Active Investment, Liquidity Externalities, and Markets for Information*

Samuel Lee
New York University

Abstract

Informed investors are a source of illiquidity, but those pursuing differently informed strategies also generate quasi-noise trading. Quasi-noise trading creates non-monotonic externalities in information choice that shape the composition of active investment and that influence investor herding, liquidity spirals, asset comovement, along with the information content of prices. These externalities also affect information markets. An information vendor with market power expands sales only to monopolize investor attention, which can make prices less informative. By contrast, vendor competition boosts quasi-noise trading, which promotes investor diversity. Finally, selling information can be a means to create liquidity for proprietary trades.

Keywords: Market liquidity, noise trading, information acquisition, information market

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Many traders in financial markets, from hedge funds to day traders, actively manage their investments to exploit private information about the value of underlying assets. Facing a plethora of relevant information, individual active investors focus on various types of information and, as a result, specialize in various trading strategies. The information choices are interdependent, because the trading strategies interact in the marketplace. This paper explores how a particular interaction channel, liquidity, affects investor information choice and, more generally, demand and supply in the market for (financial) information.

It is commonly held that active investors harm each other when they trade in the same market, because reveal each other’s information to the market and thereby erode their information advantage over uninformed investors. Consequently, each active investor’s expected profit decreases as a function of the total number of active investors (Grossman and Stiglitz (1980)). This competition effect fully characterizes the interaction between active investors even when they receive differing noisy signals, so long as the differences arise only from the noise component (i.e., misinformation). The departure point of this paper is that, when the differences originate in the fundamental component of information signals, active investors do not necessarily impede each other.

Quite to the contrary, in a world where investors choose from various information strategies, diversely informed investors can potentially benefit each other. Suppose, for example, that the values of stocks are determined by both a macroeconomic factor and firm-specific factors. Furthermore, suppose that active investors specialize in either macroeconomic information or in firm-specific information. Uninformed investors therefore participate in a market with two groups of better-informed investors: macrotraders and stockpickers. A rise in the supply of macroeconomic information, which leads to an increase in the population

\footnote{Differences in fund manager expertise (e.g., firm, industry, country, assets) are partly reflected in investment style (Chan et al. (2002), Barberis and Shleifer (2003), Goetzmann and Brown (2003)).}
of macrotraders, has two effects. On the one hand, macrotraders as a group acquire more information. This aggravates the information asymmetry in the market, creating a negative impact on market liquidity and thereby on stockpickers. On the other hand, macrotraders as a group generate a larger trading volume, revealing more of their own information to the market. This has a positive impact on market liquidity. Stockpickers do not suffer, in that the increased trading volume does not reveal their information but, in fact, benefits them in that they can better conceal their own informed trades within the trading activity generated by macrotraders. In a sense, macrotraders act as quasi-noise traders for stockpickers.

The information choices of (potential) active investors inherit the strategic interdependencies that arise from the liquidity externalities in the subsequent trading game (cf. Hellwig and Veldkamp (2009)). When evaluating a strategy, investors weigh the cost of acquiring the relevant information against the scarcity of that information relative to what other investors seek to learn. Suppose that macroeconomic information is less expensive. While macrotrading is therefore initially more attractive, the relative appeal of stockpicking rises with the number of macrotraders entering the market. Still, macrotraders may render the market too illiquid for stockpicking, even though stockpicking would be a lucrative strategy in the absence of macrotrading. Alternatively, if macrotrading attracts a sufficient number of investors, macrotrading activity may provide enough quasi-noise trading for stockpicking to become profitable (again). Thus, macrotrading can substitute for or complement stockpicking in the sense that a decrease in the cost of macrotrading can either dampen or stimulate the demand for stockpicking.
Implications for Financial Markets

In this framework, changes in information supply affect the composition of active investment, which in turn has ramifications for financial markets and corporate information policies.

**Market evolution** A general decrease in information costs leads to proliferation in investment strategies and to the emergence of new strategies. Investment strategies that are more costly to implement emerge as the number of active investors (in cheaper strategies) grows. Total trading volume increases, average trading volume decreases, and—along with the new strategies—large-volume investors continue to emerge. Simultaneously, the average (gross) return from active investment should decrease, with the “excess” returns of a growing fraction of “active” investors diminishing while newly emerging strategies generate high(er) returns. These outcomes are broadly consistent with long-run trends in trading volume and fund performance (e.g., Chordia et al. (2008b) and Barras et al. (2009)).

**Herds and spirals** Increasing and decreasing information costs can generate herding. Decreases in the cost of a particular type of information can cause some strategies to grow more common at the expense of others. Simultaneous rises in trading participation, volume, and correlations are signs of a herding *frenzy*. Increases in the cost of a particular information type can, in other circumstances, cause all strategies to wane and the least profitable ones to vanish. This can be described as a herding *panic*, as trading correlations grow but participation and volume contract. In such cases, liquidity acts as a channel of contagion. In fact, when quasi-noise trading is a mutual liquidity source, decreases in information supply trigger a vicious circle. A shock that reduces trading in a specific strategy withdraws quasi-noise trading from other strategies. The resulting contraction in the latter strategies in turn withdraws quasi-noise trading from the first one, which initiates another round of contraction. This liquidity spiral both amplifies and propagates the initial shock.
Commonalities  Variation in the supply of information, because it influences the level and composition of informed trading across various assets, affects the level of price comovement and generates liquidity comovement. Due to the non-monotonicity in externalities of information acquisition, prices of stocks followed by a greater number of traders comove more with the market unless the larger following coincides with a greater dispersion of beliefs. Furthermore, the market liquidity of more followed stocks may be more sensitive to, but also less explained by, fluctuations in aggregate liquidity. These peculiarities seem consistent with the findings of Chan and Hameed (2006), Chordia et al. (2000), and Hasbrouck and Seppi (2001).

Price efficiency  A decrease in the cost of a particular type of information can make asset prices less informative even though market efficiency improves. More investors choose to become informed, but their information choices may become less diverse. Hence, while prices incorporate more investor private information, they may in total reflect less information. Similarly, public disclosure of information may strengthen or weaken investor incentives to search for alternative information. In particular, if such disclosure merely reduces the cost of assimilating a particular type of information, prices may become less informative.

Implications for Information Markets

Liquidity externalities also play a role in the endogenous supply of information. First, the supply decisions of those who commercially provide information to investors—say, a financial newspaper, a newsletter, or electronic terminals—must take into account that investors may acquire alternative information. Second, the competitive relationship between similar information providers has spillover effects on the demand for alternative information. Finally, the choice between selling and trading on information is affected by the presence of alternative
Monopoly  Owing to the presence of alternative information types, having a monopoly on a particular type of information does not automatically confer a monopoly on investor attention. This provides a monopolistic vendor with the incentive to expand sales to crowd out those alternatives, i.e., to monopolize attention. For example, a vendor who specializes in macroeconomic data wishes to lower the price of its product so long as this fosters macrotrading at the expense of stockpicking. Such crowding-out can be achieved by way of the negative liquidity externality that macrotrading can exert on stockpicking. Thus, the optimal pricing strategy of a vendor with sufficient market power is to supply more investors with information to the extent that this reduces information diversity, thereby making the asset market more efficient but asset prices possibly less informative.

Competition  Vendors who battle for market share promote diversity even when selling the same type of information. Vendors of, say, (substitutable) macroeconomic information are more concerned with undercutting each other than with crowding out stockpicking. As their price fight propagates macroeconomic information, the volume of macrotrading grows and generates more quasi-noise trading for alternative strategies. In short, information market competition boosts financial market liquidity, which in turn attracts investment diversity. Together with the aforementioned monopoly result, this suggests that information market development can drive financial market development, and perhaps explain why stock prices comove more in emerging markets, as documented by Morck et al. (2000).

Trading plus selling  Increases in quasi-noise trading need not be a mere by-product (of competition) but can be the primary purpose behind information sales. An investor with scarce information may be willing to “hire” another investor with more common information to share this information with otherwise uninformed investors. The purpose here would
be to increase the level of quasi-noise trading to such an extent that the profitability of trading on the scarce information (plus selling the common information) increases by more than what the second investor must be paid to “compensate” for its loss. This provides a rationale for combining proprietary trading and information sales within a single financial institution; that is, the benefit of selling mundane information is to generate liquidity that enhances trading on proprietary information, information that the institution does not share with outside investors.

Related Literature

This paper belongs with the literature on endogenous information acquisition and information sales in financial markets; it revisits both themes in a setting with two distinctive features. First, investors can select among different types of information. Second, differently informed investors impose liquidity externalities on each other. The analysis builds on Subrahmanyam and Titman (1999), modifying their model to allow for information selection and sales.

Beginning with Grossman and Stiglitz (1980) and Verrecchia (1982), there is a large body of literature on endogenous information choices in financial markets.2 A number of papers examine the role of information heterogeneity. In Froot et al. (1992), short-horizon speculators can learn about various factors but optimally choose to herd on the same information. In Van Nieuwerburgh and Veldkamp (2009a,b), investors can learn about multiple assets and each optimally skews its information and portfolio choice towards a subset of the assets. In Kacperczyk et al. (2009), active investors can learn about aggregate and idiosyncratic fundamentals and optimally adjust their focus over the business cycle. As in the present paper,

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2See Veldkamp (2009).
the composition of active investment changes (over time), though it is driven by varying aggregate uncertainty rather than information supply.

Veldkamp (2006a,b) extends the focus on information types to information sales in financial markets. Due to economies of scale, information types in higher demand are sold at lower prices in competitive information markets. Veldkamp shows that this price externality in the information market causes herding frenzies and price comovement in financial markets. In the present paper, causality runs the other way: a liquidity externality in financial markets affects demand and prices in the information market. This observation is most closely related to Admati and Pfleiderer (1988a) and Fishman and Hagerty (1995a), who investigate settings with one information type. Simonov (1999) also studies information markets but focuses on direct externalities between different types of information.\(^3\)

One particular strand of the literature on endogenous information acquisition revolves around the question of whether information choices are strategic substitutes or complements. In most financial market models, information acquisition exhibits strategic substitutability, though there are exceptions. Barlevy and Veronesi (2000) argue that, under specific distributional assumptions, prices can become more information-sensitive as more agents become informed, which increases the value of becoming informed. In Li and Yang (2008), an increase in the number of informed investors may cause well-informed insiders to invest in real, as opposed to financial, assets. The exit of insiders can in turn make it attractive for yet more investors to become informed. In the present paper, complementarities in information acquisition arise across various information types; as more investors acquire one type, another type may increase in value. In Froot et al. (1992), investment decisions per se exhibit strategic complementarity, which is reflected in information choices. Hellwig and Veldkamp

\(^3\)Other papers on information sales in financial markets include Admati and Pfleiderer (1986, 1990), Garcia and Sangiorgi (2007), Boulatov and Dierker (2007), Cespa (2008), and Cespa and Foucault (2008).
(2009) show that information choices in general mirror the strategic motives of the game in which they are embedded.

This paper proceeds as follows. Section II describes the theoretical model and equilibrium in the absence of information sales. Section III discusses implications for financial markets. Section IV introduces information sales. Finally, section V presents concluding remarks. All mathematical proofs are found in the Appendix.

I Specialized Active Investment

A Informed Trading

A single asset with uncertain liquidation value \( \hat{V} \sim N(0, 2\sigma^2) \) is traded. The liquidation value is determined by a pair of fundamental factors, \( \Theta = \{A, B\} \). For simplicity, it is assumed that the factors are independent and equally important. That is, \( \hat{V} = \sum_\theta \hat{V}_\theta \) with \( \hat{V}_\theta \overset{iid}{\sim} N(0, \sigma^2) \) for \( \theta \in \Theta \).

Each active investor is informed about one of the fundamentals. That is, active investors belong to one of two classes. The size of class \( \theta \in \Theta \) is \( n_\theta \). The \( i \)th investor in class \( \theta \) receives data about \( V_\theta \) and interprets them with some idiosyncratic bias \( \tilde{\epsilon}_{i\theta} \overset{iid}{\sim} N(0, \sigma^2) \). Her information is thus a signal \( \tilde{s}_{i\theta} = \hat{V}_\theta + \tilde{\epsilon}_{i\theta} \). Intuitively, heterogeneity among active investors arises from specialization (differences in expertise) and from misjudgement (individual mistakes). Noise traders form a separate category. Their trading motives are exogenous to the model, and their total demand for (or supply of) the asset is given by \( \tilde{y} \sim N(0, \sigma^2_y) \).

Trading proceeds as in Kyle (1985). All investors non-cooperatively, simultaneously and

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\(^4\)Similar setups are used in Froot et al. (1992), Subrahmanyam and Titman (1999), Goldman (2005), and Bernhardt and Taub (2006, 2008).
anonymously submit quantity orders. All orders are batched and sent to a competitive market-maker. Upon observing the total order, the market-maker sets a uniform price at which she meets the orders. Finally, $V$ becomes public. Yet, $V_A$ and $V_B$ are not observed individually. Hence, these two components cannot be traded separately.

The probability distributions and class sizes are commonly known. All agents are risk-neutral and there is no discounting.

It is standard to solve the trading game for Bayes-Nash equilibria in linear and symmetric strategies. In such equilibria, each informed investor’s strategy $x_{i\theta} = \alpha_{i\theta} s_{i\theta}$ is linear in her signal, and the market-maker’s pricing rule $p = \lambda z$ is linear in the net imbalance of the order flow $z \equiv \sum_{\Theta} \sum_{i} n_{i\theta} x_{i\theta}(s_{i\theta}) + y$. Moreover, investors in the same class follow the same strategy, $\alpha_{i\theta} = \alpha_{\theta}$ for $\theta \in \Theta$. A strategy profile is thus a triple $(\alpha_A, \alpha_B, \lambda)$.

The “trading intensity” coefficient $\alpha_{\theta}$ denotes how much the order flow of an investor in class $\theta$ varies with her information. The “price impact” coefficient $\lambda$ denotes how sensitively the price reacts to variation in the total order flow. The inverse of the price impact, $1/\lambda$, is a measure of “market liquidity”.

**Lemma 1 (Subrahmanyam and Titman, 1999)** There exists a unique Bayes-Nash equilibrium of the trading game in linear and symmetric strategies:

\[
\alpha^*_\theta = \frac{\sigma_y}{\lambda^* [(n_\theta + 1)\sigma^2 + 2\sigma_x^2]} \quad \text{and} \quad \lambda^* = \frac{\sigma^2}{\sigma_y} \left[ \sum_{\Theta} T(n_\theta) \right]^{\frac{1}{2}} \tag{1}
\]

where

\[
T(n_\theta) \equiv \frac{n_\theta (\sigma^2 + \sigma_x^2)}{[(n_\theta + 1)\sigma^2 + 2\sigma_x^2]^2}, \quad \theta \in \Theta. \tag{2}
\]

Individual investors in the same class engage in a Cournot-type competition. Since they trade on similar information, they reinforce each other’s impact on the price. To mitigate the
cumulative impact, each of them cuts back her individual order. Thus, when the classes differ in size, individual investors in the larger class trade less intensively ($n_{A}^{*} > n_{B}^{*} \Rightarrow \alpha_{A}^{*} < \alpha_{B}^{*}$).

