(A) \).

**Proof.** For a given allocation \( A \), let \( \bar{r}(A) \) be the average total precision of traders (based on signals and prices) in equilibrium, and consider the symmetric allocation \( A' \) where all traders obtain total precision identically equal to \( \bar{r}(A) \); i.e., \( r'(A') = \bar{r}(A) \) for every \( v \). We show below that (i) such an allocation exists, and (ii) the informativeness of the equilibrium price is the same in both \( A \) and \( A' \); i.e., \( g(A) = g(A') \). Thus, a trader who does not become privately informed has the same amount of information in both allocations. The result then follows from Jensen's inequality, since, as long as the price remains equally informative, the seller's profits are an increasing and concave function of the precisions of the informed traders.

As discussed in Section 2, in equilibrium
\[ g(A) = -\left( \int_0^1 \rho \beta^*_1(A) \, dv \right)^{-1}, \] (6.4)
where \( \beta^*_1(A) \) is the regression coefficient of \( \tilde{Y}^v \) in the forecast made by
trader \( v \) (given \( \bar{P}^* \) and \( \bar{P}(A) \)), and \( r^v(A) \) is the precision of this conditional distribution. In the Appendix it is shown that for a given value of \( g(A) \), the variance of \( \bar{\varepsilon}^v \) and the total precision of trader \( v \), \( r^v(A) \), are related through

\[
\sigma^v_p = \frac{(g(A))^2 \sigma^2_{\bar{\varepsilon}}(1 - (r^v(A) - 1) \sigma^2_{\bar{\varepsilon}})}{(\sigma^2_{\bar{\varepsilon}} + (g(A))^2 \sigma^2_{Z})(r^v(A) - 1) - 1} \tag{6.5}
\]

and that \( g(A) \) and \( \bar{\varepsilon}(A) \) satisfy

\[
g(A) = \frac{1}{\rho} \left[ 1 + \frac{g(A)^2 \sigma^2_{\bar{\varepsilon}}}{\sigma^2_{\bar{\varepsilon}} + (g(A))^2 \sigma^2_{Z}} \right] - \bar{\varepsilon}(A) \left( 1 + \frac{1}{\sigma^2_{\bar{\varepsilon}} + (g(A))^2 \sigma^2_{Z}} \right)^{-1} \tag{6.6}
\]

Equation (6.6) implicitly defines the equilibrium value of \( g(A) \), and shows that \( g(A) \) depends on \( A \) only through \( \bar{\varepsilon}(A) \). Thus, the characteristics of the equilibrium price and hence the precision level obtained by the uninformed are invariant to changes in the allocation of information which leave \( \bar{\varepsilon}(A) \) constant. This implies that an allocation \( A^* \) in which all traders have conditional precision levels \( \bar{\varepsilon}(A) \) has an equilibrium price that is exactly as informative as the equilibrium price of the original allocation; i.e., it also satisfies \( r^0(A^*) = r^0(A) \). Using (6.5), this new allocation is obtained by selling to all traders \( \bar{P} = \bar{P} + \bar{\varepsilon}(A^*) \), where

\[
\text{Var}(\bar{\varepsilon}(A^*)) = \frac{(g(A))^2 \sigma^2_{\bar{\varepsilon}}(1 - (\bar{\varepsilon}(A) - 1) \sigma^2_{\bar{\varepsilon}})}{(\sigma^2_{\bar{\varepsilon}} + (g(A))^2 \sigma^2_{Z})(\bar{\varepsilon}(A) - 1) - 1} \tag{6.7}
\]

Since, by Jensen's inequality,

\[
\int_0^1 \log \left( \frac{r^v(A)}{r^0(A)} \right) dv \leq \log \left( \int_0^1 \left( \frac{r^v(A)}{r^0(A)} \right) dv \right) = \log \left( \frac{\bar{\varepsilon}(A)}{\bar{\varepsilon}(A)} \right) \tag{6.8}
\]

with strict inequality if \( \int_0^1 (r^v(A) - \bar{\varepsilon}(A))^2 dv \neq 0 \), the monopolist obtains higher profits in \( A^* \) than in A. \( \blacksquare \)

The above result asserts that any asymmetric personalized allocation can be dominated by a symmetric allocation with the property that all traders obtain, in equilibrium, the average of the precisions in the original allocation. The simplest case occurs when the monopolist is perfectly informed. Here it can be shown that Eq. (6.7) simplifies to \( \text{Var}(\bar{\varepsilon}(A^*)) = 1/\rho(1/\sigma^2_{\bar{\varepsilon}}) dv \). The new allocation simply requires selling (to all traders) signals whose precision is the average precision of the signals in the given allocation. This leaves \( g(A) \) the same, so that prices are equally informative, while profits increase. Things are not this simple when the

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Note that \( \text{Var}(\bar{\varepsilon}(A^*)) \) given in (6.7) is always non-negative since in any equilibrium 
\[ 1 + 1/(\sigma^2_{\bar{\varepsilon}} + (g(A))^2 \sigma^2_{Z}) < \bar{\varepsilon}(A) < 1 + 1/\sigma^2_{\bar{\varepsilon}}. \]
monopolist is imperfectly informed, since both \( g(A) \) and \( \bar{f}(A) \) are complex functions of the underlying parameters and a tractable solution for the personalized noise level defined in (6.7) is not available.

