Asymmetric information in financial markets *

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Abstract

These notes summarize some key contributions in the literature of asymmetric information in financial markets. It overviews competitive rational expectation models, as well as strategic market order and sequential trade models. It further provides a few applications of the theory discussed. The following set of notes are meant as a complement to the readings cited.

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1 Preliminaries

This section gives a brief overview of the course and then a simple refresher of some statistical as well as some economics concepts for what is to come in future lectures.

1.1 Overview

This course is an introduction to the literature on asymmetric information in financial markets. The focus is exclusively on “market microstructure” models - corporate finance is the other area where asymmetric information has important effects, and we shall not touch upon it.\(^1\)

The emphasis of these notes is in the actual structure of the models, and in particular their derivations. I strongly encourage you to read through the original papers to get the motivation for the models we shall study, hoping the details become simple once you have read through these notes. The homework problems are an integral part of the course. They follow the sequencing in the notes, so start on them as you read through!

A final disclaimer: I will use different notation than the original papers throughout my notes (and try to be consistent!). This should not exempt you from being able to read through others’ notation, i.e. get results from the papers themselves, rather than just from my notes (doing this would for example make some of the homework problems much easier!).

1.2 Bayesian updating

Consider a set of events \(E_1, \ldots, E_n\) and \(D\). Bayes rule states that

\[
\mathbb{P}(E_i | D) = \frac{\mathbb{P}(D | E_i) \mathbb{P}(E_i)}{\mathbb{P}(D)}
\]

If the events \(E_1, \ldots, E_n\) are a partition of the state space, then we can further write

\[
\mathbb{P}(E_i | D) = \frac{\mathbb{P}(D | E_i) \mathbb{P}(E_i)}{\sum_{i=1}^{n} \mathbb{P}(D | E_i) \mathbb{P}(E_i)}
\]

We consider next applying this result to the case where the events are realizations of normally distributed random variables, which will be our best friends in this course. In particular, consider a normally distributed random variable \(X \in \mathbb{R}^n\) with mean \(\mu_x\) and variance covariance matrix \(\Sigma_{xx} \in \mathbb{R}^{n \times n}\). Also, consider a second normally distributed random variable \(Y \in \mathbb{R}^m\) with mean \(\mu_y\) and variance covariance matrix \(\Sigma_{yy} \in \mathbb{R}^{m \times m}\). Moreover, assume that the covariance of \(X\) and \(Y\) is given by the \(n \times m\) matrix \(\Sigma_{xy}\): note that this matrix tells you in the \(i, j\) position what is the covariance between the random variables \(X_i\) and \(Y_j\). Also let \(\Sigma_{yx}\) denote the transpose of \(\Sigma_{xy}\). When we deal with a univariate random variable \(X\) we will use the notation \(\sigma_x^2\) to denote the variance of \(X\).

\(^1\)Nonetheless it should be noted that some “corporate” topics, such as disclosure, have been extensively analyzed using the class of models we will discuss.
With this notation we have that
\[ E[X|Y = y] = \mu_x + \sum_{xy} \Sigma^{-1}_{yy} (y - \mu_y); \]
and
\[ \text{var}(X|Y = y) = \Sigma_{xx} - \sum_{xy} \sum^{-1}_{yy} \Sigma_{yx}. \]

These formulas are something you should become very familiar with. For now let me just remark two things: (1) the expected value of \( X \) given \( Y = y \) is a linear function of the realization of \( Y \); (2) the conditional variance of \( X \) given \( Y \) is independent of the realization of \( Y \).

The above may look somewhat intimidating, so let us consider a couple of simple examples. First, let us consider the case where \( X \) and \( Y \) are not vectors, but simply real numbers. Then the above equations become
\[ E[X|Y = y] = \mu_x + \sum_{xy} \Sigma^{-1}_{yy} (y - \mu_y) = \mu_x + \cov(X,Y) \frac{y - \mu_y}{\text{var}(Y)} \equiv \mu_x + \beta_{xy} (y - \mu_y) \]
and
\[ \text{var}(X|Y = y) = \text{var}(X) - \frac{\cov(X,Y)^2}{\text{var}(Y)} = \text{var}(X) (1 - \rho^2) \]
where \( \beta_{xy} \) is the regression coefficient of \( X \) on \( Y \), and \( \rho \) denotes the correlation between \( X \) and \( Y \). In words, one updates on \( X \) using a realization on \( Y \) more strongly the higher the regression coefficient \( \beta_{xy} \) is, and how much you learn about \( X \) from \( Y \) is simply determined by the correlation between these two random variables.\(^2\)