Investors from different classes do not compete in the above sense, since their trades are uncorrelated ex ante. They nonetheless affect each other. Since the market-maker expects informed orders from either class, both classes contribute to illiquidity in the market [via the subfunctions $T(\cdot)$]. Market liquidity is thus a channel for interclass externalities. These externalities arise because the investors, even if they possess unrelated information, trade in the same market.

As in single-class models, the relation between the number of informed traders and market liquidity is ambiguous. The cumulative order flow of greater numbers of investors conveys more (precise) information, as idiosyncratic biases tend to offset each other. At the same time, the higher number of traders simply leads to a larger, more volatile trading volume. These two effects have countervailing consequences for market liquidity. Since the information effect gradually vanishes, $T(\cdot)$ is strictly quasi-concave and has an interior maximum in $\mathbb{R}^{+}$.

**Corollary 1** $\lambda^{*}$ is unimodal in $n_{\theta} \geq 0$ for all $\theta \in \Theta$.

The corollary states that the externality that a given investor class imposes on all other investors can increase or decrease with the size of the class.\(^5\) (This also implies that changes in $n_{A}$ and $n_{B}$ can have opposite effects on market liquidity.) The non-monotonicity has interesting consequences for the formation of investor classes, to which we turn in the next section.

The assumption that two investor classes receive independent information is not critical. What matters is that competition is stronger within a class than across classes, and that

\(^5\)A similar non-monotonicity arises when investors are risk-averse (Subrahmanyam (1991)).
a class as a whole affects liquidity in a non-monotonic way. By isolating these effects, the independence assumption merely accentuates the results. Nor are the externalities unique to the above market microstructure. For example, the results in Bernhardt and Taub (2006) imply that, if the factors are independent, the interaction between investor classes is the same for quantity orders and price-quantity schedules.⁶

B Information Choice

Rather than being endowed with information, investors must now actively acquire information. Formally, the model is extended to include stage 0, in which investors either choose to remain uninformed or conduct a fundamental analysis of the asset. Each active investor can specialize only in the analysis of one fundamental factor, and a truthful exchange of private signals among investors is not enforceable.⁷ These assumptions are discussed in section C.

Fundamental analysis is costly. To produce information about a factor $\theta \in \Theta$, an investor must incur some fixed cost $c_\theta > 0$ to gather and interpret data. For instance, one can think of $c_A$ as the cost of a macroeconomic analysis and $c_B$ as the cost of a company-specific analysis. Accordingly, active investors can be thought of as macrotraders and stockpickers.

Investors choose their specialization to maximize expected profit

$$\pi_\theta(n_\theta, n_{\theta'}) = \rho_\theta(n_\theta, n_{\theta'}) - c_\theta$$

⁶As regards the interaction within a class, investors who submit price-quantity schedules compete more fiercely than investors who submit pure quantity orders. This merely reinforces the (information and trading volume) effects of growing class size on liquidity. The main insight that one investor class can benefit from the growth of another investor class should also hold in a competitive rational expectations model. The price—being a weighted average of the information of all investor classes—should become more biased towards the information of the growing class. This would benefit investors in other classes, as they learn more from the price while their own information is revealed less.

⁷Hong et al. (2007) also study a model in which asset values depend on several factors but investors update their beliefs over the class of single-factor models. In their words, the investors make “simple forecasts”.

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where \( \rho_\theta(n_\theta, n_{\theta'}) \) denotes investor \( \theta \)'s expected trading profit (gross of information costs). A pure-strategy subgame perfect equilibrium is defined by a pair of investor classes \((n_A, n_B)\) in conjunction with Lemma 1 such that (i) no active investor would rather join the other class or remain uninformed, and (ii) no uninformed investor prefers to become informed. There is an infinite population of investors who can become informed. Normalizing their outside option to 0, this implies that the expected profit of any investor in equilibrium must be 0. That is, \( \pi_\theta (n_\theta, n_{\theta'}) = 0 \) for all \( \theta \in \Theta \) in equilibrium.\(^8\)

**Quasi-noise trading**

Because investors in the same class compete with each other, their (expected) trading profits decrease with the size of their own class. This competition effect is illustrated by the downward sloping curve in Figure 1. The effect of class size on trading profits across investor classes is ambiguous, as illustrated by the U-shaped curve in Figure 1.

At first glance, the intuition behind the U-shape comes from Corollary 1. If the growth of an investor class primarily makes its order flow more informative, the price impact increases. This in turn induces investors in the other class to trade less intensively, thus reducing their trading profits. By contrast, if the order flow primarily gains in magnitude, the price impact decreases and investors in the other class gain through more intensive trading.

However, since the price impact is the same for all investors, this cannot explain why expected profits respond differently to changes in class size. Indeed, the key is not that a growth in macrotrading affects the price impact, but that it increases competition more for macrotraders than for stockpickers. The bottom line is therefore that the two classes trade on different types of information, so that the trading activity of one class is orthogonal to

\(^8\)To simplify matters, let us ignore integer problems and treat \( n_\theta \) as a continuous variable.
(some of) the information exploited by the other class. In this sense (alone), it is as if the two investor classes serve as noise traders for each other: Increased macrotrading provides greater camouflage for stockpicking insofar as the increased trading volume impairs the market-maker’s ability to extract the information contained in stockpicking trades. In response, stockpickers trade more aggressively and make higher expected profits. This explains why $\pi_B$ is increasing in $n_A$ once the latter becomes sufficiently large.

**Crowding out**

Let us restrict attention to cases where each trading strategy is per se profitable, i.e., $\pi_{\theta}(1,0) > 0$ for all $\theta \in \Theta$. Without loss of generality, let $c_A \leq c_B$. Thus, in terms of the previous example, a single investor prefers macroeconomic information over company-specific information, and either of these over no information. In Figure 1, the single investor’s preferences are reflected by the diverging but positive intercepts.

A single trader chooses macrotrading over stockpicking. The presence of such a trader not
only decreases the profitability of macrotrading but also the profitability of stockpicking (for other investors), as reflected in Figure 1 by the initial decline in both curves. A second active investor then faces the following trade-off: while macroeconomic information is cheaper, macrotrading is more competitive. The second investor therefore prefers macrotrading only if the cost difference $c_B - c_A$ exceeds the difference in trading profits $\rho_B(1,1) - \rho_A(2,0)$. In this fashion, every investor weighs the cost of a trading strategy against its scarcity (or competitiveness).

As macrotrading propagates, macrotraders compete each others’ profits away, so that stockpicking becomes relatively more attractive, as reflected in Figure 1 by the falling margin $\pi_A(n_A,0) - \pi_B(1,n_A)$. At some point, the “marginal” investor prefers either to stay out of the market or to become a stockpicker. Graphically, stockpicking is chosen only if $\pi_A(n_A,0)$ and $\pi_B(1,n_A)$ intersect before $\pi_A(n_A,0)$ reaches zero, as is the case in Figure 1. This intuition is captured in the following result.

**Proposition 1** Let $\mathcal{R}_A \equiv \{n_A : \pi_A(n_A,0) \leq 0\}$ and $\mathcal{R}_B \equiv \{n_A : \pi_B(1,n_A) \leq 0\}$. In equilibrium, $(n^*_A,n^*_B) = (n^0_A,0)$ if and only if $\mathcal{R}_B \neq \emptyset$ and $\min \mathcal{R}_A \in \mathcal{R}_B$. Otherwise, there exists a unique equilibrium $(n^*_A,n^*_B)$ with $n^*_A \geq n^*_B > 0$.

The cheaper trading strategy is always more prevalent. Less obvious is that under certain circumstances the other strategy, despite generating profits for a single trader, is not pursued in equilibrium. This occurs precisely when the illiquidity created by macrotrading renders stockpicking unprofitable, even though the underlying information is unrelated.

**Substitute or complement?**

Proposition 1 says that stockpicking is crowded out whenever the (unique) root of $\pi_A(n_A,0)$ falls into a region $\mathcal{R}_B$ where $\pi_B(1,n_A)$ is negative. Now suppose that $\mathcal{R}_B$ is non-empty.
Because the root of $\pi_A(n_A, 0)$ can be shifted by varying $c_A$, it follows that there exists a cost range $[c_A, \bar{c}_A]$ such that crowding out occurs whenever $c_A \in [c_A, \bar{c}_A]$. Intuitively, one can choose the cost of macrotrading such that stockpicking becomes unprofitable. The impact of changing the cost of macrotrading outside of this range is also interesting.

**Proposition 2** Lowering the cost of $A$-information decreases the amount of $B$-trading above some threshold $\underline{c}_A$ but increases it below some threshold $\bar{c}_A$.

The proposition implies that macrotrading acts as a *substitute* for stockpicking when $c_A \geq \bar{c}_A$ but as a *complement* to stockpicking when $c_A \leq \underline{c}_A$ (Figure 2). When it is difficult to obtain macroeconomic information, the volume of macrotrading is small and makes the market very illiquid. This discourages trading on even more inaccessible company-specific information. By contrast, when macroeconomic information is easily accessible, the large macrotrading volume provides quasi-noise trading, which raises the profitability of stockpicking.
The following analysis focuses on the case $\mathcal{R}_B \neq \emptyset$. This is less restrictive than it seems, as the analysis can be extended to more than two trading strategies. The assumption $\mathcal{R}_B \neq \emptyset$ simply states that some strategies are crowded out under some cost schedules.

C Robustness

This section discusses several issues related to the robustness of the above information equilibrium. These are not critical to an understanding of the main analysis, which continues in Section II with a discussion of the economic implications of the Propositions 1 and 2.

Cognition

The above analysis assumes that investors can process information about at most one fundamental factor. Such “limits in processing (receiving, storing, retrieving, transmitting) information” (Williamson (1981), 553) may arise from bounds on cognitive capacity, which induce decision-makers to focus on a subset of information.\(^9\)

Nevertheless, the “pure specialization” equilibrium in Proposition 1 exists even if investors can process information about both factors without additional difficulty. Suppose that investors could produce a compound signal $s_{ABi} = \sum_{\theta} (V_\theta + \epsilon_\theta)$ at cost $c_{AB} = c_A + c_B$. Though there may now exist other equilibria, pure specialization remains an equilibrium in this setting: Given competitive (i.e., zero-profit) pure specialization, no uninformed investor finds it worthwhile to enter with either type of information and, by the same token, no active investor finds it worthwhile to incur the cost of acquiring a compound signal. Furthermore,\(^9\)

\(^9\)This assumption is akin to the notion of rational inattention. Theoretical studies of rational inattention in financial markets include Moscarini (2004), Peng (2005) and Peng and Xiong (2006). Evidence of inattention among investors is reported in Huberman (2001), Huberman and Regev (2001), Massa and Simonov (2005), and Hong et al. (2007). For more general treatments of rational inattention, see Simon (1957), Kahneman (1973), Sims (2003, 2006) and Gabaix et al. (2006).
if the cost of processing information is convex so that \( c_{AB} > c_A + c_B \) (which seems reasonable), becoming a “generalist” loses further appeal and specialization becomes a more likely equilibrium outcome.

**Communication**

The present model does not permit communication among investors. Since neither the factor realizations \( V_0 \) nor the error terms \( \epsilon_i \) are individually revealed, investors can lie about their signals without ever being detected. They can always claim to have made an “honest” mistake. As a result, investors cannot credibly share information, as they might shirk the effort to acquire information or communicate false information to trade privately.

However, even if investors could communicate credibly—e.g., by producing hard information at no cost—communication may not arise in equilibrium. In a similar setting, Colla and Mele (2008) show that information sharing, because it dilutes each investor’s “monopoly” power, arises only if the initial correlation between signals is sufficiently high. This finding suggests that, in the present model where information about different fundamental factors is uncorrelated, communication across investor classes should be unattractive.

Communication becomes even less attractive when it is costly. Just as with information acquisition by an individual, communication among individuals typically requires effort (Dewatripont and Tirole (2005)). The sender must exert effort to provide an intelligible message, and the receiver must exert effort to comprehend the sender’s message. Extending bounded rationality to include communication costs lends further credence to the idea of specialized investors.
Information

We could also consider a setting where investors can mix information about different factors, while choosing the precision of each type of information (cf. Goldmann (2005), Kacperczyk et al. (2009)). Because of the competition effect, any two investors prefer to be as different as possible, and therefore specialize in distinct factors. Similarly, many investors temper competition by choosing different combinations of information, weighing the cost of a particular type of information against its scarcity. Liquidity externalities would still exist such that changes in the cost of one type of information, and the resulting changes in its demand and in trading behavior, should continue to have a positive or negative impact on the demand for the other type of information.

Endogenous noise trading

Active investors profit at the expense of noise traders. This is because not only do noise traders have less information but they also must trade for reasons other than information (e.g., hedging or liquidity needs). However, the assumption that their willingness to trade is entirely insensitive to the level of active investment and to market liquidity is clearly a simplification. Noise traders might be more willing to trade when market liquidity, $1/\lambda$, is higher. Such a positive relationship between market liquidity and noise trading reinforces the liquidity externalities in the model. Consider a decrease in the cost of macroeconomic information. If the growth in macrotrading improves market liquidity (complement region), stockpicking expands. This effect is reinforced if the rise in market liquidity also attracts more noise traders. Conversely, if market liquidity deteriorates (substitute region), stockpicking contracts. Here again, the effect is reinforced if the drop in market liquidity also reduces noise trading.
Correlated fundamentals

When the fundamental components, $V_A$ and $V_B$, are correlated, there is also a competition effect across groups insomuch as (i) investors in one class are able to use their own signals to make inferences about the signals in the other class and (ii) trades of one class partly reveal the information of the other class. Hence, an increase in the size of one class partly dilutes the value of the information held by the other class. However, as long as the components are not perfectly correlated, the information in the two classes possesses some distinct component. This component gains in value as the interclass competition effect weakens with larger class sizes. As a result, there is still a non-monotonicity (Subrahmanyam and Titman (1999)).

Skill heterogeneity

Some investors may be better at acquiring particular information than others, e.g., because of geography or familiarity (Malloy (2005), Massa and Simonov (2006), Van Nieuwerburgh and Veldkamp (2009a)). Accordingly, one could assume that investors in the model have different cost schedules for information. Different strategies would then attract investors from different pools. This would leave the results virtually unchanged, unless one also assumes that the different pools have finite size. The latter case would, intuitively, lead to a combination of the effects in Veldkamp (2006a,b) and the present paper.

II Composition of Active Investment

Financial markets experience not only differential information shocks, which affect the cross-section of asset prices, but also differential shocks to the supply of information, which affect the composition of active investment. Examples include changes in uncertainty about
the economic environment, communication technologies, media coverage, government transparency, corporate disclosure policies, or accounting standards. In this model, an amenable way to study information supply shocks is to view them as shocks to information costs.\textsuperscript{10}

According to Proposition 2, shocks to the information costs of one investor class have ramifications for other investor classes. The following, mainly informal discussion highlights linkages and commonalities that such spillover effects can generate with respect to trading volumes, profits, prices and liquidity.