An important corollary of Proposition 6.1 is that if personalization is possible, the seller can always do strictly better than restricting the fraction of informed traders (as is done with photocopied signals for parameters in regions A and B of Fig. 1). First, the proposition applies directly to allocations where no photocopied noise is added (as in region A), since they are special cases of an asymmetric personalized allocation with infinite noise added to some signals. If the seller adds some photocopied noise and sells to only a fraction of the traders (as in region B), then Proposition 6.1 implies that he can do strictly better by selling to everyone signals with the same level of photocopied noise and also some additional personalized noise. (To see this interpret the parameter \( \sigma^2_n \) in the proof as the total variance of the noise in the signal; i.e., that of \( \theta \) + \( \bar{q} \)). Note that we know from Proposition 5.1 that the seller can do strictly better by adding only personalized noise, even selling to the same fraction of traders. Here we see another way to dominate photocopied allocations for parameters in region B.

As discussed in the previous section, an important difference between personalized and photocopied allocations is the amount of correlation between the signals sold and the equilibrium price given the seller's information. Personalized signals are independent of the equilibrium price given the seller's information; photocopied signals are not; they are simply more precise than the price. Inspect again the expressions for the value of information in (5.7) and (5.8). For a fixed \( \lambda \), the value of a photocopied signal declines to zero as \( \sigma_Z/\rho \) approaches zero, no matter how much photocopied noise is added, and even if the amount of photocopied noise is made a function of \( \sigma_Z/\rho \). This follows because the extra term \( (\lambda^2 \rho^2/\sigma_Z^2) \) in the denominator of (5.7) (due to the conditional correlation between the signal and the price) is independent of \( s \) and grows to infinity as \( \sigma_Z/\rho \) approaches zero. Thus, when the externality induced by price information becomes intense, adding more photocopied noise will not help. The seller must resort to restricting the fraction of informed traders, as can be seen in Fig. 1 (regions A and B). In the expression for the value of a personalized signal (5.8) the term \( (\lambda^2 \rho^2/\sigma_Z^2) \) does not appear. Adding personalized noise turns out to be always a better way to fight the externality than is restricting the fraction of informed traders. In particular, when \( \sigma_Z/\rho \) is small the seller simply increases the amount of noise in each personalized signals sold to all.

To find the optimal personalized allocation it remains to determine the common variance of the added error terms. Maximizing profits is equivalent to maximizing the value to each trader of signals of the form
A monopolistic market for information

\( Y^r = F + U + \tilde{e} \), where \( \tilde{e} \) are i.i.d., and equilibrium reflects the fact that all the traders are symmetrically informed. The choice variable for the seller is \( s^* = \text{Var}(\tilde{e}^a) \), which now determines the allocation completely. The next proposition provides the optimal level of added noise in the case of personalized signals. The main idea of the proof is to find the optimal value of \( g(A) \), the coefficient on the supply in the normalized price, and then to recover the optimal value of \( s_p \) from the equilibrium conditions. Since \( g(A) \) determines (uniquely) the informativeness of the price, when the seller chooses its value he is choosing how much information to provide, free of charge, to potentially uninformed traders; i.e., how much information to give away through equilibrium prices. As is shown below, the optimal choice of \( g(A) \) takes a particularly simple form.

**Proposition 6.2.** A monopolist selling signals with independent (and identically distributed) additional noise terms \( \tilde{e} \) maximizes profits by setting \( s_p = \text{Var}(\tilde{e}^a) \) to be \( s^*_p \), where

\[
s^*_p = \max \left( 0, \frac{(\rho/\sigma_Z) \sqrt{1 + \sigma_Z^2 - \sigma_Z^4}}{1 + \sigma_Z^2(1 + \sigma_Z^2)} \right). \tag{6.9}
\]

The resulting normalized equilibrium price is given by

\[
\bar{p}_N(s^*_p) = F + U - \frac{\sqrt{1 + \sigma_Z^2}}{\sigma_Z} \tilde{Z}, \quad \text{if } s^*_p > 0;
\]

\[
= \bar{F} + \bar{U} - \frac{\sigma_Z^2}{\rho} \tilde{Z}, \quad \text{otherwise}.
\tag{6.10}
\]