For example, consider the case where \( Y = X + \epsilon_i \), where \( \epsilon_i \sim N(0, \sigma^2_\epsilon) \), i.e. where the signal \( Y \) is equal to the actual realization of \( X \) plus some noise. Also, let \( X \sim N(\mu_x, \sigma^2_x) \). The above formulas imply that
\[ E[X|Y_i = y_i] = \mu_x + \frac{\sigma^2_x}{\sigma^2_x + \sigma^2_\epsilon} (y_i - \mu_x) \]
and
\[ \text{var}(X|Y_i = y_i) = \sigma^2_x \left( \frac{\sigma^2_\epsilon}{\sigma^2_\epsilon + \sigma^2_x} \right). \]

One can think of the above equations for the conditional expectation of \( X \) as a weighted average of the unconditional (ex-ante) mean \( \mu_x \) and the actual signal realization \( y_i \).

### 1.3 Asset pricing with CARA preferences

We will be concerned with asset pricing with asymmetric information. It turns out the most tractable class of models is one in which agents have CARA preferences and the assets have normally distributed payoffs. The main reason is that under these assumptions one can (1) easily compute updated beliefs using the results from the previous section; (2) easily compute...
asset demands due to the properties of CARA preferences.

Recall that in asset pricing theory we start with some preferences $u(W)$ defined over final wealth (in one-period models), where agents maximize $\mathbb{E}[u(W)]$ given their beliefs. CARA preferences stand for constant-absolute-risk-aversion, and are the case where $u(W) = -e^{-\tau W}$, where $\tau$ denotes the coefficient of risk-aversion of an agent.

Let’s consider the most typical investment model with CARA preferences. In particular, assume that there are two assets available for investment. One asset is risk-less and has a payoff $R_f$. The second asset is risky, and has payoff $X \sim \mathcal{N}(\mu, \sigma^2)$. Assume an agent has an endowment $e_0$ of the bond (riskless asset), and $z$ units of the stock. This agent is considering investing in these assets by buying $\theta$ units of the risky asset, and $\gamma$ units of the bond. Let the price of the stock be denoted by $P_x$, and normalize the price of the bond to 1 (this is without loss of generality in this one-period model). The final wealth this agent will have is

$$W = \gamma R_f + \theta X = (e_0 + P_x(z - \theta))R_f + \theta X$$

where we have used the budget equation $P_x \theta + \gamma = P_x z + e_0$. Note that we can write $W = W_0 + \theta(X - P_x R_f)$, where $W_0$ is the “initial wealth” of an agent.

The problem we are trying to solve is

$$\max_{\theta} \mathbb{E}[u(W_0 + \theta(X - P_x R_f))] = \mathbb{E}[-\exp(-\tau(W_0 + \theta(X - P_x R_f)))]$$

The following fact is useful: if $Y \sim \mathcal{N}(\mu, \sigma^2)$, then $\mathbb{E}[e^Y] = \exp(\mu + \frac{1}{2} \sigma^2)$. Therefore we have the problem of a CARA agent in this setup reduces to

$$\max_{\theta} \mathbb{E}[u(W_0 + \theta(X - P_x R_f))] = -e^{-\tau(W_0 - \theta P_x R_f)} e^{-\tau \mu_x + \frac{1}{2} \tau^2 \sigma^2 x}$$

It is easy to see that the optimal demand for the risky asset for this agent is

$$\theta = \frac{\mu_x - P_x R_f}{\tau \sigma^2 x}.$$  

Note how simple this demand equation is. Also, note how the demand for the risky asset is independent of the endowment (initial wealth) of the agent.

The above is the analysis of one single agent optimization problem. If there are $I$ agents, each with risk aversion $\tau_i$, for $i = 1, \ldots, I$, then each agent would have a similar demand for the risky stock, and aggregate demand can be written out as

$$\sum_{i=1}^{I} \frac{\mu_x - P_x R_f}{\tau_i \sigma^2 x} = \eta I \left( \frac{\mu_x - P_x R_f}{\sigma^2 x} \right)$$

where $\eta = (1/I) \sum_{i=1}^{I} 1/\tau_i$ is the harmonic mean of the agents risk-aversion parameters, which loosely can be thought of as aggregate risk-tolerance.

If the total supply of the risky asset is $ZI$, so that $Z$ denotes the per-capita supply, then
we have, by equating demand and supply (market clearing), that the equilibrium price is given by

\[ P_x = \left( \mu_x - \frac{Z \sigma^2 x}{\eta} \right) / R_f \]

Note: (1) equilibrium price is expressed above as a simple function of the model’s primitives; (2) the price of the stock can be viewed as expected value minus a risk-premium term; (3) the risk-premium term increases in the risk of the asset \( \sigma^2_x \) and its per-capita supply (more supply means more risk to share), and it is decreasing in the number of agents and the aggregate risk-tolerance.