A Evolution of Active Investment

A continuous decrease in the cost of an investment strategy can move the market from the substitute region, through the crowd-out region and to the complement region (Figure 2). During this process, the number of active investors steadily increases, but the number of investment strategies evolves non-monotonically. In the substitute region, expansions of one strategy come at the expense of the other strategy, so that active investment becomes less diverse. By contrast, in the complement region, such expansions supply quasi-noise trading, which encourages investors to pursue the other strategy. When both strategies are widespread, expansions become mutually reinforcing.

Now consider multiple possible investment strategies. The preceding arguments imply that, if a market can accommodate only a few investment strategies, it attracts only the less expensive strategies. The more expensive strategies emerge only when trading in the less expensive strategies grows so large that it yields too small trading profits but generates sufficient quasi-noise trading.

\textbf{Implication 1 (Costly strategies)} Costlier active investment strategies should emerge in

\textsuperscript{10}Alternatively, one could model these shocks as changes in the precision of the signals. For given costs, a decrease (increase) in precision is similar to a decrease (increase) in access to information.
markets that already accommodate a high level of active trading (in cheaper strategies).

This differs from models with a single information type, where the use of costlier strategies coincides with lower levels of active trading. The above implication challenges the view that active investors only harm each other, but it is consistent with the view that more liquid markets attract more informed trading (Chordia et al. (2008a)). What stands out is that expensive investment strategies arise when they are able to feed on liquidity provided by other active investors.

The composition of active investment also influences the distribution of trading volume across investors.\footnote{Because of differences in trading intensity across investment strategies, order flow composition (i.e., the distribution of order sizes) may convey more information than the mere net order imbalance. The information content of the composition would itself depend crucially on the (assumed) composition of pure noise trading. Analyzing such a model is beyond the scope of this paper. One conjecture is that, to the extent that order sizes reveal information, investors cannot fully exploit the benefits of a scarcer strategy because they might have to “mimic” smaller order sizes in order not to expose themselves excessively.} For a given information supply, the expected size of an active investor’s trade is determined by her trading intensity ($\alpha$). As the overall cost of information decreases, total trading volume increases. The cross-sectional changes are less uniform. Average trading volume in existing strategies decreases because of growing competition. However, the market also attracts new strategies, which are adopted by fewer investors trading larger volumes on average.

**Implication 2 (Trading volume)** As information costs decrease, total trading volume rises, average trading volume falls, though some large-volume trading continues to emerge.

This pattern is broadly consistent with the trends documented by Chordia et al. (2008b): While the total stock trading volume at the New York Stock Exchange has steadily grown over the past several decades, the average order size has steadily decreased, although activity in large-volume trades has persisted. These authors also report that the growth in trading
volume coincides with an increase in the production of private information. These trends could in part be the result of improvements in the information environment, leading to more (competitive) active investment, decreasing order sizes for conventional investment strategies, and the continuous emergence of new investment strategies traded in larger volumes.

In the model, the distribution of trading volume is intrinsically linked to returns from active investment.

**Implication 3 (Performance)** *As information costs decrease, average trading profits fall and a larger number of investors earns smaller profits, but high-profit investors continue to emerge.*

As information becomes more accessible, active investment spreads such that competition drives down gross returns from trading. At the same time, some investors adopt new, more costly strategies that are subject to less competition and hence promise higher gross returns. Thus, performance falls on average and becomes more skewed in the cross-section. Consistent with these predictions, Barras et al. (2009) find, in a large U.S. sample, that the proportion of skilled (positive alpha) funds significantly decreased from 1996 to 2006, while the number of active funds dramatically increased over this period. Furthermore, their analysis indicates that larger and older funds consist of far more unskilled funds than smaller and newer funds.

**B Herding Behavior and Liquidity Spirals**

The relationship between information supply and investment diversity implies that episodes of increased correlated trading can result from sudden shifts in information supply. For instance, a surge in the supply of macroeconomic information can move a relatively inactive market from the substitute region into the crowd-out region. This boosts macrotrading at the
expense of stockpicking, resulting in more—but also more correlated—trading. Conversely, a drop in the supply of macroeconomic information can move a relatively active market from the complement region into the crowd-out region. In this case, liquidity and trading contract, while trading converges on macroeconomic information.

**Implication 4 (Herding episodes)** A positive shock to information supply can lead to an increase in correlated trading coupled with an increase in trading activity. A negative shock to information supply can lead to an increase in correlated trading coupled with a decrease in trading activity.

These effects occur because the ease of obtaining information affects both the propensity to become informed (and hence to trade actively) and the specific information choice adopted by investors. On the surface, these herding episodes may appear as if shifts in “confidence” cause *frenzy* or *panic*; either phenomenon may affect multiple assets, as discussed below in subsection C.

Implication 4 could be tested by studying how shocks to information supply—as measured, for example, by Ljungqvist and Kelly (2009)—impact trading volume and distribution, along with trade correlation. Recent evidence corroborates the view that correlated trades are driven by common information. Dorn et al. (2008) find that correlated market orders by retail speculators lead returns, which suggests that these orders carry information. Feng and Seasholes (2004) find that investors in nearby geographic locations submit correlated trades and react similarly to public information. These authors conclude that the correlation reflects common information.

Ljungqvist and Kelly (2009) document that shocks to information supply cause changes in liquidity. In the present framework, this affects not only uninformed investors, but also active investors who use other types of information. This latter spillover effect can trigger a
vicious circle, wherein mutually reinforcing reductions in quasi-noise trading harm different strategies.

**Implication 5 (Liquidity spirals)** In markets that attract multiple investment strategies, a negative shock to the information supply of traders in one strategy can cause a contraction in active trading across all strategies, via liquidity as the channel of contagion.

For instance, suppose that macrotrading and stockpicking supply each other with quasi-noise trading (i.e., $T'(n_A^*) < 0$ and $T'(n_B^*) < 0$). A sudden increase in the cost of, say, macroeconomic information can trigger a “liquidity spiral”: The immediate effect of the shock is a drop in macrotraders’ expected profits and, consequently, a contraction in macrotrading. However, this contraction withdraws quasi-noise trading from stockpickers so that stockpicking, too, contracts. This in return withdraws quasi-noise trading from the (remaining) macrotraders, spiraling into another round of contractions in trading volume. Ultimately, the market reaches a new equilibrium with fewer active traders and a lower total trading volume. Thus, liquidity spirals amplify the shock on the directly affected investor class, but also propagate it across other investor classes.\footnote{This is rather different from the liquidity spirals in Brunnermeier and Pedersen (2008)’s model, in which liquidity spirals occur because market liquidity affects investor funding capacity, which in turn affects investor trading activity and hence market liquidity. This creates contagion across investors. In the present model, funding liquidity plays no role. Instead, interdependence arises from quasi-noise trading.} Also, liquidity spirals are closely linked to herding panics insomuch as they can cause some strategies to disappear completely.

\section*{C Price and Liquidity Commonalities}

Liquidity externalities channel shocks not only across investment strategies, but also across assets, provided that such asset values share common determinants. Consider three fundamental factors, $\Theta = \{A, B, C\}$, and two separately traded assets, $\mathcal{M} = \{1, 2\}$, with
liquidation values
\[ \tilde{V}_1 = \tilde{V}_A + \tilde{V}_B, \]
\[ \tilde{V}_2 = \tilde{V}_A + \tilde{V}_C. \]

where \( A \) can be thought of as a macro factor, and \( A \)-investors as macrotraders. As in the single-asset case, a shock to the asset-specific investment strategy \( B \) in market 1—because of liquidity externalities—affects macrotraders. But here, as the shock to market 1 alters the composition of active investment, it may also affect trading in market 2 including strategy \( C \). That is, shocks to the supply of information, even if they are relevant only to asset-specific factors, affect investors pursuing multi-asset strategies and create a ripple effect across assets.

Of course, a more direct source of commonalities is shocks to the supply of information about common factors. Consider a simple example: All factors are i.i.d., all factor signals equally precise, and all asset-specific signals equally costly. Trading occurs simultaneously with a separate competitive market-maker for each asset, who sees only her own order flow. Noise trading induces market-wide movements.\(^{13}\) A measure of price comovement is the average correlation coefficient \( \bar{\rho}_{a,M} \equiv \frac{1}{2} |\rho_{1M}| + \frac{1}{2} |\rho_{2M}| \), where \( \rho_{a,M} \) denotes the correlation coefficient between the price of asset \( a \in M \) and the market index \( p_M \equiv p_1 + p_2 \). Propositions 1 and 2 imply that \( \bar{\rho}_{a,M} < 1 \) if and only if \( c_A \) is outside some interval \([c_A, \bar{c}_A]\) (see Appendix B). This intuition is simple. Prices incorporate idiosyncratic information, and hence comove less, only outside the crowd-out region.

The brief example above illustrates that the level of price comovement depends on the composition of active investment, which in turn depends on the distribution of information costs. Furthermore, the example highlights that this relationship can be non-monotonic. At

\[^{13}\text{The assumption that noise traders induce market-wide movements is not important; it merely neutralizes a source of noise that is orthogonal to the impact of } c_A \text{ on the level of price comovement. Similarly, information "spillovers" between market-makers would simply introduce an additional channel for price comovement, from which we here abstract.}\]
a broad level, this implies that, overall, higher levels of information supply or (exogenous) noise trading, insofar as they enable markets to accommodate more investment strategies, decrease price comovement. This suggests the following testable prediction for individual assets and aggregate markets.

**Implication 6 (Price comovement)**  *Price comovement decreases with market capitalization, free float, and improvements in the overall information environment.*

This may in part explain cross-country and time-series differences in price comovement documented, for example, by Morck et al. (2000) and Campbell et al. (2001). Studies by Bushman et al. (2004) and Hameed et al. (2005) suggest that comovement patterns may indeed depend on characteristics of the information environment. In this vein, Section III explores possible links between price comovement and information markets. As regards non-monotonicity, Chan and Hameed (2006) find suggestive evidence in a sample of emerging markets. They report that a stock’s comovement with the market increases with analyst coverage *unless* increased analyst coverage coincides with larger forecast dispersion. This is consistent with the notion that increased information supply can initially reduce information diversity but later spur the use of more diverse information sources.\(^{14}\)

Variation in information supply is also a source of variation in liquidity. In the above example, Lemma 1 implies that, for all \(c_A \in [0, \infty)\), \(\partial \lambda_1 / \partial c_A = \partial \lambda_2 / \partial c_A \neq 0\) (see Appendix B). That is, variation in the supply of macroeconomic information causes comovement in liquidity. Conversely, variation in the supply of asset-specific information introduces idio-

\(^{14}\)To derive a cross-sectional prediction, we could add a more idiosyncratic asset, \(\hat{V}_3 = \hat{V}_D + \hat{V}_E\), to the above model. Suppose that factors \(D\) and \(E\) are i.i.d., so that asset 3 is insensitive to factor \(A\). If macrotrading attracts many investors, asset 3 not only covaries less with the market, but is also traded by fewer active investors than assets 1 and 2. The difference in comovement is especially large when macrotrading crowds out strategies \(B\) and \(C\). However, this difference shrinks when macrotrading becomes so widespread that it attracts strategies \(B\) and \(C\) again.
syncracies in liquidity variation. The sensitivity of liquidity to common factors can therefore vary across assets. Some stocks attract more (diverse) strategies because they accommodate more noise trading and in turn more quasi-noise trading. (That is, these stocks are more likely to be in the complement region.) For such stocks, liquidity is likely to be higher and more idiosyncratic. These stocks are also likely to respond more strongly to common liquidity shocks insofar as they are amplified by liquidity spirals.

Implication 7 (Liquidity commonality) The liquidity of assets that accommodate more quasi-noise trading and more strategies (i) is higher, (ii) reacts more sensitively to (market-wide) liquidity shocks, and (iii) exhibits more idiosyncratic variation.

This may in part explain cross-sectional differences in liquidity commonality, as documented by Chordia et al. (2000), Hasbrouck and Seppi (2001), and Huberman and Halka (2001). The implication is, for example, consistent with the observed differences between large and small stocks, under the presumption that larger stocks are traded by a larger number and greater variety of active investors.

D Information Role of Prices

A central role of prices is to aggregate dispersed information (Hayek (1945)). The economic literature offers three concepts to describe how well prices fulfill this role.

Market efficiency How much private information do prices reveal (Fama (1970))?

Price informativeness How precisely do prices predict intrinsic asset values?

Allocative efficiency How well do prices aid resource allocation (Tobin (1982))?
Market efficiency reflects how much of the private information acquired by active investors is captured in market prices. This can be measured via the total loss that uninformed noise traders suffer as a result of trading with better-informed parties. In contrast, price informativeness depends on how much total information—rather than private information—is impounded in the price. This is measured by residual uncertainty with respect to asset values, $\text{Var}(\tilde{V} | P)$.

In a single-factor model, shocks to information supply typically affect all three kinds of information efficiency in the same direction. In particular, a decrease in the cost of information implies that, as the number of active investors increases, investors as a group possess more information and more of that information becomes revealed in the price due to increased competition. Thus, both market efficiency and price informativeness improve. In addition, allocative efficiency improves insofar as decision-makers condition their actions on information contained in the price.

In a multi-factor model, the effects of shocks to information supply are less uniform.

**Implication 8 (Information and prices)** A decrease in the cost of information improves market efficiency, but it need neither improve price informativeness nor allocative efficiency.

The implications concerning market efficiency and price informativeness are formally established in Appendix C. The following comparative statics provide some intuition: For either type of information, price informativeness increases with the number of informed investors ($\partial \text{Var}(\tilde{V} | P) / \partial n_0 < 0$). However, the marginal increase in informativeness decreases with the number of informed investors ($\partial^2 \text{Var}(\tilde{V} | P) / \partial n_0^2 < 0$). For a given investor population, $n$, this means that price is most informative under a balanced information structure, $n_A = n_B = n/2$. Or putting it differently, skewing the information distribution, all else
This is why a decrease in the cost of, say, macroeconomic information need not increase price informativeness when macrotrading crowds out stockpicking. On the one hand, price aggregates macroeconomic information more precisely due to growth in macrotrading. On the other hand, macrotrading may not grow enough (due to the competition effect) for the gain in macroeconomic information to compensate for the loss in asset-specific information. That is, the loss in diversity may outweigh the overall growth in active trading, and the price may reflect less total information, although it does reflect more private information.

This finding has close antecedents in the literature. Fishman and Hagerty (1995b) show that insider trading can discourage investors from acquiring information, while Morrison (2004) shows that exogenous increases in the number of active investors induce each individual investor to acquire less precise information. In either case, prices can become less informative due to the loss of information acquisition. Dow and Gorton (1997) contend that market efficiency and price informativeness are, more generally, distinct concepts. In a stark example, these authors point out that, absent information acquisition, markets are maximally efficient but prices are minimally informative. Implication 8 complements these insights by showing that a lower information cost—despite increasing information acquisition—can reduce price informativeness due to a loss in diversity.