**Proof.** In the Appendix it is shown that for a given allocation \( A \) of information where added error terms are independent, the total precisions of informed and uninformed traders are related to \( g(A) \) through

\[
r^v(A) = 1 + \frac{1}{\sigma_Z^2 + s^*_p (g(A))^2 \sigma_Z^2 / (s^*_p + (g(A))^2 \sigma_Z^2)} \tag{6.11}
\]

and

\[
r^\sigma(A) = 1 + \frac{1}{\sigma_Z^2 + (g(A))^2 \sigma_Z^2}. \tag{6.12}
\]

To determine the optimal \( s_p \), it is necessary to specify the equilibrium relation between \( g(A) \) and \( s_p \). Using Eq. (6.6) and the fact that in the symmetric allocation \( r(\tilde{e}) = r^v(\tilde{e}) \) for every \( v \), we get the following cubic equation in \( g(A) \):

\[
\sigma_Z^2 s_p + (g(A))^2 \sigma_Z^2 (s_p + s_p) + \rho (g(A))^3 \sigma_Z^2 = 0. \tag{6.13}
\]
Viewed as an equation for the equilibrium level of \( g(A) \), it can be shown, using standard techniques, that for every set of parameters \( \sigma^2, s_p, \sigma_z^2 \) and \( \rho \), Eq. (6.13) has a unique real root. A key observation for our purposes, however, is that the parameter \( s_p \) enters (6.13) linearly. It follows that for given \( \sigma^2, \sigma_z^2 \) and \( \rho \), there is a one-to-one relationship between \( s_p \) and \( g(A) \), where \( A \) is a symmetric personalized allocation. We can now formulate the seller's problem as that of choosing \( g \) subject to constraints that guarantee that \( s_p \geq 0 \). The functional relation \( s_p(g) \) is obtained by rearranging (6.13):

\[
s_p(g) = \frac{-\sigma^2 g^2 \sigma_z^2 - \rho g^3 \sigma_z^3}{g^2 \sigma_z^2 + \sigma_p^2}.
\]  
(6.14)

Thus, the constraint on \( g \) is \( g \leq -\sigma_p^2/\rho \). Substituting \( s_p(g) \) into (6.12) and then taking the ratio of the two expressions, we obtain

\[
\frac{r^p(g)}{r^o(g)} = 1 - \frac{g \sigma_z^2}{\rho(1 + \sigma_z^2 + g^2 \sigma_z^3)}.
\]  
(6.15)

It is now straightforward to show that \( g = -\sqrt{1 + \sigma_p^2/\sigma_z} \) maximizes the seller's profits. Substituting the value in (6.14) yields the optimal \( s_p \) if the value is nonnegative. Otherwise the monopolist adds no noise and sells \( F + \theta \) to all.

Several properties of the optimal solution should be noted. First, the seller does not add any noise if his own information is very noisy and supply variation is relatively large. (The point at which the seller stops adding personalized noise is a solution of the quadratic equation in \( \sigma_p^2 \) that equates the second expression in (6.9) to zero.) Also, while the optimal value of \( s_p \) depends on all the parameters of the model (\( \sigma^2, \sigma_z, \) and \( \rho \)), the optimal choice of \( g \) is independent of \( \rho \) and, more interestingly, it is inversely proportional to \( \sigma_z \). This implies that in the allocation that is optimal for the seller, if \( s_p^* > 0 \), the precision of the price as a signal is independent of \( \sigma_z^2 \) and \( \rho \); from (6.12) we get for the optimal allocation

\[
r^o(s_p^*) = 1 + \frac{1}{1 + 2\sigma_z^2}.
\]  
(6.16)

This means that the seller neutralizes the effect of the exogenous supply noise and traders' risk tolerance on the asset price, and brings the entire market to a benchmark information level that is only a function of his own information. When \( \sigma_z^2 = 0 \), we get \( s_p^* = \rho/\sigma_z \) and \( r^o(s_p^*) = 2 \). That is, the seller doubles the precision of uninformed traders relative to the prior distribution. Finally, the amount of noise added in the optimal personalized allocation is always at least as large as that in the optimal photocopied allocation; i.e., \( s_p^* \geq s_x^* \).
Figure 2 illustrates the seller's optimal allocations under photocopying and personalization for various parameters of the model. The curve that defines region $D_2$ divides the parameter space according to whether personalized noise is added. As is clear from our results, the set of parameters for which personalized noise is added by the seller (all except those in $D_2$) strictly includes those for which photocopied noise is added (B and C) as well as those where the fraction of traders who buy the photocopied signals is restricted (A and B). Note that the optimal personalized allocation strictly dominates the optimal photocopied allocation also in the region $D_1$, which is not covered by either Proposition 5.1 or Proposition 6.1.

