TODAY

1. **Introduction**
   - Course
   - Facility
   - Students

2. **Asset Pricing**
   - Standard paradigm
   - HARA/GARA example
   - Information and asset prices
Q: What do we mean by asymmetric information?

- Not all agents have access to the same facts.

- Different agents vs. different \( P \text{ri}os \).

\[ E[u(w)] \]

- Savage's [1961]), we'll assume homogenous priors.

- Standard Agent Picking a regime assume symmetric information.
Q: What is the impact of asymmetric info in financial markets?

- Fama (prof. form: $\frac{\text{info}}{\text{risk}}$) (who loses?)
- Akerlof’s lemons: used cars market

\[
\text{Good cars} \quad \frac{\$1k}{\frac{1}{2}} \quad \frac{1}{2} \quad \frac{\$6k}{\frac{1}{2}}
\]

Only bad cars get offered in eqm. (1970s)

- Market breakdown (no trade)

- Liquidating - sell slowly and “quickly”

- “Market depth” $\frac{\partial D}{\partial S}$ (prices)

- $\frac{D}{\partial S}$ (trade)
- standard paradigm (competitive EOBs)

\[ \max \ E(u_i) \text{ (prices at given)} \]

Structures of the licenses (4+)

1. Intro
2. Competitive Models (Tue. Wed.)
3. Strategic Models (Thu.)
4. Bid-Ask Spreads (Fri.)
STANDARD ASSET PRICING PARADIGM

(a) Set of Agents: preferences \( (w(w)) \), beliefs, endowments.

(b) Set of Securities (market structure)

How do we determine asset prices?

"Competitive eqn":

1. \( \max_{\theta} E[F(w(w))] \) s.t. budget constraint, optimization

"Walrasian eqn":

2. Eqn in each market: "Demand = supply"
What assumption do we typically make?

(a) on securities - complete markets (= lots of securities)

No free + market (RV)

(b) no price impact, i.e. we assume that agents take prices as given (fixed) when maximizing

(c) on preference - representative agent?

\[ \text{so } \alpha E[n(t)] \]

What do we need to get a representative agent?
Set of agents: preferences \((\mathcal{U}(W))\), beliefs, endowments.

\(\Rightarrow\) \&

Representative agent.

\(\Rightarrow\) Can we have different beliefs? No (exception case)

Can we have \(n\) others? No (Henri class)

\(\Rightarrow\) Everyone is the same.
LEARNING EXAMPLE

All agents will have CARA preferences $v_i(w) = -e^{-\alpha w}$.

N agents, each endowed with $e_{0i}$ bonds and $z_i$ units of risky stock:

- Bond: risk-free, $P_0 = 1$, payoff $P_T = 1 + \text{risk-free rate}$
- Stock: risky, payoff $X_i = X_i N(\mu, \sigma^2)$

Q: What is the price of stock in this model?

Supply of risky units: $\sum_{i=1}^{N} z_i = \frac{e_{0i}}{P_0}$

Demand of risky units?
Agent: $\gamma_i$ in bonds
\[ w_i = \gamma_i \frac{P}{x} + \theta_i X \]

Budget constraint:
\[ \gamma_i x + \theta_i P_x = c_{oi} + \theta_i P \]
\[ \Rightarrow \gamma_i = \frac{c_{oi}}{x} + (\frac{\gamma_i - \theta_i}{\theta_i}) P_x \]

\[ \Rightarrow w_i = \left( c_{oi} + (\gamma_i - \theta_i) P_x \right) \frac{P}{x} + \theta_i X \]
\[ \Rightarrow w_i = \left( c_{oi} + \frac{\gamma_i - \theta_i}{\theta_i} P_x \right) \frac{P}{x} + \theta_i \left( X - \frac{P_x P}{x} \right) \]
\[ \Rightarrow w_i = w_{oi} + \theta_i \left( X - \frac{P_x P}{x} \right) \]
\[
\max_{\theta_i} \mathbb{E}(\mathcal{L}(\theta_i)) = \log \left[ -e^{-\mathbb{E} \left[ -e^{-\mathbb{E}(\mathcal{L}(\theta_i) + \Theta \mathcal{R}_x)} \right]} \right] \\
= -\mathbb{E}(\mathcal{L}(\theta_i) + \Theta \mathcal{R}_x) + \frac{1}{2} \mathbb{E}[\Theta^2 \mathcal{R}_x^2] \\
= -\mathbb{E}(\mathcal{L}(\theta_i) + \Theta \mathcal{R}_x) + \mathbb{E}[\mathcal{L}(\theta_i) + \Theta \mathcal{R}_x] \\
= \mathbb{E}(\mathcal{L}(\theta_i)) - \frac{1}{2} \mathbb{E}[\Theta^2 \mathcal{R}_x^2] \\
= \mathbb{E}(\mathcal{L}(\theta_i)) - \frac{1}{2} \mathbb{E}[\Theta^2 \mathcal{R}_x^2] \\
\]
\[
\begin{align*}
\frac{y}{z} & \quad \Rightarrow \quad Q = \frac{L_x - P \cdot R_x}{2 \cdot R_x^2} \\
\frac{z}{z} & = \frac{R_x}{R_x} = \frac{P}{}
\end{align*}
\]

\[
\frac{z}{z} = \frac{L_x - P \cdot R_x}{Q \cdot R_x^2} = \frac{1}{z} = \frac{1}{z}
\]

\[
\text{solve for } R_x: \quad R_x = \left( \frac{L_x - P \cdot R_x}{Q \cdot R_x^2} \right) \cdot \frac{1}{z} = \frac{1}{z}
\]
Tomorrow: take this example and change

\[ X \sim N(\mu_x, \sigma_x^2) \quad (\text{for}) \]

\[ y_i = x + \varepsilon \quad (\text{in other cases}) \]
BAYESIAN UPDATING

Probability space \( (\Omega, \mathcal{F}, P) \)

Take a set of events \( E_1, \ldots, E_n, D \) (\( n - 1 \) events), s.t.
\( E_1, \ldots, E_n \) be a partition;

\[
P(E | D) = \frac{P(D | E_i) P(E_i)}{\sum_{i=1}^{n} P(D | E_i) P(E_i)}
\]
"work-holds" is the Gaussian setup (normal distribution)

Given $X \sim N(m_x, \Sigma_{xx})$, $Y \sim N(m_y, \Sigma_{yy})$

$\text{Cov}(X, Y) = \Sigma_{xy}$

$\text{Var}(X \mid Y = y) = \mu_x + \Sigma_{xy} \Sigma_{yy}^{-1} (y - \mu_y)$

$\text{Var}(X \mid Y = y) = \Sigma_{xx} - \Sigma_{xy} \Sigma_{yy}^{-1} \Sigma_{yx}$
What if \( x \perp \epsilon \) are ind\(^2\)? We shouldn't! \( \mathbb{E}_x = 0 \). ✓

Does risk assessment depend on realization \( \mathcal{F} = \{ y \} \)?

\( \text{It does no.} \) \( \text{var}(x | y) \) ind\(^2\) of \( \mathcal{F} \)!

Example. \( x \in \mathbb{R}, x \sim N(\mu_x, \sigma_x^2) \)

Signal \( y = x + \epsilon \)

\( \theta \sim \mathcal{E}(x | y) \) ?

\( \text{var}(x | y) \) ?