The implication concerning allocative efficiency relies on a heuristic argument. With multiple factors, some types of information are more relevant for allocative decisions than others. For instance, Holmström and Tirole (1993) and Faure-Grimaud (2004) show that stock prices can enhance the incentives of corporate insiders provided that they are information.

\[<\text{equal, reduces price informativeness.}^{15}\]

While the optimality of the balanced structure is specific to the assumption that both types of information are equally valuable, the argument is more general.
tive about insider effort.\footnote{See also Edmans (2009) and Edmans and Manso (2009). A complementary strand of research explores the role of stock prices in guiding future investment decisions (e.g., Subrahmanyam and Titman (1999, 2001), Dow et al. (2006), Goldstein and Gümbel (2008)). There is empirical evidence in support of this view (Wurgler (2000), Baker et al. (2003), Durnev et al. (2004), Chen et al. (2007)).} Suppose that asset-specific factors are informative about insider effort, whereas macroeconomic factors reflect events beyond insider control. Stock prices are less useful for incentives when macrotrading crowds out asset-specific strategies—irrespective of market efficiency or price informativeness. Since the macro component of the stock price reflects luck, a surge in macrotrading can confound the price as a signal of effort.

Public disclosures can be interpreted as positive shocks to information supply. The preceding discussion challenges the view that public disclosure obviates private efforts to acquire information and always makes prices more informative. Disclosure of one type of information may trigger the acquisition of other types of information. Alternatively, if disclosure merely reduces rather than eliminates the cost of assimilating one type of information, it may skew information choices, which in turn can make prices less informative.\footnote{This view is in the spirit of Fishman and Hagerty (1989) and Kim and Verrecchia (1994) in that disclosure facilitates information acquisition by active investors. Disclosure can alternatively be viewed as a public signal (e.g., Diamond (1985), Foster and Viswanathan (1993)).} Finally, managers may use selective disclosure to strategically bias investor information acquisition activities.

### III Markets for Financial Information

The preceding analysis focuses on exogenous changes in the supply of information. Alternatively, one can analyze the endogenous supply of information, for example, from commercial vendors. Subsections A and B discuss pricing incentives under monopoly and competitive conditions. Subsection C considers a situation in which two investors, each with distinct expertise, agree to sell part of their information.
θ-information. In stage 0, each investor chooses between remaining uninformed, purchasing a subscription, or producing information privately at cost \( c_\theta \). In stage 1, the vendors communicate the promised information to their subscribers, while non-subscribing active investors produce information privately. All active investors—including the subscribers—interpret their information with some error.\(^\text{18}\) News vendors add neither bias nor noise to their product, and their marginal cost of communication is negligible.

### A Market Power and Crowding Out

To begin to address the role of competition in the market for information, let us first analyze the case of a monopolistic vendor. The monopolistic setting highlights, in the extreme, how an information vendor with market power affects the information efficiency of financial markets. The assumption of market power in markets for information is by no means implausible.\(^\text{19}\) Due to high fixed but low marginal costs, the production and dissemination of information exhibits substantial economies of scale. Market power is salient in the media industry also because media ownership confers private benefits of control, which promotes ownership concentration (Demsetz (1983), Djankov et al. (2003)). Entry regulation and political capture place further restrictions on competition in many countries (Besley and Prat (2006)).

The monopolistic supply of any product is typically smaller than competitive supply. Admati and Pfleiderer (1988a) show that the monopolist’s disincentive to expand supply is

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\(^{18}\)The idea behind this last assumption is that news vendors provide raw data, which investors must interpret to make informed investment decisions. Strictly speaking, information cost should therefore comprise the costs of both acquiring data and of interpreting it. While allowing for interpretation errors, the model de facto normalizes interpretation cost to zero. This simplification does not affect the conclusions.

\(^{19}\)Before Reuters and Thompson merged in 2007, Bloomberg, Reuters and Thompson accounted for a combined market share of about two-thirds of the US financial information services industry. Notwithstanding, compared to other countries, the US financial information services industry is arguably the most competitive.
particularly strong in the case of financial information. The reason is for this is that infor-
mation is sold to investors who compete over trading profits, so that new subscribers’ profits
come at the expense of existing subscribers’ profits. Thus, the value of information critically
relies on scarcity, which is eroded by sales: the more people who own the information, the
less it is worth.

Monopolizing attention

Admati and Pfleiderer (1988a) demonstrate this logic in a model with a single type of in-
formation. These authors derive the stark result that a risk-neutral monopolist does not
wish to sell information to more investors than it needs to—if possible, to no more than
one investor. For the same reason, a monopolist in the present framework does not expand
supply if it sells both types of information. The monopolist would set \( p_A = c_A \) and \( p_B = c_B \),
and the market would de facto be the same as without the information vendor.

The absence of worthwhile information that is not, or cannot be, sold (by the monop-
olist) is an important presumption insofar as it implies that the monopolist, for any price
\( p_A \leq c_A \), absorbs all actual demand. This is to say that the vendor not only monopolizes
the sale of information but also, by construction, investor attention. In contrast, when mar-
ketable information represents only a subset of all investment-relevant information, even a
monopolistic vendor may have to strive for attention.\(^{20}\)

To analyze such a setting, consider a partial information market in which only information
about factor \( A \) is marketable. First, we derive the optimal form of sale and price structure.

\(^{20}\)Information is less marketable when it is difficult to assimilate or when truthful communication is non-
credible (Allen (1990), Michaely and Womack (1999)). For example, Goetzmann et al. (2004) find that a
(movie) script that is less fact-checkable by hard information is more difficult to sell. Regarding financial
information, Roll (1988) concludes in his pioneering study on price comovement that “the financial press
misses a great deal of relevant information generated privately” (564).
Lemma 2 A direct sale of unlimited subscriptions at a single price is optimal.

Consider any price-quantity schedule posted by a monopolist. Since investors are symmetric within each investor class, the highest subscription price paid in equilibrium must equal the expected trading profit of a subscriber. At all lower prices, subscribers pay less than their reservation price, and the monopolist would earn more by raising the price. Binding quantity restrictions are therefore suboptimal from the vendor’s point of view, while slack restrictions are unnecessary.

Indirect sales can serve as a means to control the number of active investors, and hence to curb competition (Admati and Pfleiderer, 1990). Here, this benefit does not arise because investors can self-produce information. Suppose that the vendor sets up a fund. The fund’s optimal trading strategy is that of a single investor. Once a sufficient number of investors sign up for the fund, it becomes worthwhile for other investors to compete with the fund by trading on self-produced information. The fund’s expected profit is \( \rho_A(n_A^*, n_B^*) - c_A \). If the vendor instead sells the data at \( p_A = c_A \), it extracts the expected trading profits of all \( A \)-investors and earns \( n_A^* \rho_A(n_A^*, n_B^*) - c_A \).\(^{21}\)

Given Lemma 2, the monopolist chooses a subscription price \( p_A \in [0, c_A] \) to maximize its total profit \( \Pi(p_A) = n_A^*(p_A)p_A - c_A \). The number of subscriptions, \( n_A^*(p_A) \), is endogenously determined as a function of \( p_A \). Subscriptions are sold until the subscribers’ expected profits reach zero. Thus, as mentioned above, the monopolist extracts the expected trading profits

\(^{21}\)The argument abstracts from (management) fees that are contingent on realized trading profits (cf. Biais and Germain (2002)). When fees and payouts are restricted to proceeds from trading, it is unclear whether there exists a fee schedule that can make the fund manager both more aggressive and better off than a single proprietary trader in this setting (see Appendix B). A full analysis of such a contracting problem, in the presence of alternative investment strategies, is beyond the scope of this paper.
Figure 3: Monopolizing attention

of all the subscribers:

$$\Pi(p_A) = n_A^*(p_A)\rho(n_A^*(p_A), n_B^*(p_A)) - c_A.$$  \hspace{1cm} (5)

This highlights not only that $p_A$ determines $n_A^*$ and $n_B^*$ but also that maximizing the vendor’s profit is equivalent to maximizing total expected trading profits from strategy $A$.

**Proposition 3**  For $c_A > c_A$, the monopolist lowers $p_A$ until $n_B^*(p_A) = 0$.

Unlike in Admati and Pfleiderer (1988a), the monopolist must battle for attention, since investors can turn to $B$-information. This affects the monopolist’s pricing decision for two reasons. First, expenditures to acquire $B$-information do not translate into revenues for the monopolist. Second, the presence of $B$-investors reduces the trading profits of the monopolist’s subscribers. This gives the monopolist incentives to crowd out $B$-investors or, more precisely, to sway them toward becoming subscribers. Once strategy $B$ is entirely crowded out, the monopolist has no incentive to further lower the price (see Figure 2).

Relative to the outcome in the absence of information sales, the monopolist increases
both the trading volume and the number of active investors, which improves market efficiency. At the same time, by monopolizing attention, the monopolist effectively induces herd behavior and potentially decreases price informativeness (Implication 8). Thus, the impact of information sales on financial markets is not necessarily benign: a vendor with market power increases supply only if it crowds out investor incentives to seek alternative types of information.

**Reuters in its early days**

An anecdote about *Reuters* in its early days illustrates this crowding-out rationale. In the 1850s, *Reuters* controlled telegraph lines as well as the right to circulate news received from ships of the *Austrian Lloyd’s*. For a given market (readership), the ideal buyer for these services might have been a single newspaper at a price that reflects the exclusivity. However, many newspapers at the time collected their own foreign news through correspondent networks. Major newspapers, like *The Times*, initially resented any dependence on *Reuters*, in particular because of the view that the short telegraph dispatches differed in nature from correspondent reports; “*The Times* kept a correspondent at the [war] front, the famous W. H. Russell, who wrote long, mailed dispatches. These made a great impression by revealing military shortcomings, but they were not intended to give the latest news.”

*Reuters* therefore adopted a pricing strategy aimed at replacing the newspapers’ own networks. To this end, it offered all the main London newspapers subscriptions to its international news dispatches at £30 per month, significantly less than a newspaper’s cost of running a correspondent network. However, at this price, *Reuters* had to attract a critical number of daily newspapers to make the service profitable.

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*22* The historical account is taken from Read (1999, 24f).
While *The Times* continued to snub *Reuters*, relying on its own network, others began to subscribe to the *Reuters* service. As more newspapers gained access to *Reuters* dispatches, the value of running a correspondent network deteriorated: “Good though its own network was, *The Times* needed to know each evening what telegrams from Reuter were likely to appear in the columns of its competitors next morning, even though it did not necessarily want to print the telegrams itself.” Eventually, *The Times* took out a *Reuters* subscription. By 1861, *Reuters* had won over almost all London newspapers as customers.

The success of *Reuters* gave each newspaper cheaper access to foreign news, but it also made foreign news coverage more homogenous. At times, the overseas sections of all newspapers were identical, merely displaying the same telegraph dispatches. Arguably, the short dispatches crowded out more complex information contained in correspondent reports.

B Competition and Liquidity-Driven Diversity

It is apparent that introducing a vendor who sells other types of information promotes diversity.\(^{23}\) Surprisingly, so does competition by a vendor who sells the same type of information. To address the role of competition in information markets, which are characterized by high fixed and entry costs, let us model a contested monopoly in which the incumbent vendor is threatened by the entry of a competitor—while keeping the assumption that some information is non-marketable.

\(^{23}\) Though, even if we introduce a *B*-vendor into the monopoly setting, predatory pricing by the *A*-vendor remains a possible equilibrium. For example, \(p_A\) may be set such that \(\min R_A = \max R_B\), and \(n_A\) is just small enough for the market not to reach the complement region. At this point, a single investor finds *B*-trading unprofitable. Selling information about *B* to more than one investor is more profitable, if crowding out *A*-investors improves the liquidity for *B*-investors. Yet, at \(\min R_A = \max R_B\), the *A*-strategy is a complement to the *B*-strategy, so that expanding the supply of *B*-information to crowd out *A*-investors reduces liquidity for *B*-investors. Thus, a *B*-vendor may not fare better than a single *B*-investor, and stays out of the market.
A contestable market

Consider the following changes to the model: There is an incumbent vendor who incurs a non-negligible fixed cost \( K_I \in [K_L, K_H] \) of running its sales operation. The true cost \( K_I \) is privately observed, while the distribution of \( K_I \) is commonly known. A rival vendor can enter the market if it incurs an up-front (sunk) cost \( S \). Once the rival enters the market, its operating costs are \( K_E \). For simplicity, let \( K_E \) be commonly known, and let \( K_E < K_L \).

This parsimoniously captures the idea that the incumbent’s market power is curbed by the potential entry of a competitor.

The entry game takes place in stage \(-2\) and proceeds as follows. First, the incumbent commits to a subscription price \( p_I \), which is renegotiable in stage \(-1\) subject to consensus. Second, after observing \( p_I \), the rival decides whether or not to enter the market at cost \( S \). Third, if the rival enters, the vendors engage in price competition in stage \(-1\). Otherwise, the game proceeds as in the monopoly case except that the incumbent cannot raise the price above its pre-commitment level \( p_I \).

The rival enters when it expects post-entry profits to at least match its entry cost \( S \). If the rival enters, price competition pushes the price to the incumbent’s break-even point. The expected post-entry revenue is therefore \( E(K_I | p_A^I) + c_A \), and the rival enters (only) when \( E(K_I | p_A^I) + c_A \geq S + K_E + c_A \). If the incumbent wishes to preempt entry, it must choose \( p_A^I \) to signal that its own operating costs \( K_I \) do not exceed \( S + K_E \).

**Lemma 3** The following is a Pareto-dominant Perfect Bayesian Equilibrium: For \( E(K_I) \leq S + K_E \), the incumbent is uncontested and sets the monopoly price. For \( E(K_I) > S + K_E \), the incumbent is contested and deters entry if \( K_I \leq K_I^* \) but loses the market if \( K_I > K_I^* \).

\[^{24}\text{Introducing uncertainty or private information about } S \text{ or } K_E, \text{ or allowing for } K_E \geq K_L, \text{ makes the extension more realistic, but the mechanics and basic intuition behind the results remain the same. Note also that this setup includes Bertrand competition as a special case (} S = 0 \text{ and } K_E = K_I. \)\]
where $K_I^* < K_H$. When the incumbent is contested, the equilibrium price is lower than the monopoly price and (weakly) increasing in $S$ and $K_E$.

If entry is not worthwhile for a rival when facing an incumbent of average efficiency, the incumbent is de facto a monopolist who need not be concerned with signaling high efficiency. Entry is deterred without further ado. But if average efficiency is not enough to deter entry, the incumbent may wish to signal high efficiency, in order to inform the rival that post-entry profit is low. Such a signal is to commit to a price that a less efficient incumbent could not afford to mimic. Hence, a sufficiently efficient incumbent ($K_I \leq K_I^*$) commits to a low price and successfully defends the market. If the incumbent is too inefficient ($K_I > K_I^*$), it is unable to deter entry and loses the market to the rival.