It is useful to review the development in the previous two sections. First we showed (Proposition 5.1) that adding photocopied noise can always be strictly dominated by adding personalized noise; even if the seller is restricted to selling to a fixed fraction of the market, selling personalized signals leads to higher profits than selling a signal with added photocopied noise. This applies to regions B and C, where noise is added in the optimal photocopied allocation. We then showed (Proposition 6.1) that if personalization is possible the seller always prefers to sell identically distributed signals to all the traders. This strengthens the domination result, since it implies that adding personalized noise is not only better than adding photocopied noise but it is also better than restricting the fraction of informed traders. The implication is that personalization strictly dominates in region A and, since it is proved for every value of $\sigma_z$, also in
region B. (Note that region B is covered by both results.) Finally, we found the optimal level of personalized noise, which shows that the seller can obtain strictly higher profits also for a subset of the parameters in region D, namely $D_1$. For the remaining parameters, those in $D_2$, no personalized noise is added in the optimal allocation, and the two allocations coincide—the seller sells his signal as is to all traders.

Our results imply that the seller never wishes to add some photocopied noise and also some personalized noise. It is easy to show that the optimal profit function in the case of personalized signals is strictly decreasing in $\sigma_p^2$. Adding any photocopied noise is equivalent to voluntarily increasing $\sigma_\theta^2$, and is therefore suboptimal.21

A final comment concerning comparative statics. The ratio $\sigma_z/\sigma_p$ determines how well the price is able to transmit information from the informed to the uninformed traders. For the allocations we have analyzed, a decrease in $\sigma_z$ unambiguously lowers profits, since the price at which any particular piece of information can be sold decreases with a reduction in $\sigma_z$. An increase in $\sigma_p$ has two effects. It causes the equilibrium price to be a more informative signal, which tends to lower the value of information. At the same time, it increases the amount a trader is willing to pay for a given percentage increase in precision. Without further analysis it is not clear which effect dominates.

Observe that $\rho$ enters the profit function in precisely the same manner as the fraction of informed traders. In both the photocopied and the personalized allocations, profits can be written as a function of $\lambda \rho$. It follows immediately that when $\rho$ increases the seller’s profits cannot decrease, since $\lambda$ can always be adjusted to keep the product $\lambda \rho$ constant. We have shown previously that for all positive $\rho$, profits obtained by (optimal) personalization are strictly increasing in $\lambda$. We can turn this statement around to conclude that for any fixed $\lambda$ (and, specifically, for $\lambda = 1$) profits are always strictly increasing in risk tolerance $\rho$. This is not true in the photocopied case. When $\lambda^*_p \leq 1$, an increase in $\rho$ is met with a proportional reduction in $\lambda$, and profits are constant. Only when $\lambda^*_p = 1$ are profits strictly increasing in the level of risk tolerance.

21 This observation, incidentally, suggests another proof for Proposition 5.1. One can interpret $\delta$ in this section as the total common precision; i.e., the sum of the seller signal error and any photocopied noise he may have added. Then Propositions 6.1 and 6.2 can be interpreted as providing the optimal solution for a given level of added photocopied noise. It is possible to show directly that, once personalization is done in an optimal manner, the optimal amount of photocopied noise to add is zero.
7. General Allocations and Viability

So far we have only examined a particular family of feasible allocations, where added noise could either be common to all buyers or completely personalized (or a combination of both). There are, however, many other feasible allocations. For example, the seller may sell a finite number of different photocopied signals, each to a fraction of the market. More interestingly, he may sell signals whose errors are conditionally correlated given his information, yet they are not photocopied versions of the same signal. We now address the more general problem the seller faces, where the choice is among all feasible allocations, including, in particular, those considered in the previous sections. We also discuss issues of viability; i.e., the amount of discrimination that is required to implement various allocations.

First, let us ask whether any other allocations can produce the same profits as the personalized allocation. Recall that the key element in the personalized allocation is that the signals and the equilibrium prices are conditionally independent of each other given the seller's information $F + \theta$. Thus, it is natural to look for other allocations where this is also true. Consider the following allocation: let $\tilde{e}$ have variance $\sigma^2_\tilde{e}$, the optimal noise level for the personalized signals found in Proposition 6.1. Now suppose that half of the traders observe photocopied versions of $F + \theta + \tilde{e}$, while the other half observes photocopied versions of the signal $F + \theta - \tilde{e}$. It is easy to see that the seller's profits in this allocation are equal to those obtained in the personalized allocation. Clearly, there exist many other feasible allocations that yield this profit level. The intuition suggested by our previous discussion is that, while adding noise in order to protect the value of information from the externalities may be desirable, the seller prefers to add noise in such a way that additional noise terms cancel out in the price, so that the realizations of any added noise do not affect the asset price.

It is easy to identify one potential implementation problem with the allocation where a fraction of the traders privately observe $F + \theta + \tilde{e}$ and others observe $F + \theta - \tilde{e}$. A trader who observes $F + \theta + \tilde{e}$, where $\text{Var}(\tilde{e}) > 0$ may value the signal $F + \theta - \tilde{e}$ much more highly than a trader who only observes the price. This is because the price is conditionally independent of $F + \theta - \tilde{e}$ given $F + \theta$, while observing both $F + \theta + \tilde{e}$ and $F + \theta - \tilde{e}$ enables the trader to unlock the seller's information $F + \theta$. The two signals are therefore natural complements. In the extreme case, where $\sigma^2_\theta = 0$, observing $F + \tilde{e}$ and $F - \tilde{e}$ provides perfect information, which has infinite value to the individual competitive trader, assuming that the price is not fully revealing. Thus, to implement this allocation, the seller may need to prevent buyers from buying two signals.

In [2] we define an allocation to be viable if it is consistent with a non-
discriminatory market for information; i.e., if there exist prices for the set of marketed signals such that each trader (weakly) prefers the signals assigned to him in the allocation to any other bundle given the prices for the signals (and assuming that the informational content of asset prices does not change). When an allocation of information is viable, the holders of each bundle of information value it, in equilibrium, at least as highly as those who do not have it. Clearly, the allocation where a single photocopied signal is held by a fraction of a homogeneous set of traders is viable. It can be supported by setting the price of the signal equal to the signal's value (to an uninformed trader) at the given equilibrium. As discussed above, some allocations that yield profits equal to those in personalized allocations are not viable.

The following proposition implies that the personalized allocations considered in the previous sections are viable. As a result, the seller does not need to discriminate in order to implement them. If each personalized signal in the allocation is priced according to its value to (potentially) uninformed traders, then no trader wishes to obtain two signals. The proposition is an application of Proposition 8.2 in [2]. The proof is straightforward, using properties of normal variables, and the value of information function.

**Proposition 7.1.** Suppose $\bar{Y}_1$ and $\bar{Y}_2$ are identically and, conditional on $\bar{F} + \bar{B}$, independently distributed, and suppose that the equilibrium price is conditionally independent of $\bar{Y}_1$ and $\bar{Y}_2$ given $\bar{F} + \bar{B}$. Then for every risk tolerance coefficient $\rho$, the value of $\bar{Y}_2$ to a trader who observes only the equilibrium price is higher than its value to a trader who observes both price and $\bar{Y}_1$.

Can the monopolist do better (with or without discrimination) than in the personalized allocation? We conjecture that the answer is negative. We do not have an argument that applies to all feasible allocations. However, we can show that personalized allocations dominate a large class of allocations, including many that have not been explicitly considered in our analysis so far. For example, suppose the seller divides the market into $N$ equal subsets and sells each subset photocopied versions of a signal that is conditionally independent of and identically distributed with the signal of other subsets. Then it can be shown that the seller's profits are the same as they would be in an allocation in which all traders observe identically distributed signals where some common photocopied noise and some personalized noise are added. Such allocations were shown above to be dominated by a completely personalized allocation. We also conjecture that the personalized allocation is the only one that is viable among the optimal allocations for all the parameters of the model.
Note that if the number of traders is finite, or a finite number of signals is sold to different subsets of traders, then even if the signals are conditionally independent, the added noise terms do not disappear from the equilibrium price entirely. The results in [2] imply that such allocations need not be viable (and are certainly not viable if externalities are sufficiently intense). Thus, if the seller must choose only among viable allocations, he may not be able to achieve the same profits as with perfect discrimination. It can be shown, however, that the personalized allocation is viable if the number of traders is large enough.

8. Discussion and Concluding Remarks

We have shown that, in order to overcome the dilution in the value of information due to its leakage through informative prices, a monopolistic seller of information may prefer to sell noisier versions of the information he actually has. Moreover, to obtain higher profits, it is desirable to sell different signals to different traders, so that the added noise realizations do not affect equilibrium prices. One way of doing so, which does not require discrimination, is to sell identically distributed personalized signals to each of a large number of traders.

Do we observe sellers adding noise to the information they sell? First consider the photocopied case. A simple interpretation of photocopied signals is that they correspond to conventional newsletters sent to investors. There are, however, other ways photocopied information is sold, and one of the most important is through mutual funds. If traders are identical, they choose the same portfolio for the same realization of a piece of shared photocopied information. Thus, it seems desirable to have a fund that would implement this choice. This minimizes various transaction costs and also prevents resale and further leakage of the information. Our results show that no matter how the information is sold, a seller of photocopied information does not want to sell a signal that is too precise. Although this implies that a seller may want to add noise before selling the information, it has more important implications for the production of information when production is costly. Clearly, resources will not be spent to collect information so precise that the seller then chooses to add photocopied noise. Hence, if information is costly to the seller, we will never observe photocopied noise being added. Combined with assumptions about the cost of collecting information, our results can be used to predict the precision level of information produced and sold.

Now consider personalized signals. Investment advisory services and newsletters that pass their information to subscribers through telephone calls can easily personalize their information since they can give distinct
signals to each client. Signals may be personalized in other, less direct ways. For example, the seller may provide information that is vague and open to interpretation, so that the buyers themselves make personal, independent, errors of interpretation. When done in this way, personalization is truly costless.