When the incumbent is contested, the equilibrium price is below the monopoly price irrespective of whether the incumbent enters. To deter the rival, the incumbent must commit to a lower price. In case of entry, competition drives the price down to the incumbent’s break-even price. Since lower levels of $S$ or $K_E$ increase contestability, they decrease the (expected) equilibrium price and the cut-off type $K_I^*$. Intuitively, when entry is cheaper or the rival more efficient, the deterrence price is lower and the incumbent more often fails to deter entry. Empirically, the entry cost $S$ may consist of two components: technological expenditures, $S_1$, and the cost of overcoming regulatory “red tape”, $S_2$.\footnote{Djankov et al. (2002) document that regulatory entry barriers tend to be higher in countries with less democratic governments (cf. http://www.doingbusiness.org/). Djankov et al. (2003) show that the media sector in such countries is often concentrated and government-controlled.} Contestability would then be a function of technological progress, $(S_1 K_E)^{-1}$, and market openness, $S_2^{-1}$.\footnote{Djankov et al. (2002) document that regulatory entry barriers tend to be higher in countries with less democratic governments (cf. http://www.doingbusiness.org/). Djankov et al. (2003) show that the media sector in such countries is often concentrated and government-controlled.}
While two quarrel, a third rejoices

It is noteworthy that, when the market is contested, price policy is geared toward defeating rival vendors and less toward crowding out sources of alternative information. Indeed, vendors supplying information about $A$ compete in perfect substitutes, whereas information about $B$ represents an imperfect substitute. Competition is thus “hierarchical” in the sense that vendors first and foremost compete with each other, while concerns about $B$-investors play a secondary role for their pricing incentives.

Being neglected benefits $B$-investors, since competition among vendors creates a positive liquidity spillover effect: As contestability increases, information about factor $A$ becomes cheaper, which propagates active investment in strategy $A$. The associated rise in trading volume generates more quasi-noise trading for alternative investment strategies and, as a result, promotes active investment in strategy $B$. This spillover effect is most pronounced when the rival vendor actually enters the market. The entry of new information vendors—as, for example, the entry of Bloomberg in the 1980s—can thus have dramatic effects on financial market liquidity and on both the level and composition of active trading.

**Proposition 4** For $E(K_I) > S + K_E$, $n_B$ is (weakly) increasing in $(S_1 K_E)^{-1}$ and $S_2^{-1}$.

Competition among vendors is beneficial not only because it disseminates information more effectively, but also because it encourages investors to seek alternative information. This lends support to the notion that competitive media can be instrumental in improving the quality of financial markets. Empirically, one could test predictions regarding the composition of active trading by exploring whether measures of progress in information technologies or measures of information market openness predict (i) greater information market competition, (ii) lower information market prices, (iii) a more diverse universe of active investment
strategies, and (iv) lower levels of price comovement. D’Avolio et al. (2002) discuss related informational implications of technological progress on financial market development. Alternatively, one could study how cross-country or time-series variation in the competitiveness of information markets relates to the cross-section of asset returns (cf. Fang and Peress (2009)), the behavior and performance of active investors (cf. Kacperczyk and Seru (2007)), or the predictive power of financial news (cf. Tetlock et al. (2008)).

Key to Proposition 4 are liquidity externalities in asset markets. In contrast, Veldkamp (2006a,b) focuses on price externalities that arise in competitive information markets; since information in higher demand is cheaper, investors wish to acquire the same type of information. This leads to the opposite prediction when the number of potential active investors is finite, namely that information market competition exacerbates price comovement. In Simonov (1999), competing vendors sell information signals that are per se complements or substitutes. Such direct information externalities do not exist in the present model.
C Proprietary Trading and Information Sales

Above, the liquidity spillover is the unintended consequence of competition among vendors. In other situations, the purpose of information sales may be precisely to generate quasi-noise trading. This may be the case for financial institutions that engage in proprietary trading activities and, at the same time, supply information to other investors. At first glance, it seems puzzling that active traders distribute information, as this activity creates competitors. However, information sales may be driven by liquidity concerns.

Consider two investors, 1 and 2, endowed with information about A and B, respectively. Information about A is also held by \( n_A - 1 \) other investors, whereas information about B is exclusive to investor 2. Investor 2 can “hire” investor 1 for wage \( W \) to sell—or even to give away—information about A to other investors. The question is whether there exists a wage \( W \) such that both investors benefit from such an arrangement.

**Proposition 5** There exists a non-empty interval \( N_A \) such that, when \( n_A \in N_A \), the two investors benefit from the following agreement: investor 2 trades on strategy B and pays \( W > 0 \) to investor 1, which in return shares its A-data with infinitely many other investors.

The intuition behind this result is straightforward. The presence of \( n_A - 1 \) other investors pursuing the A-strategy reduces the expected profit of both investor 1 and investor 2. While giving away information about A for free eliminates investor 1’s profit, it also eliminates any negative externality that the A-investors exert on investor 2’s profit \([T(\infty) \to T(0)]\). So long as \( \pi_B(1,0) > \pi_B(1,n_A) + \pi_A(n_A,1) \), there are gains from trade that the two investors can share. Intuitively, by flooding the market with information about A, they create a “herd” of A-investors among which investor 2 can “hide”. In analogy, for a financial institution, a benefit of supplying many investors with mundane information may thus be increased quasi-
noise trading, which in turn facilitates trading on proprietary information, i.e., information that the institution does not share with outside investors.

This contrasts with other explanations for combining trading and information sharing. In Benabou and Laroque (1992), the release of information by a well-informed investor is meant to (sometimes) manipulate other investors and to exploit their misinformation through proprietary trades. Proposition 5 presumes truthful information sales; those who receive, or buy, the information are not deceived in any way. The seller's gains stem from increased liquidity rather than from price manipulation. This liquidity motive is also present in Mendelson and Tunca (2004), in whose model a well-informed investor truthfully releases part of its information to increase participation—i.e., liquidity provision—from uninformed noise traders. In Proposition 5, the release serves to increase participation—i.e., quasi-noise trading—from informed traders. Another difference is that the release (like the monopoly sale) is predatory insofar as it is designed to eliminate returns from a “rival” trading strategy. Finally, Biais and Germain (2002) study the combination of proprietary trading and indirect information sales, i.e., selling shares of an actively managed fund. They show how an indirect sale can be structured to maximize combined profits from proprietary and fund trades.

IV Concluding Remarks

This paper examines a financial market in which fundamentals are driven by several factors, and active investors choose about which factor to acquire information, i.e., which investment strategy to follow. The central questions are: How do differently informed investors interact when trading in the same market? How does this interaction affect such investors' information choice? Contrary to common wisdom, active investors in this setting can benefit from an increase in trading on the part of other active investors, so long as they follow different
strategies. On the one hand, trading in any active strategy creates informational frictions that reduce market liquidity. On the other hand, investors who trade on different information generate \textit{quasi}-noise trading for each other. A given investment strategy can therefore enhance or impair another strategy; that is, a decrease in the cost of one type of information can decrease or increase the demand for another type of information.

This paper discusses how these externalities in information choice affect the composition of active investment and, as a result, herding behavior, trading volume, liquidity spirals, price and liquidity comovement, and the information content of prices. It also examines how these externalities affect information markets. Market power allows an information vendor to monopolize investor attention, i.e., to crowd out the demand for alternative information by exploiting negative externalities through its pricing policy. Such information sales increase active trading and market efficiency, but they reduce information diversity and possibly price informativeness. By contrast, competition among (identical) vendors generates, as a by-product, positive externalities. Such competition propagates cheap information, which in turn boosts quasi-noise trading to the benefit of alternative investment strategies. For this reason, a financial institution may supply investors with mundane information to deliberately generate quasi-noise trading that enhances its own proprietary trading strategies.

The analysis presented in this paper can be extended in several directions. For instance, investors might acquire different types of information in different degrees. This would allow for hierachical information structures among active investors and for a distinction between “generalists” and “specialists”. One could also consider dynamic issues. Because of liquidity externalities, active investors may want to “time” not only noise trades (Admati and Pfleiderer, 1988b) but also—\textit{depending} on their volume—differently informed trades. Similarly, investors may have incentives to coordinate the timing of their information choices.
Appendix A: Proofs

Proof of Lemma 1

Proof of Proposition 1
The main proof further below makes use of the following auxiliary results.

Lemma 4 For $n \geq \max(n_\theta, n_{\theta'}) \geq \min(n_\theta, n_{\theta'}) \geq 0$,

(a) $\pi_\theta(n_\theta, \cdot)$ decreases in $n_\theta$.
(b) $\pi_\theta(\cdot, n_{\theta'})$ has a unique minimum in $\mathbb{R}^+$,
(c) $\pi_\theta(1, n-1) > \pi_\theta(n_\theta, n-n_\theta)$ for all $n_\theta > 1$.

Proof of Lemma 4. Given that $c_\theta$ is a constant, we need only consider the behavior of $\rho_\theta(n_\theta, n_{\theta'})$ or, more precisely,

\[
\rho_\theta(n_\theta, n_{\theta'}) = E[(\bar{X} - \bar{Y})\hat{s}_{i,theta}] = E[(\bar{X} - \lambda\bar{Z})\hat{s}_{i,theta}]
\]

\[
= \alpha_\theta E(\bar{X}\hat{s}_{i,theta}) - \lambda E \left[ \sum_{l=1}^{n_\theta} \alpha_{\theta} \hat{s}_{i,theta} + \sum_{l=1}^{n_{\theta'}} \alpha_{\theta'} \hat{s}_{i,theta'} + \tilde{y} \right] \alpha_{\theta'} \hat{s}_{i,theta'}
\]

\[
= \alpha_\theta \left[ E \left( \bar{X}\hat{s}_{i,theta} \right) + E \left( \bar{X}\hat{s}_{i,theta'} \right) \right] - \lambda \left[ \sum_{l=1}^{n_\theta} \alpha_{\theta}^2 E(\hat{s}_{i,theta}) + \sum_{l=1}^{n_{\theta'}} \alpha_{\theta'} \alpha_{\theta} E(\hat{s}_{i,theta'}) + \alpha_{\theta} E(\hat{\bar{X}}\hat{s}_{i,theta'}) \right].
\]  \tag{6}

Since $E \left( \bar{X}\hat{s}_{i,theta} \right) = \sigma^2$, $E \left( \bar{X}\hat{s}_{i,theta'} \right) = 0$, $E \left( \hat{s}_{i,theta} \right) = \sigma^2 + \sigma_z^2$, $E \left( \hat{s}_{i,theta'} \right) = \sigma^2$, $E \left( \hat{s}_{i,theta'} \hat{s}_{i,theta} \right) = 0$, and $E \left( \hat{\bar{X}}\hat{s}_{i,theta} \right) = 0$ for all $\theta, \theta' \neq \theta, i, l \neq i$, this becomes

\[
\rho_\theta(n_\theta, n_{\theta'}) = \alpha_\theta \sigma^2 (1 - \lambda n_\theta \alpha_\theta) = \alpha_\theta \sigma^2 \left[ \frac{\sigma^2 + 2\sigma_z^2}{(n_\theta + 1) \sigma^2 + 2\sigma^2} \right]
\]  \tag{7}

where the last equality follows from Lemma 1. Further substituting for $\alpha_\theta$ and then $\lambda$ from Lemma 1 yields

\[
\rho_\theta(n_\theta, n_{\theta'}) = \left[ \frac{\sigma^2 + 2\sigma_z^2}{(n_\theta + 1) (\sigma^2 + 2\sigma^2)} + T_{\theta'} \right]^{-1/2}.
\]  \tag{8}

Now define $A \equiv [\rho_\theta(n_\theta, n_{\theta'})]^{-2}$ so that, by construction, $A$ and $\rho_\theta(n_\theta, n_{\theta'})$ are inversely related. Using (8), we get

\[
A = \frac{n_\theta \left( \sigma^2 + \sigma_z^2 \right)^2}{\sigma^2 \sigma_{\theta'} \sigma^2 + \sigma_{\theta'}^2 \sigma^2} + \frac{\left( \sigma^2 + 2\sigma_z^2 \right)^2}{\sigma^2 \sigma_{\theta'}^2 \sigma^2 + \sigma_{\theta'}^2 \sigma^2} \cdot T_{\theta'}
\]
and can make the following observations: $A$ strictly increases in $n_\theta$, which proves part (a). Furthermore, recall that $T_{\theta'}$ has a unique maximum. For a given $n_\theta$, that maximum translates into a unique maximum of $A$ in $n_{\theta'}$. This in turn corresponds to a unique minimum of $\rho_\theta(n_\theta, n_{\theta'})$ in $n_{\theta'}$, which proves part (b).

The proof of part (c) proceeds in steps (i)-(iii). For use below, let us define $\rho_\theta^0(x) \equiv \rho_\theta(x, n - x)$.

(i) We want to show that $\rho_\theta^0(n_\theta) > \rho_\theta^0(n - n_\theta)$ for $n_\theta \in (0, n/2)$. For a given population $n$, it follows from Lemma 1 that $n_\theta < n_{\theta'} \Rightarrow \alpha_\theta > \alpha_{\theta'} \Rightarrow \rho_\theta^0(n_{\theta'}) > \rho_\theta^0(n_{\theta'})$ (★). Simple inspection of the formulae in Lemma 1 further shows that, for a given population $n$, a trader of class $\theta$ for $n_\theta = x$ faces exactly the same decision problem as would a trader of class $\theta'$ for $n_{\theta'} = x$. That is, $\rho_\theta^0(x) = \rho_{\theta'}^0(x)$. In conjunction with (★), this “symmetry” implies that $\rho_\theta^0(n_\theta) > \rho_\theta^0(n - n_\theta)$ for $n_\theta \in (0, n/2)$.

(ii) We want to show that $\rho_\theta^0(n_\theta)$ is monotonically decreasing for $n_\theta \in [0, n/2]$. Consider the situation $(n_\theta, n_{\theta'}) = (n/2, n/2)$, where—by the same symmetry argument as in step (i)—$\rho_\theta^0(n/2) = \rho_{\theta'}^0(n/2)$ and $T(n_{\theta'}) = T(n_{\theta'})$. Now, first suppose that $T'(n/2) > 0$. Since $T(\cdot)$ is concave at this point, if we start decreasing $n_\theta$ from $n_\theta = n/2$, $\sum_\theta T(n_\theta)$ and hence $\lambda$ will decrease. (In fact, once $n_\theta$ is sufficiently low, both $T(n_\theta)$ and $T(n_{\theta'})$ will decrease if $n_\theta$ drops further.) In conjunction with the fact that this reduces competition in class $\theta$, the decrease in $n_\theta$ increases the expected profit in class $\theta$. Alternatively, suppose that $T'(n/2) < 0$. Again, consider a decrease in $n_\theta$. From part (a) of the lemma, we know that, if we were to keep $n_{\theta'}$ constant, this would increase the expected profit in class $\theta$. This positive effect is reinforced by the fact that the additional increase in $n_{\theta'}$, for $T'(n/2) < 0$, further decreases $\lambda$. Thus, in either case, $\rho_\theta^0(n_\theta)$ is monotonically decreasing for $n_\theta \in [0, n/2]$.