The implications of our results for the production of information when signals can be personalized are quite different than those discussed above for the photocopied case. When the seller can sell personalized signals, profits are always increasing in the precision of the information. This is unlike the photocopied case where there is no value in collecting information to improve precision beyond a certain point. Again, with some additional assumptions about the cost of production, our results can be used to determine both the precision of the information the seller collects and the amount of the personalized noise added.

The distinction between photocopying and personalization may be related to the organizational structure of a firm selling information. A firm that gathers and sells information through a centralized unit naturally photocopies information, while a firm whose research is decentralized, with many units independently gathering information and selling it to local clienteles, naturally sells personalized signals. The optimality of personalization suggests that the decentralized structure may be preferable. Of course, the technology for information production and for communication within the organization will also be important.

This paper is a small step toward an understanding of markets for information such as the markets of investment newsletters and advising services. Many important issues remain to be explored. For example, in our basic model we have assumed that resale of information is not possible. The model is actually consistent with traders reselling the information they purchase. Suppose each trader sells or shares the information with a finite number of traders. This would correspond to another model of the same kind, in which each “trader” (potential buyer of information) is more risk tolerant, since he represents a group of traders. This is related to the general issue of the timeliness of information; i.e., more timely information is, in many contexts, more valuable. A complete model of this, which must obviously be dynamic in nature, represents an important direction for future research.

Another interesting issue is the sale of information in markets where traders differ in their degrees of risk tolerance. In the context of a homogeneous market and under the assumption that signals can be personalized, we have shown that the seller’s optimal allocations are such that

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22 We are grateful to Steve Ross for suggesting this personal interpretation of our analysis.

23 We are grateful to the referee for suggesting this interpretation of our analysis.
all traders are treated symmetrically; i.e., all buy identically distributed
signals. It can be shown that this remains true in markets where buyers
have varying levels of risk tolerance if the seller can fully discriminate. The
seller would still choose to sell identically distributed personalized signals to
the entire market. Discrimination would be done by charging traders with
different preferences different prices, with the more risk tolerant paying
higher prices.

If the seller cannot discriminate, complex adverse selection problems
arise when buyers differ. It is easy to see that pricing would have to be such
that some surplus is left to the risk tolerant traders. Moreover, the seller
may now sell different signals to induce self selection. The use of per-
sonalized signals may actually become problematic, since by combining a
number of signals traders can create more precise signals for themselves.
This limits the ability of the seller to offer nonlinear pricing schemes, unless
he can make sure that each trader buys at most one signal.

It is important to note that, while a monopolist selling a standard com-
modity to a homogeneous market always treats buyers symmetrically, this
result is simply not true for general models with externalities, and par-
ticularly in the context of markets in information. We have seen that in our
model, a seller of a single photocopied signal may not wish to sell to the
entire market. Two types of traders, informed and uninformed, are created
in an otherwise homogeneous economy. Moreover, it is easy to construct
other examples where a seller of information wishes to discriminate among
identical buyers.\textsuperscript{24}

Our analysis has been solely concerned with the case of a single seller. In
a market with many sellers and many assets, competition among sellers
and substitution effects among assets and information signals are impor-
tant. How these markets operate is an exciting topic for future research.

Finally, casual observation suggests that many newsletters are published
by investment and brokerage houses, who also trade in the securities. As
we have pointed out, this may introduce moral hazard problems. We also
observe bundling of information sales with other services (e.g., brokerage).
This may be explained by attempts to induce more active trading, but it
may also be the case that information is certified by having the seller take
\textit{ex post} publicly observable positions in the securities market. It would be

\textsuperscript{24} For example, suppose a seller of information knows a parameter of demand in a Cournot
duopoly game between two identical firms. It is easy to specify parameters (e.g., with linear
demand and constant marginal cost of production) such that the seller would prefer to sell to
only one of the firms rather than to both. This is because the uninformed firm will be very
timid in the subsequent game against the informed firm. As a result, the value of the infor-
mation to a single firm, if it is the only one informed, is higher than the combined value to
both firms if they are both informed. Similar considerations arise in [11].
interesting to analyze a model which gives rise to some of these phenomena, and where the trade-offs between selling information and trading on it can be seen clearly.

APPENDIX

Proof of Proposition 4.1. 1. Proof that there is a unique $m^* > 0$ that maximizes (4.9). An interior optimal solution $m$ solves

$$
\frac{\partial m \log(1 + 1/(m(2 + m)))}{\partial m} = \log \left(1 + \frac{1}{m(2 + m)}\right) - \frac{2}{(1 + m)(2 + m)} = 0 \tag{A1}
$$

That (A1) has a unique solution follows from the following facts, which the reader can verify

(a) $\lim_{m \to 0} \frac{\partial m \log(1 + 1/(m(2 + m)))}{\partial m} = \infty$

(b) $\lim_{m \to \infty} \frac{\partial m \log(1 + 1/(m(2 + m)))}{\partial m} = 0 \tag{A2}$

(c) $\frac{\partial^2 m \log(1 + 1/(m(2 + m)))}{\partial m^2} = \frac{2(m^2 - 2)}{m(m + 1)^2(m + 2)^2}$

Since the second derivative is negative and then positive as $m$ increases, (A1) can be satisfied only at one point, $m^*$. For $m$ less than $m^*$, (4.9) is increasing in $m$, and for $m > m^*$, it is decreasing.