(iii) Thus, note that $\rho_\theta^0(1) > \rho_\theta^0(x) > \rho_\theta^0(n - x)$ for all $x \in (1, n/2)$. The first inequality follows from step (ii), while the second inequality follows from step (i). This proves part (c).

**Main proof.** Parts (a) and (b) of Lemma 4 respectively imply that $\pi_A(n_\lambda, 0)$ has a unique root and that $\pi_B(1, n_\lambda)$ can have—and has at most—two roots. For use below, let us therefore define $n_\lambda^A \equiv \min \mathcal{R}_A = \min\{n_\lambda : \pi_A(n_\lambda, 0) \leq 0\}$ and $[n_\lambda^A, \pi_\lambda] \equiv \mathcal{R}_B = \{n_\lambda : \pi_B(1, n_\lambda) \leq 0\}$.

**Preliminary.** We want to show that $\pi_A(n_\lambda, 0) = \pi_B(1, n_\lambda)$ has a unique solution. That they must cross at least once follows from parts (a) and (b) of Lemma 4. To show that they cross at most once, it suffices to show that $\rho_A(n_\lambda, 0) = \rho_B(1, n_\lambda)$ has a unique solution. Using (8), these functions are given by

\[
\rho_A(n_\lambda, 0) = \frac{\sigma_\theta \sigma_\lambda^2 (\sigma^2 + 2\sigma_\lambda^2)}{[(n_\lambda + 1) \sigma^2 + 2\sigma_\lambda^2] \sqrt{n_\lambda (\sigma^2 + \sigma_\lambda^2)}}
\]

\[
\rho_B(1, n_\lambda) = \frac{\sigma_\lambda \sigma_\lambda^2 (\sigma^2 + 2\sigma_\lambda^2)}{2(\sigma_\lambda^2 + \sigma_\lambda^2)^{3/2} \sqrt{((n_\lambda + 1) \sigma^2 + 2\sigma_\lambda^2)^2 + 4n_\lambda (\sigma^2 + \sigma_\lambda^2)^2}}
\]

Equating these expressions and rearranging yields

\[
1 = \frac{((n_\lambda + 1) \sigma^2 + 2\sigma_\lambda^2)^2 \sqrt{n_\lambda}}{2(\sigma_\lambda^2 + \sigma_\lambda^2)^{3/2} \sqrt{((n_\lambda + 1) \sigma^2 + 2\sigma_\lambda^2)^2 + 4n_\lambda (\sigma^2 + \sigma_\lambda^2)^2}}
\]
We square and take the inverse on both sides, and further rearrange to get

\[
4 (\sigma^2 + \sigma_A^2) \left( \frac{1}{(n_A + 1) \sigma^2 + 2\sigma_A^2 n_A} + \frac{1}{(n_A + 1) \sigma^2 + 2\sigma_A^2 n_A} \right) = 1.
\]

The left-hand side goes to infinity for \( n_A \to 0 \) and strictly decreases in \( n_A \). Thus, \( \pi_A(n_A, 0) = \pi_B(1, n_A) \) has exactly one solution, to the left of which \( \pi_A(n_A, 0) > \pi_B(1, n_A) \) and to the right of which \( \pi_A(n_A, 0) < \pi_B(1, n_A) \).

**First part:** We start with statement for the case in which \( \min \mathcal{R}_A \in \mathcal{R}_B \neq \emptyset \). Consider first the sufficient condition, \( \mathcal{R}_A \leq n_A^0 \leq \mathcal{R}_A \Rightarrow (n_A^0, n_B^0) = (n_A^0, 0) \). It is straightforward to verify that the inequality \( \mathcal{R}_A \leq n_A^0 \) is equivalent to the condition \( \pi_B(1, n_A^0) \leq \pi_A(n_A^0, 0) = 0 \). We now check different candidate equilibria in steps (i)-(iv):

(i) Note that \( (n_A^0, 0) \) trivially satisfies the free-entry condition.

(ii) Conjecture an equilibrium with \( n_A > n_A^0 \) and \( n_B \geq 0 \). For all \( n_A > n_A^0 \), \( \pi_A(n_A, n_B) < \pi_A(n_A^0, n_B) < \pi_A(n_A^0, 0) = 0 \).

That is, \( A \)-traders would incur a loss. Hence this cannot be an equilibrium.

(iii) Conjecture an equilibrium with \( n_A < n_A^0 \) and \( n_B = 0 \). Clearly, \( \pi_A(n_A, 0) > \pi_A(n_A^0, 0) \). Hence, this cannot be an equilibrium.

(iv) Conjecture an equilibrium with \( n_A < n_A^0 \) and \( n_B \geq 1 \). Since \( n_A < n_A^0 \), there is less competition in class \( A \) than in the case of \( (n_A^0, 0) \). This requires that \( \lambda \) must be higher for \( (n_A, n_B) \) than for \( (n_A^0, 0) \) in order for the zero-profit condition \( \pi_A(n_A, n_B) = 0 \) to hold. However, this implies that \( B \)-investors not only face (weakly) more competition but also a less liquid market for \( (n_A, n_B) \) than for \( (n_A^0, 1) \). This in turn implies \( \pi_B(n_B, n_A) < \pi_B(1, n_A^0) \leq 0 \). Hence, this cannot be an equilibrium.

Finally, consider the necessary condition, \( (n_A^*, n_B^*) = (n_A^0, 0) \Rightarrow \mathcal{R}_A \leq n_A^0 \leq \mathcal{R}_A \). If the first inequality were violated, \( n_A^0 \) would lie to the right of the intersection of \( \pi_A(n_A, 0) \) and \( \pi_B(1, n_A) \). The same would be true, if the second inequality were violated. In either case, then, it follows from the preliminary step above that we would have \( \pi_A(n_A^0, 0) < \pi_B(1, n_A^0) \). This in turn would imply that \( (n_A^0, 0) \) cannot be an equilibrium as \( A \)-investors would, on the margin, switch to the \( B \)-strategy.

**Second part:** We now turn to the statement for the case in which \( \min \mathcal{R}_A \notin \mathcal{R}_B \) or \( \mathcal{R}_B = \emptyset \). We must show that, when \( n_A^0 \notin (\mathcal{R}_A, \mathcal{R}_A) \), there exists a unique pair \( (n_A^*, n_B^*) \) that satisfies \( n_A^* \geq n_B^* > 0 \) and \( \pi_\theta(n_A^*, n_B^*) = 0 \) for \( \theta = A, B \). The proof proceeds in steps (i)-(vi):

(i) Note that neither \( (0, 0) \), \( (0, n_B) \) nor \( (n_A, 0) \) can be an equilibrium. This follows respectively from \( \pi_A(1, 0) > 0 \) for \( \theta = A, B \), \( \pi_A(1, n_B - 1) > \pi_B(1, n_B - 1) > \pi_B(n_B, 0) \) due to \( c_A \leq c_B \) and part (c) of Lemma 4, and the necessary condition in the first part of the proof.

(ii) Consider an arbitrary population \( n \) such that \( \pi_B(x, n - x) > \pi_A(n, 0) \) for some \( x > 0 \). Gradually increase \( x \) so that \( \pi_A \) and \( \pi_B \) change continuously. For \( x = n - 1 \), we know that \( \pi_B(n - 1, 1) \leq \pi_A(n, n - 1) \). This follows from \( \pi_B(n - 1, 1) < \pi_B(1, n - 1) \leq \pi_A(n, n - 1) \), where the first inequality follows from part (c) of Lemma 4 and the second inequality follows from \( c_A \leq c_B \). Thus, as \( x \) goes from \( 0 \) to \( n \), the profit functions \( \pi_B \) and \( \pi_A \) must intersect.
(iii) As both classes grow, the investors’ total expected profits decrease (and eventually become negative). To see this, write these profits as

\[\frac{n_A}{(n_A + 1)^2} \left( \frac{\sigma_A^2}{\sigma_A^2 + \sigma_B^2} \right)^{1/2} \left( \frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2} \right)^{1/2} - n_A c_A \]

\[+ n_B \frac{\sigma^2 \sigma_y (\sigma^2 + 2 \sigma^2)}{((n_B + 1) \sigma^2 + 2 \sigma^2)} \left( \frac{n_A (\sigma^2 + \sigma^2)}{((n_A + 1) \sigma^2 + 2 \sigma^2)} \right)^{1/2} \left( \frac{n_B (\sigma^2 + \sigma^2)}{((n_B + 1) \sigma^2 + 2 \sigma^2)} \right)^{1/2} - n_B c_B = \]

\[\frac{\sigma^2 \sigma_y (\sigma^2 + 2 \sigma^2)}{((n_A + 1) \sigma^2 + 2 \sigma^2)} \left( \frac{n_A (\sigma^2 + \sigma^2)}{((n_A + 1) \sigma^2 + 2 \sigma^2)} \right) - \frac{n_B (\sigma^2 + 2 \sigma^2)}{((n_B + 1) \sigma^2 + 2 \sigma^2)} \left( \frac{n_A (\sigma^2 + \sigma^2)}{((n_A + 1) \sigma^2 + 2 \sigma^2)} \right)^{1/2} - n_A c_A - n_B c_B \]

Now consider the effect of increasing both \(n_A\) and \(n_B\). It is straightforward to verify that the marginal increase in the first term, the total expected trading profits, is decreasing in both \(n_A\) and \(n_B\). Moreover, the term converges to zero as both \(n_A\) and \(n_B\) approach \(\infty\). By contrast, the last two terms, the total information costs, increase linearly. Thus, as \(n_A\) and \(n_B\) grow, total expected profits reach zero and then become negative. This implies that there exists a population size \(n^*\) such that the “balancing” exercise in (ii) leads to an intersection of the profit functions where \(\pi_B(n_A^*, n_B^*) = \pi_A(n_A^*, n_B^*) = 0\). That is, a zero-profit equilibrium exists.

(iv) We now show that the zero-profit equilibrium \((n_A^*, n_B^*)\) is unique. First, consider any structure in which both groups are larger than under \((n_A^*, n_B^*)\). From the arguments in (iii), it then follows that overall profits must be smaller, which in turn implies that at least one of the groups makes a negative profit. Second, consider any structure in which both groups are smaller than under \((n_A^*, n_B^*)\). From the arguments in (iii), it then follows that overall profits must be larger, which in turn implies that at least one of the groups makes a positive profit. Third, consider any structure in which one group is larger and one group is smaller than under \((n_A^*, n_B^*)\). If \(\lambda\) increases (weakly), the larger group makes negative profits [because of the higher competition and the (weakly) lower liquidity relative to \((n_A^*, n_B^*)\)]. Conversely, if \(\lambda\) decreases, the smaller group makes positive profits [because of the lower competition and the higher liquidity relative to \((n_A^*, n_B^*)\)]. Thus, neither case admits a zero-profit equilibrium.

Proof of Proposition 2

It suffices to show that, in equilibrium, \(n_A\) is monotonically decreasing in \(c_A\). Starting from an equilibrium \((\pi_A^* = \pi_B^* = 0)\), consider an exogenous increase in \(c_A\). If \(n_B^*\) and \(n_A^*\) were to remain unchanged, the expected profit in class \(A\) would turn negative, whereas expected profits in class \(B\) would be unaffected.

First, let us keep the total number of investors \(n\) fixed. Then, \(A\)-investors prefer to switch to the \(B\)-strategy. By the “balancing” process described in step (iii) of the second part of the previous proof, there exists a new distribution, \(n_A^*\) and \(n_B^*\) such that \(n_A^* + n_B^* = n\), such that the expected profits in the two classes are equal again, \(\pi_A^* = \pi_B^*\). Since the cost \(c_A\) has increased and the “balancing” process started from \(\pi_A^* = \pi_B^* = 0\), the new equated profit levels for both investor classes must be negative, \(\pi_A^* = \pi_B^* < 0\). That is, there are too many investors in the market.
Second, by step (iv) of the second part of the previous proof, a decrease in the total investor population only keeps the expected profits at 0 in both investor classes if both classes, A and B, shrink. Hence, class A must further shrink. Overall, as a result of the increase in \( c_A \), class A must shrink because A-investors either switch to the B-strategy or exit the market.

Analogous arguments apply in the case of an increase in \( c_B \).

**Proof of Lemma 2**

First, note that price discrimination without rationing at each price is equivalent to selling unlimited subscriptions at the lowest offered price. Second, consider price-quantity schedules of the following form: the vendor determines a set of prices \( p_A^n > p_A^{n-1} > \cdots > p_A^1 \) and the maximum number of subscriptions \( y_A^n, y_A^{n-1}, \ldots, y_A^1 \) sold at each of these prices. Suppose that a quantity restriction is binding in the sense that more traders would like to purchase a subscription at that price, say \( p_A^i \). On the one hand, if it is unprofitable for any additional trader to purchase data at \( p_A^{i+1} \), the information vendor fares better by increasing \( y_A^i \) and hence the number of subscriptions sold at \( p_A^i \). On the other hand, if there are traders who purchase data at \( p_A^{i+1} \), the information vendor fares better by setting \( y_A^i = 0 \). To see this, note that all A-traders make the same trading profit, irrespective of the individual price paid for the data. Thus, if some traders do not incur a loss when buying data at \( p_A^{i+1} \), the \( y_A^i \) traders that buy data at \( p_A^i \) make a (total) profit of at least \( y_A^i (p_A^i - p_A^{i+1}) \). When \( y_A^i = 0 \), these traders would be willing to buy subscriptions at \( p_A^{i+1} \). Thus, having a binding quantity restriction is not optimal, whereas quantity restrictions that are not binding are unnecessary.

Finally, consider the indirect sale of information, for example, via a fund. In this case, the vendor trades on behalf of its subscribers, and a contract prespecifies a fixed subscription fee and a profit-sharing rule. If only a fixed fee is paid, the fund’s trading strategy and hence the fund’s expected profit is equivalent to that of a single trader. That is, the seller is better off directly selling the data (to many traders). Alternatively, suppose that the sharing rule induces the seller to trade as aggressively as \( n_A \) traders, where \( n_A \) is arbitrary. In this case, the fund’s expected profit will be equal to the combined expected profit of \( n_A \) individual traders with exactly the same signal. It can be shown that competition is fiercer for any number of traders with “photocopied” errors than for the same number of traders with “personalized” errors (cf. Admati and Pfleiderer, 1986). This is because, in the first case, the traders commonly know that all of them submit perfectly correlated orders, perfectly reinforcing each other’s impact on the price. As a result, selling data to \( n_A \) traders that interpret the data differently generates a higher expected profit than trading as aggressively as \( n_A \) traders with the same signal. Since the vendor extracts the entire trading profits of its subscribers, it is more profitable to sell the data directly to \( n_A \) traders.

**Proof of Proposition 3**

Suppose that \( n_A^0(c_A) < n_A \) and define \( \overline{p}_A \) by \( n_A^0(\overline{p}_A) = n_A \). Now consider any price \( p_A \in (\overline{p}_A, c_A] \). For any such price, \( n_A^0(p_A) < n_A \), both types of data will be acquired in equilibrium, and the monopolist’s gross profit (which is equal to A-traders'
total trading profits) is given by

\[
\Pi^A_{n_A, n_B} = \frac{n_A}{n_A + 1} \frac{(\sigma^2 + 2\sigma^2)}{(\sigma + 2\sigma^2)} \left( \frac{n_A (\sigma^2 + \sigma^2)}{(n_A + 1)\sigma^2 + 2\sigma^2} \right) + \frac{n_B}{n_B + 1} \left( \frac{(n_A + 1)\sigma^2 + 2\sigma^2}{(n_B + 1)\sigma^2 + 2\sigma^2} \right)^{1/2}
\]

\[
= \frac{\sigma^2 + 2\sigma^2}{(\sigma + 2\sigma^2)} \left( \frac{(n_A + 1)\sigma^2 + 2\sigma^2}{n_A (\sigma^2 + \sigma^2)} + \frac{n_B}{n_B + 1} \left( \frac{(n_A + 1)\sigma^2 + 2\sigma^2}{(n_B + 1)\sigma^2 + 2\sigma^2} \right)^{1/2} \right) \quad \text{(12)}
\]

Now suppose that the monopolist reduces the price to \( p_A \), thereby crowding out all \( B \)-traders. The monopolist’s gross profit in this case is given by

\[
\Pi^A_{p_A, 0} = \frac{n_A}{n_A + 1} \frac{\sigma^2 + 2\sigma^2}{(\sigma + 2\sigma^2)} \left( \frac{n_A (\sigma^2 + \sigma^2)}{(n_A + 1)\sigma^2 + 2\sigma^2} \right)^{1/2}
\]

\[
= \frac{\sigma^2 + 2\sigma^2}{(\sigma + 2\sigma^2)} \left( \frac{n_A (\sigma^2 + \sigma^2)}{(n_A + 1)\sigma^2 + 2\sigma^2} \right)^{1/2}
\]

\[
= \frac{\sigma^2 + 2\sigma^2}{(\sigma + 2\sigma^2)} \left( \frac{n_A (\sigma^2 + \sigma^2)}{(n_A + 1)\sigma^2 + 2\sigma^2} \right)^{1/2} \quad \text{(13)}
\]

From the proof of Proposition 2, we know that a decrease in the price of \( A \)-information increases the number of \( A \)-traders, so that \( p_A > n_A \).

We now need to show that \( \Pi^A_{p_A, 0} > \Pi^A_{n_A, n_B} \) or, equivalently, that

\[
\frac{\Pi^A_{p_A, 0}}{\Pi^A_{n_A, n_B}} = \frac{\sigma^2 + 2\sigma^2}{(\sigma + 2\sigma^2)} \left( \frac{(n_A + 1)\sigma^2 + 2\sigma^2}{n_A (\sigma^2 + \sigma^2)} \right) \left( \frac{n_A (\sigma^2 + \sigma^2)}{(n_A + 1)\sigma^2 + 2\sigma^2} \right)^{1/2}
\]

\[
= \frac{n_B}{n_B + 1} \left( \frac{(n_A + 1)\sigma^2 + 2\sigma^2}{n_A (\sigma^2 + \sigma^2)} + \frac{n_B}{n_B + 1} \left( \frac{(n_A + 1)\sigma^2 + 2\sigma^2}{(n_B + 1)\sigma^2 + 2\sigma^2} \right)^{1/2} \right) \quad \text{(14)}
\]

is larger than 1. This condition can be written as

\[
1 + \frac{n_B}{(n_A + 1)\sigma^2 + 2\sigma^2} \geq \frac{n_A}{(n_B + 1)\sigma^2 + 2\sigma^2}.
\]

The value of the left-hand side is greater than 1. The value of the right-hand side is smaller than 1 if \( \frac{n_A}{(n_B + 1)\sigma^2 + 2\sigma^2} > \frac{n_B}{(n_A + 1)\sigma^2 + 2\sigma^2} \), or equivalently \( T(p_A) > T(n_A) \). This is true because, by the definition of \( p_A \), \( T(n_A) \) is increasing in \( n_A \) for \( n_A < p_A \). Thus, the monopolist fares better by crowding out \( B \)-information.
Proof of Lemma 3

For notational convenience, define $K^{+} = S + K_{E}$. If $E(K_{I}) \leq S + K_{E}$, any uninformative precommitment price preempts entry. That provided, it is clearly a Perfect Bayesian Equilibrium for all incumbent types to choose the monopoly price. It is also straightforward to see that the pooling equilibrium with the monopoly price maximizes the incumbent’s profit.

However, if $E(K_{I}) > S + K_{E}$, a pooling price does not preempt entry. Therefore, some (of the more efficient) incumbent types have an incentive to reveal their type in order to deter the challenger. Let us conjecture a Perfect Bayesian equilibrium such that all types below a cut-off type $K^{+}_{I}$ preempt entry by setting a uniform price $p^{+}_{A}$ and all types above $K^{+}_{I}$ surrender the market. Incentive-compatibility requires that

$$n^{*}_{A}(p^{+}_{A})\rho(n^{*}_{A}(p^{+}_{A}), n^{*}_{B}(p^{+}_{A})) - c_{A} - K_{I} < 0 \quad \text{for all } K_{I} > K^{+}_{I} \tag{16}$$

and that

$$n^{*}_{A}(p^{+}_{A})\rho(n^{*}_{A}(p^{+}_{A}), n^{*}_{B}(p^{+}_{A})) - c_{A} - K_{I} \geq 0 \quad \text{for all } K_{I} \leq K^{+}_{I}. \tag{17}$$

This trivially implies that $p^{+}_{A}$ must satisfy

$$n^{*}_{A}(p^{+}_{A})\rho(n^{*}_{A}(p^{+}_{A}), n^{*}_{B}(p^{+}_{A})) - c_{A} - K^{+}_{I} = 0. \tag{18}$$

To deter entry, the cut-off value must further satisfy

$$E(K_{I} \leq K^{+}_{I}) \leq K^{+}_{I}. \tag{19}$$

Since $E(K_{I}) > S + K_{E}$ implies $K^{+}_{I} < E(K_{I}) < K_{H}$ and since $K_{I}$ is continuously distributed, there exists a unique $K^{+}_{I} \in [K_{L}, K_{H}]$ such that all $K^{+}_{I} < K^{+}_{I}$ satisfy this condition. Furthermore, since $p^{+}_{A}$ increases in the cut-off value $K^{+}_{I}$, the incumbent wishes to set the cut-off value as high as possible, that is, to $K^{+}_{I} = K^{+}_{I}$. To see this, note that all types above $K^{+}_{I}$ earn zero profits in any Perfect Bayesian equilibrium, and that all types below $K^{+}_{I}$ prefer a higher cut-off value not only because it preempts entry for more incumbent types but also because it increases the precommitment price and hence the profit of any incumbent type. If there is entry, the price is set to incumbent’s break-even price.

Lower $S$ or $K_{E}$ make it less likely that the information market is uncontestable ($E(K_{I}) \leq S + K_{E}$). When the market is contestable, lower $S$ or $K_{E}$ decrease $K^{+}_{I}$ and thereby also the equilibrium price. To see this, first note that $K^{+}_{I}$ increases in $S$ and $K_{E}$ (by definition: $K^{+}_{I} = S + K_{E}$), that $K^{+}_{I}$ increases in $K^{+}_{I}$ (by definition: $E(K_{I} \leq K^{+}_{I}) = K^{+}_{I}$), and that the equilibrium price $p^{+}_{A}$ increases in $K^{+}_{I}$ (by definition: $n^{*}_{A}(p^{+}_{A})\rho(n^{*}_{A}(p^{+}_{A}), n^{*}_{B}(p^{+}_{A})) = K^{+}_{I} + c_{A}$). By implication, $K^{+}_{I}$ and $p^{+}_{A}$ increase in $S$ and $K_{E}$. \(\blacksquare\)
Proof of Proposition 4

By Proposition 3, the monopolist reduces the price just until it reaches the crowd-out region. Hence, any further decrease in the price can only increase the number of B-investors. From Lemma 3, we know that \(K^+_B\) increases in \(S_1, S_2, \) or \(K_E\) and hence that \(p_A\) is (weakly) smaller when \(S_1, S_2, \) or \(K_E\) are smaller. This in turn implies that \(n_B\) is higher when \(S_1, S_2, \) or \(K_E\) are smaller. 

Proof of Proposition 5

It suffices to show that there are aggregate gains from trade that the two investors can achieve by entering into the contract. Without the contract, their joint expected trading profit is given by the sum

\[
\rho_B(1, n_A) + \rho_A(n_A, 1) = \frac{\sigma_y \sigma^2 (\sigma^2 + 2\sigma_t^2)}{(2\sigma^2 + 2\sigma_t^2)^2} [T(1) + T(n_A)]^{-1/2} + \frac{\sigma_y \sigma^2 (\sigma^2 + 2\sigma_t^2)}{(\sigma^2 (n_A + 1) + 2\sigma_t^2)^2} [T(1) + T(n_A)]^{-1/2} \]

\[= \frac{\sigma_y \sigma^2 (\sigma^2 + 2\sigma_t^2)}{(2\sigma^2 + 2\sigma_t^2)^2} \frac{[T(1) + T(n_A)]^{-1/2}}{(n_A + 1) \sigma^2 + 2\sigma_t^2} \] 

\[
[1 + \frac{T(n_A)}{T(1)}]^{1/2} > 1 + \frac{4(\sigma^2 + \sigma_t^2)^2}{(2\sigma^2 + 2\sigma_t^2 + \sigma^2 n_A)^2} \]

\[1 + \frac{T(n_A)}{T(1)} > 1 + \frac{8(\sigma^2 + \sigma_t^2)^2}{(\sigma^2 + 2\sigma_t^2 + \sigma^2 n_A)^2} + \frac{16(\sigma^2 + \sigma_t^2)^4}{(\sigma^2 + 2\sigma_t^2 + \sigma^2 n_A)^4} \quad (22)\]

Finally, substituting for \(T(\cdot)\) and rearranging yields

\[4n_A - 8 > \frac{16(\sigma^2 + \sigma_t^2)^2}{(\sigma^2 + 2\sigma_t^2 + \sigma^2 n_A)^2} \quad (23)\]
The left-hand side is increasing in \( n_A \), whereas the right-hand side is decreasing in \( n_A \). Hence, this inequality is satisfied for sufficiently high \( n_A \). ■

**Appendix B: Commonalities**

**Price comovement**

As mentioned in the text, price comovement can be measured by the average (absolute) correlation coefficient between individual asset prices and the market index:

\[
\hat{\rho}_M = |\mathcal{M}|^{-1} \sum_{a \in \mathcal{M}} |\rho_{a,M}|
\]  

(24)

where

\[
\rho_{a,M} \equiv \frac{\text{Cov}(p_a, p_M)}{\sqrt{\text{Var}(p_a) \text{Var}(p_M)}}
\]  

and

\[
p_M = \sum_{a \in \mathcal{M}} p_a.
\]  

(25)

The average correlation coefficient is typically a good approximation of the \( R^2 \) in a regression.

Each \( A \)-investor trades in both assets. Let \( \alpha_{a,A} \) denote \( A \)-investors’ trading intensity when trading in asset \( a \). By definition of the market makers’ pricing functions,

\[
p_1 = \lambda_1 \left( \sum_{i=1}^{n_A} x_{1A}(\tilde{s}_{iA}) + \sum_{l=1}^{n_B} x_B(\tilde{s}_{lB}) + \tilde{y} \right) \quad \text{and} \quad p_2 = \lambda_2 \left( \sum_{i=1}^{n_A} x_{2A}(\tilde{s}_{iA}) + \sum_{l=1}^{n_C} x_C(\tilde{s}_{lC}) + \tilde{y} \right)
\]  

(26)

which implies the following market index

\[
p_M = \lambda_1 \left( \sum_{i=1}^{n_A} x_{1A}(\tilde{s}_{iA}) + \sum_{l=1}^{n_B} x_B(\tilde{s}_{lB}) + \tilde{y} \right) + \lambda_2 \left( \sum_{i=1}^{n_A} x_{2A}(\tilde{s}_{iA}) + \sum_{l=1}^{n_C} x_C(\tilde{s}_{lC}) + \tilde{y} \right).
\]  

(27)

Price variances are given by

\[
\text{Var}(p_1) = (\lambda_1)^2 \text{Var} \left( \sum_{i=1}^{n_A} x_{1A}(\tilde{s}_{iA}) + \sum_{l=1}^{n_B} x_B(\tilde{s}_{lB}) + \tilde{y} \right) = (\lambda_1)^2 \text{Var} \left( \sum_{i=1}^{n_A} \alpha_{1A} \tilde{s}_{iA} + \sum_{l=1}^{n_B} \alpha_B \tilde{s}_{lB} + \tilde{y} \right)
\]

\[
= (\lambda_1)^2 \left[ \text{Var} \left( \sum_{i=1}^{n_A} \alpha_{1A} \left( \bar{V}_A + \tilde{e}_i \right) \right) + \text{Var} \left( \alpha_B \sum_{l=1}^{n_B} \left( \bar{V}_B + \tilde{e}_l \right) \right) + \sigma_y^2 \right]
\]

\[
= (\lambda_1)^2 \left[ (n_A \alpha_{1A})^2 \sigma^2 + (\alpha_{1A})^2 n_A \sigma_x^2 + (n_B \alpha_B)^2 \sigma^2 + (\alpha_B)^2 n_B \sigma_x^2 + \sigma_y^2 \right]
\]  

(28)

and, analogously,

\[
\text{Var}(p_2) = (\lambda_2)^2 \left[ (n_A \alpha_{2A})^2 \sigma^2 + (\alpha_{2A})^2 n_A \sigma_x^2 + (n_C \alpha_C)^2 \sigma^2 + (\alpha_C)^2 n_C \sigma_x^2 + \sigma_y^2 \right].
\]  