2. Proof that the constraint $s_x \geq 0$ is binding when $\sigma^2 > s^2_x$. The monopolist chooses $\lambda_x$ and $s_x$ to solve

$$
\max_{\lambda_x s_x \geq 0 \text{ s.t. } s_x \leq 1} \lambda_x \rho \log \left(1 + \frac{s_x + \sigma^2 \rho}{(\lambda_x \rho / \sigma_Z)^2(1 + s_x + \sigma^2 \rho) + (s_x + \sigma^2 \rho)^2}\right) \tag{A3}
$$

Again let $m = \lambda_x \rho / \sigma_Z$. Then an equivalent statement of the problem is

$$
\max_{m \geq 0} m \log \left(1 + \frac{1}{m(2 + m) + (\sigma^2 + s_x - m)^2 / (\sigma^2 + s_x)}\right) \tag{A4}
$$

We now argue that it is optimal to set $s_x = 0$. First assume that $\sigma_Z / \rho < 1/m^*$ so that $\lambda^*_x < 1$ and thus $s^*_x = \lambda^*_x \rho / \sigma_Z$. Fix $s_x > 0$ and assume
that \( m' \) maximizes (A4) subject to \( s_x \) fixed at its positive value. It is easy to show that \( m^* \leq m' \leq \sigma_\rho^2 + s_x \). There are two possibilities:

(i) \( m^* < \sigma_\rho^2 \leq m' \leq \sigma_\rho^2 + s_x \)

(ii) \( m^* \leq m' < \sigma_\rho^2 < s_x \).

If (i) holds, then the seller can increase profits by setting \( s_x \) equal to zero and setting \( m = \sigma_\rho^2 \). This follows from the following chain of inequalities, one of which must hold strictly:

\[
\sigma_\rho^2 \log \left(1 + \frac{1}{\sigma_\rho^2(2 + \sigma_\rho^2)}\right) \geq m' \log \left(1 + \frac{1}{m'(2 + m')}\right)
\]

\[
\geq m' \log \left(1 + \frac{1}{m'(2 + m')} + \frac{(\sigma_\rho^2 + s_x - m')^2}{(\sigma_\rho^2 + s_x)}\right).
\]

(A5)

The first inequality holds because \( \partial m \log(1 + 1/(m(2 + m))) / \partial m \) is negative for \( m > m^* \). The second is due to the additional non-negative term in the denominator.

If (ii) holds, the monopolist can increase profits by setting \( s_x = 0 \) and \( m = m' \). In this case profits are increased because

\[
\frac{1}{\sigma_\rho^2}(\sigma_\rho^2 - m')^2 < \frac{1}{\sigma_\rho^2 + s_x}(\sigma_\rho^2 + s_x - m')^2
\]

(A6)

whenever \( m' \leq \sigma_\rho^2 \).

Thus, for both (i) and (ii), the monopolist optimizes by setting \( s_x = 0 \) and \( \lambda_x = \min(\sigma_\rho^2 m^*(\sigma_\rho^2), 1) \), where \( m^*(\cdot) \) is the function defined in the statement of the proposition.

Now suppose, \( \sigma_z / \rho \geq 1/m^* \), so that \( \lambda_z = 1 \) and thus \( s_z = 1/\sigma_z \). If \( m^* > 1/\sigma_z \), profits are maximized by setting \( \lambda_z = 1 \) and \( s_x = 0 \). That \( s^* = 0 \) follows from the fact that \( \sigma_\rho^2 > s_z^*(\lambda_z) \) for all \( \lambda_z \in [0, 1] \) when \( m^* > 1/\sigma_z \) and \( \sigma_\rho^2 > s_z^* \). To show that \( \lambda_z = 1 \) it is necessary to show that

\[
m \log \left(1 + m/(m(2 + m) + (\sigma_\rho^2 - m)/\sigma_\rho^2)\right)
\]

(A7)

is increasing in \( m \) whenever \( m < 1/\sigma_z \) and \( \sigma_\rho^2 > 1/\sigma_z \). We can rewrite (A7) as

\[
\log \left(1 + \frac{1}{m(2 + m)}\right) + m \log(g(m, \sigma_\rho^2))
\]

(A8)
where
\[
g(m, \sigma^2_Z) = \left(1 + \frac{1}{m(2+m)}\right)^{-1} \left(1 + \frac{1}{m(2+m) + (\sigma^2_Z - m)^2/\sigma^2_Z}\right).
\]

Since the first term is increasing in \(m\) for \(m < 1/\sigma_Z\) (\(< m^*\)), it is sufficient to show that \(\delta g(m, \sigma^2_Z)/\delta m\) is positive for \(m < 1/\sigma_Z^*\) and \(\sigma^2_Z > 1/\sigma_Z\). This is easily established since
\[
\frac{\delta g(m, \sigma^2_Z)}{\delta m} = \frac{2(1 + \sigma^2_Z)(\sigma^2_Z - m)(\sigma^2_Z^2 + \sigma^2_Z(2m + m^2) + m^2(\sigma^2_Z - m))}{(1 + m)^2(\sigma^2_Z + m^2\sigma^2_Z + m^2)^2}
\]
(A9)

which is positive for \(\sigma^2_Z > m\).

**Proof of Proposition 5.1.** As before, let \(r'(A)\) equal the total precision attained by trader \(v\). Then equilibrium requires that
\[
\alpha_1'(A) = \rho'(\beta_1'(A) r'(A) \left(\int_0^1 (1 - \beta_2'(A)) \, dv\right))^{-1}
\]
(A10)
\[
\alpha_2(A) = \left(\int_0^1 \rho(1 - \beta_2'(A) r'(A)) \, dv\right)^{-1},
\]
(A11)

where \(\beta_1'(A)\) and \(\beta_2'(A)\) are the regression coefficients of the private signal and the normalized prices, respectively, in trader \(v\)'s forecast of \(\bar{P}\); i.e.,
\[
\mathbb{E}[^\beta'(\bar{P} \mid \bar{Y}, \bar{P}_N)] = \beta_0' + \beta_1'(A) \bar{Y} + \beta_2'(A) \bar{P}_N.
\]

Let \(\Omega\) be the variance-covariance matrix of \((\bar{Y}, \bar{P}_N)\). It is easy to see that
\[
\Omega = \begin{pmatrix}
1 + \sigma^2_Z + s_p^2 & 1 + \sigma^2_Z
\end{pmatrix} \begin{pmatrix}
1 + \sigma^2_Z
\end{pmatrix}
\]
(A12)

From regression theory for normal variables, we have
\[
\begin{pmatrix}
\beta_1'(A) \\
\beta_2'(A)
\end{pmatrix} = (\Omega)^{-1} \begin{pmatrix}
1 \\
1
\end{pmatrix},
\]
(A13)

and
\[
\begin{pmatrix}
r'(A)
\end{pmatrix} = \left(1 - (1 1)(\Omega)^{-1} \begin{pmatrix}
1 \\
1
\end{pmatrix}\right)^{-1}
\]
(A14)

Using (A13) and (A14) we obtain
\[
\beta_1'(A) = \frac{(g(A))^2 \sigma^2_Z/(s_p^2 + (g(A))^2 \sigma^2_Z)}{1 + \sigma^2_Z + s_p^2(g(A))^2 \sigma^2_Z/(s_p^2 + (g(A))^2 \sigma^2_Z)}
\]
(A15)
A MONOPOLISTIC MARKET FOR INFORMATION

\[ r^*(A) = 1 + \frac{1}{\sigma^2 + \sigma^2_p (g(A))^2 \sigma^2_2 / (\sigma^2_p + (g(A))^2 \sigma^2_2)} \quad (A16) \]

\[ \beta_1(A) r^*(A) = \frac{(g(A))^2 \sigma^2_2 / (\sigma^2_p + (g(A))^2 \sigma^2_2)}{\sigma^2 + \sigma^2_p (g(A))^2 \sigma^2_2 / (\sigma^2_p + (g(A))^2 \sigma^2_2)} \quad (A17) \]

If we solve (A16) for \( s_p^* \) we obtain

\[ s_p^* = \frac{(g(A))^2 \sigma^2_2 (1 - (r^*(A) - 1) \sigma^2_2)}{(\sigma^2 + (g(A))^2 \sigma^2_2)(r^*(A) - 1) - 1} \quad (A18) \]

Substitution of this into (A17) and subsequent simplification produces

\[ \beta_1(A) r^*(A) = -\left(1 + \frac{1 + \sigma^2_2}{(g(A))^2 \sigma^2_2} \right) r^*(A) \left(1 + \frac{\sigma^2_2}{(g(A))^2 \sigma^2_2} \right) \quad (A19) \]

From this it follows that \( g(A) \) can be written as a function of the average precision, \( \bar{r}(A) \), because

\[ g(A) = -\left(\int_0^1 \rho \beta_1(A) r^*(A) \, dv \right)^{-1} \]

\[ = \frac{1}{\rho} \left[ \left(1 + \frac{1 + \sigma^2_2}{(g(A))^2 \sigma^2_2} \right) - \bar{r}(A) \left(1 + \frac{\sigma^2_2}{(g(A))^2 \sigma^2_2} \right) \right]^{-1} \quad (A20) \]

The rest of the argument is in the text.

REFERENCES