(29)
The variance of the market index is

\[
\text{Var}(p_M) = \text{Var}\left( \sum_{i=1}^{n_A} x_{1A}(\bar{s}_{1A}) + \sum_{i=1}^{n_A} x_{2A}(\bar{s}_{2A}) + (\lambda_1 + \lambda_2) \bar{y} + \lambda_1 \sum_{l=1}^{n_B} x_B(\bar{s}_{1B}) + \lambda_2 \sum_{l=1}^{n_C} x_C(\bar{s}_{1C}) \right) \\
= \text{Var}\left( \sum_{i=1}^{n_A} x_{1A}(\bar{s}_{1A}) + \sum_{i=1}^{n_A} x_{2A}(\bar{s}_{2A}) \right) + \text{Var}\left( (\lambda_1 + \lambda_2) \bar{y} \right) \\
+ (\lambda_1)^2 \left[ (\lambda_1 n_B \alpha_B)^2 \sigma^2 + (\alpha_B)^2 n_B \sigma_y^2 \right] + (\lambda_2)^2 \left[ (\lambda_1 \alpha_C)^2 \sigma^2 + (\alpha_C)^2 n_C \sigma_y^2 \right] \\
= (\lambda_1 \alpha_{1A} + \lambda_2 \alpha_{2A})^2 \left[ (n_A)^2 \sigma^2 + (\alpha_B)^2 n_B \sigma_y^2 \right] + (\lambda_1 + \lambda_2)^2 \sigma_y^2 \\
+ (\lambda_1)^2 \left[ (\lambda_1 \alpha_{1A})^2 \sigma^2 + (\alpha_B)^2 n_B \sigma_y^2 \right] + (\lambda_2)^2 \left[ (\lambda_1 \alpha_{1A})^2 \sigma^2 + (\alpha_C)^2 n_C \sigma_y^2 \right]
\]

(30)

The covariances between individual asset prices and the market index are given by

\[
\text{Cov}(p_{1,M}) = \text{Cov}\left( \sum_{i=1}^{n_A} x_{1A}(\bar{s}_{1A}) + \sum_{i=1}^{n_A} x_{2A}(\bar{s}_{2A}) + (\lambda_1 + \lambda_2) \bar{y} + \lambda_1 \sum_{l=1}^{n_B} x_B(\bar{s}_{1B}) + \lambda_2 \sum_{l=1}^{n_C} x_C(\bar{s}_{1C}) \right) \\
= \text{Cov}\left( \sum_{i=1}^{n_A} x_{1A}(\bar{s}_{1A}) + \sum_{i=1}^{n_A} x_{2A}(\bar{s}_{2A}) \right) + \lambda_1 \sum_{i=1}^{n_B} \alpha_B \bar{s}_{1B} + \lambda_1 \bar{y} + (\lambda_1 \alpha_{1A} + \lambda_2 \alpha_{2A}) \sum_{i=1}^{n_B} \bar{s}_{1A} + \lambda_1 \sum_{l=1}^{n_B} \alpha_B \bar{s}_{1B} + (\lambda_1 + \lambda_2) \bar{y} \\
= \text{Cov}\left( \sum_{i=1}^{n_A} x_{1A}(\bar{s}_{1A}) + \sum_{i=1}^{n_A} x_{2A}(\bar{s}_{2A}) \right) + \lambda_1 \sum_{i=1}^{n_B} \alpha_B \bar{s}_{1B} + \lambda_1 \bar{y} + \lambda_1 \sum_{l=1}^{n_B} \alpha_B \bar{s}_{1B} \\
+ \lambda_2 \sum_{i=1}^{n_A} \alpha_A \bar{s}_{1A} + \lambda_1 \sum_{i=1}^{n_A} \alpha_A \bar{s}_{1A} + \lambda_2 \sum_{l=1}^{n_B} \alpha_B \bar{s}_{1B} + (\lambda_1 + \lambda_2) \bar{y} \\
= (\lambda_1 \alpha_{1A}) (\lambda_1 \alpha_{1A} + \lambda_2 \alpha_{2A}) \left[ (n_A)^2 \sigma^2 + n_A \sigma_y^2 \right] + (\lambda_2 \alpha_{1A})^2 \left[ (n_B)^2 \sigma^2 + n_B \sigma_y^2 \right] + (\lambda_1 + \lambda_2) \sigma_y^2
\]

(31)

and, similarly,

\[
\text{Cov}(p_{2,M}) = (\lambda_2 \alpha_{2A}) (\lambda_1 \alpha_{1A} + \lambda_2 \alpha_{2A}) \left[ (n_A)^2 \sigma^2 + n_A \sigma_y^2 \right] + (\lambda_2 \alpha_{1A})^2 \left[ (n_C)^2 \sigma^2 + n_C \sigma_y^2 \right] + (\lambda_1 + \lambda_2) \sigma_y^2.
\]

(32)

The correlation coefficients are thus

\[
\rho_{1,M} = \frac{1}{\sqrt{(\lambda_1)^2 \left[ (n_A \alpha_{1A})^2 \sigma^2 + (\alpha_A)^2 n_A \sigma_y^2 + (\alpha_B)^2 n_B \sigma_y^2 + (\alpha_C)^2 n_C \sigma_y^2 \right]}} \\
\sqrt{(\lambda_1 \alpha_{1A}) (\lambda_1 \alpha_{1A} + \lambda_2 \alpha_{2A}) \left[ (n_A)^2 \sigma^2 + n_A \sigma_y^2 \right] + (\lambda_1 \alpha_B)^2 \left[ (n_B)^2 \sigma^2 + n_B \sigma_y^2 \right] + (\lambda_1 \alpha_C)^2 \left[ (n_C)^2 \sigma^2 + n_C \sigma_y^2 \right]}
\]
Given the assumption that Liquidity commonality

Intuitively, price movements in this case result exclusively from the (market-wide) trades of a decrease in strategies would (weakly) increase, as would then the number of active investors in those strategies. Similarly, when through which the strategies affect each other) must increase in equilibrium; otherwise, the profitability of the asset-specific strategies would (weakly) increase, as would then the number of active investors in those strategies. Similarly, when a decrease in \( c_A \) crowds out the asset-specific strategies. Crowding out immediately implies that \( \lambda^* \) (which is the only channel through which the strategies affect each other) must increase in equilibrium; otherwise, the profitability of the asset-specific strategies would (weakly) increase, as would then the number of active investors in those strategies. Similarly, when \( c_A \geq \tau_A \), a decrease in \( c_A \) must increase equilibrium liquidity; otherwise, the number of investors in the asset-specific strategies would not increase. 

**Liquidity commonality**

Given the assumption that \( c_A \leq c_B = c_C \) and that the two asset markets are otherwise isomorphic, investment strategy \( A \) either crowds out both or none of the other strategies. When \( c_A \geq \tau_A \), either market is in the substitute region. Here, a decrease in \( c_A \) crowds out the asset-specific strategies. Crowding out immediately implies that \( \lambda^* \) (which is the only channel through which the strategies affect each other) must increase in equilibrium; otherwise, the profitability of the asset-specific strategies would (weakly) increase, as would then the number of active investors in those strategies. Similarly, when \( c_A \geq \tau_A \), a decrease in \( c_A \) must increase equilibrium liquidity; otherwise, the number of investors in the asset-specific strategies would not increase.
Appendix C: Information role of prices

Market efficiency

We want to show that market efficiency increases when $c_A$ decreases. It is straightforward to see that this is the case in the crowd-out region and the complement region, as in either case the size of both investor classes (weakly) increases when $c_A$ decreases. It remains to show that the previous statement is also true in the substitute region. Specifically, let us show that, for $c_A \leq c_B$ and $n_A < n_A^*$, total trading profits are lower under $(\overline{n}_A, 0)$ than under $(n_A, n_B)$. First, note that $\pi_A(n_A + n_B, 0) < \pi_A(n_A, n_B) + \pi_B(n_B, n_A)$ where the inequality follows from Lemma 4 (c), and the equality follows provided that $(n_A, n_B)$ denotes an equilibrium outcome. These relations imply that

\[(n_A + n_B) \pi_A(n_A + n_B, 0) < n_A \pi_A(n_A, n_B) + n_B \pi_B(n_B, n_A).\]  

(34)

This inequality can be rearranged to

\[
(n_A + n_B) [\rho_A(n_A + n_B, 0) - c_A] < n_A [\rho_A(n_A, n_B) - c_A] + n_B [\rho_B(n_B, n_A) - c_B]
\]

\[
(n_A + n_B) \rho_A(n_A + n_B, 0) - (n_A + n_B) c_A < n_A \rho_A(n_A, n_B) + n_B \rho_B(n_B, n_A) - n_A c_A - n_B c_B
\]

\[
(n_A + n_B) \rho_A(n_A + n_B, 0) < n_A \rho_A(n_A, n_B) + n_B \rho_B(n_B, n_A) - n_B (c_B - c_A)
\]

(35)

which—due to $c_B > c_A$—implies

\[
(n_A + n_B) \rho_A(n_A + n_B, 0) < n_A \rho_A(n_A, n_B) + n_B \rho_B(n_B, n_A)
\]

(36)

where the left-hand side and the right-hand side represent total trading profits for $(n_A + n_B, 0)$ and $(n_A, n_B)$ respectively.

Finally, it is well-known that $n \rho_A(n, 0) < (n') \rho_A(n', 0)$ if $n' > n$ (e.g., Admati and Pfeiderer, 1988a). Since $n_A > n_A + n_B$, it follows that

\[
\overline{n}_A \rho_A(\overline{n}_A, 0) < (n_A + n_B) \rho_A(n_A + n_B, 0) < n_A \rho_A(n_A, n_B) + n_B \rho_B(n_B, n_A)
\]

(37)

which proves that trading profits are smaller for $(\overline{n}_A, 0)$ than for $(n_A, n_B)$. The inequality $\overline{n}_A > n_A + n_B$ follow from the fact that the total number of active investors, $n$, is strictly decreasing in $c_A$, which follows from the last parts of the proof of Proposition 2.

Price informativeness

First, note that $\text{Var}(\tilde{V} | p) = \text{Var}(\tilde{V} | z)$ as the price is based exclusively on order flow information. This conditional variance is given by $\text{Var}(\tilde{V} | z) = \text{Var}(\tilde{V})(1 - \rho_z^2)$ where
A measure of price informativeness. Suppose that participation constraint of a single investor is binding, i.e.

\[ \rho_{V,z}^2 = \frac{\text{Cov}(\hat{V}, \hat{z})^2}{\text{Var}(V)\text{Var}(\hat{z})} = \frac{\text{Cov}(\hat{V}, \hat{z})}{\text{Var}(V)} = \frac{(n_A \alpha_A + n_B \alpha_B) \sigma^2}{2\sigma^2} \lambda \]

\[ = \frac{1}{2} \left( \frac{n_A}{\sigma^2(n_A + 1) + 2\sigma_t^2} + \frac{n_B}{\sigma^2(n_B + 1) + 2\sigma_t^2} \right) \lambda \]

\[ = \frac{1}{2} \sum_{\theta = A,B} \frac{n_\theta \sigma^2}{\sigma^2(n_\theta + 1) + 2\sigma_t^2} \]  

(38)

Note that

\[ \frac{\partial \rho_{V,z}^2}{\partial \theta} = \frac{\sigma^2 (\sigma^2 + 2\sigma_t^2)}{[\sigma^2(n_\theta + 1) + 2\sigma_t^2]^2} > 0 \quad \text{and} \quad \frac{\partial \rho_{V,z}^2}{\partial \theta} = -\frac{2\sigma^4 (\sigma^2 + 2\sigma_t^2)}{[\sigma^2(n_\theta + 1) + 2\sigma_t^2]^3} < 0. \]  

(39)

We now show that price informativeness can decrease when B-information is crowded out. Define \( I(n_A, n_B) = 2\rho_{V,z}^2 \) as a measure of price informativeness. Suppose that \( n_A^0(c_A) < n_A \), and denote the equilibrium information structure by \((n_A, n_B)\).

Price informativeness is then given by

\[ I(n_A, n_B) = \sum_{\theta = A,B} \frac{n_\theta \sigma^2}{\sigma^2(n_\theta + 1) + 2\sigma_t^2}. \]  

(40)

Comparing this to the information structure \((n_A, 0)\) with price informativeness

\[ I(n_A, 0) = \frac{n_A \sigma^2}{\sigma^2(n_A + 1) + 2\sigma_t^2}. \]  

(41)

it remains to show that \( I(n_A, 0) < I(n_A, n_B) \) is feasible.

To this end, consider a numeric example where, for a given \( c_A, n_B = 1 \). This is true in equilibrium if the participation constraint of a single B-investor is binding, i.e. \( \rho_B(1, n_A) = c_B \) or

\[ \frac{\sigma^2 \sigma_y (\sigma^2 + 2\sigma_t^2)}{(2\sigma^2 + 2\sigma_t^2)^2} \left[ \frac{1}{4 (\sigma^2 + \sigma_t^2)} + \frac{n_A (\sigma^2 + \sigma_t^2)}{[(n_A + 1)\sigma^2 + 2\sigma_t^2]^2} \right]^{-1/2} = c_B. \]  

(42)

When the participation constraint of an A-trader is also binding,

\[ \frac{\sigma^2 \sigma_y (\sigma^2 + 2\sigma_t^2)}{((n_A + 1)\sigma^2 + 2\sigma_t^2)^2} \left[ \frac{n_A (\sigma^2 + \sigma_t^2)}{[(n_A + 1)\sigma^2 + 2\sigma_t^2]^2} + \frac{1}{4 (\sigma^2 + \sigma_t^2)} \right]^{-1/2} = c_A. \]  

(43)

the structure \((n_A, n_B)\) is an equilibrium.

Suppose that these conditions hold for \( c_A \), and consider a small change to the cost of A-information to \( c'_A < c_A \). This will increase \( n_A \), which in turn will crowd out the B-investor. The new equilibrium number of A-investors is determined by

\[ \frac{\sigma^2 \sigma_y (\sigma^2 + 2\sigma_t^2)}{((n_A + 1)\sigma^2 + 2\sigma_t^2)^2} \left[ \frac{n_A (\sigma^2 + \sigma_t^2)}{[(n_A + 1)\sigma^2 + 2\sigma_t^2]^2} \right]^{-1/2} = c'_A. \]  

(44)
Let us now compute the equilibrium information structure(s) for \( \sigma^2 = 1, \sigma^2 = 10, \sigma^2 = 10, c_A = 1/2 \) and \( c_A' = 1/(2.1) \). (Here, the choice of \( c_B \) gives us a degree of freedom to ensure that \( n_B = 1 \).) Using condition 43, we can determine the equilibrium number of \( A \)-investors for \( c_A \),

\[
\frac{210}{(n_A + 21)^2} \left( \frac{11n_A}{(n_A + 21)^2} + \frac{1}{44} \right)^{-1/2} = \frac{1}{2},
\]

which yields \( n_A = 12.236 \). Similarly, we can compute the number of \( A \)-investors that enter once the \( B \)-investor has been crowded out via condition 44,

\[
\frac{210}{21\sqrt{11}} = (n_A + 21) \sqrt{n_A},
\]

as \( n_A = 14.238 \). The \( B \)-investor will indeed drop out because the maximum of \( T(n_A) \) is at \( n = 21 \) in this case. That is, the increase in \( n_A \) occurs in a range where a decrease in the cost of \( A \)-information crowds out \( B \)-investors.

Price informativeness in this example drops from

\[
I(n_A, 1) = \frac{12.236}{(12.236 + 1) + 20} + \frac{1}{(1 + 1) + 20} = 0.41361
\]

(47)

to

\[
I(2A, 0) = \frac{14.238}{(14.238 + 1) + 20} = 0.40405.
\]

(48)

That is, the price becomes less informative.
References


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