For your records I include one last mathematical result that we may use at some point (it includes the preceding calculations as special cases). If we have a random variable \( X \sim N(0, \Sigma) \), with \( \Sigma \in \mathbb{R}^{n \times n} \) positive definite, then

\[ \mathbb{E} \left[ \exp \left( X^\top A X + b^\top X \right) \right] = |I - 2\Sigma A|^{-1/2} \exp \left( \frac{1}{2} b^\top (I - 2\Sigma A)^{-1}\Sigma b \right) ; \quad (1) \]

where \( A \) is a \( n \times n \) matrix and \( b \) is \( n \)-dimensional vector.

## 2 Competitive rational expectations models

The literature on asymmetric information in finance started with competitive models. We start by reviewing one of the classic papers on asymmetric information in financial markets, that of Grossman and Stiglitz (1980).

### 2.1 Prices as signals

This paper was one of the first ones to study the issue of information acquisition and information revelation by prices. It basically extends the symmetric information equilibrium introduced in the first set of notes to allow for heterogeneous information. The main question addressed by rational expectations models is what happens when people with different information decide to trade. How market prices are affected by agents’ information affects how the traders can infer information from market prices. The fundamental insight is that prices serve two purposes: they clear markets and they aggregate information. This dual role makes the behavior of prices and markets much more complex than that assumed in the standard asset pricing paradigm.

Grossman and Stiglitz (1980) work in a large economy, where a fraction \( \lambda \) of agents have private information, whereas a fraction \( 1 - \lambda \) is uninformed. The per-capita supply of the risky asset is \( Z \), which is assumed to be random, namely \( Z \sim N(0, \sigma^2_z) \). There is a risky asset with payoff \( X \sim N(0, \sigma^2_x) \). There is also a risk-free asset with price normalized to one and gross return \( R_f = 1 \). The random variables \( Z \) and \( X \) are uncorrelated.

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3Note the case where \( X \) has a non-zero mean can be obtained by decomposing \( X = \mu_x + \hat{X} \), and applying the theorem to \( \hat{X} \) after simplifying the quadratic term.
The informed agents get signals of the form $Y = X + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. Note how all agents get the same signal about the risky asset. The uninformed agents get to observe price, but otherwise do not have any information about the risky asset payoff. Agents can also invest in a risk-free security, whose return we normalize to 1. All agents are assumed to have CARA preferences with risk aversion parameter $\tau$. Since initial wealth is irrelevant, we assume that all agents have zero initial wealth.

In order to solve for equilibrium asset prices in this economy, we follow what is typically known as the 5-step approach:

1. First conjecture a functional form for the price function. In our case we will guess that price will be of the form
   \[ P_x = a + bY - dZ; \] (2)
   for some real numbers $a, b, d$.

2. Derive beliefs for the agents as a function of the price coefficients. Here we will use our Bayesian updating formulas from section 1.2.

3. Derive the optimal demands for the agents, given their (endogeneously determined) beliefs.

4. Impose market clearing in the stock, and solve for the market clearing price.

5. Impose rational expectations, i.e. make sure the conjecture pricing formula coincides with the actual pricing formula. Typically this can be done by “matching coefficients.”

We should note at this point that this 5-steps are equivalent to the usual notion of equilibrium in asset pricing: (a) agents’ trading strategies are optimal (given prices and their information sets); (b) markets clear (demand equals supply). The reason we have to be extra careful in our equilibrium definition is because prices now play the double role of (i) market clearing (as usual); (ii) price revelation (since traders have information).

Given the price function (2), let’s derive the beliefs of the agents. First note that the conjectured functional form makes price a noisy version of the signal that informed agents get, i.e. by observing price these agents do not update their beliefs about $X$. The uninformed agents, on the other hand, do learn something from observing price.

For the informed agents, using our results from the previous lecture we have that
\[
\mathbb{E}[X | Y, P_x] = \mathbb{E}[X | Y] = \mu_x + \frac{\text{cov}(X, Y)}{\text{var}(Y)} (Y - \mathbb{E}[Y])
\]
\[
= \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\epsilon^2} Y
\]
\[
\text{var}(X | Y_i, P_x) = \sigma_x^2 - \frac{\sigma_x^4}{\sigma_x^2 + \sigma_\epsilon^2} = \sigma_x^2 \left( \frac{\sigma_\epsilon^2}{\sigma_x^2 + \sigma_\epsilon^2} \right)
\]
For the uninformed, again using our results from the previous lecture we have that
\[ E[X|P_x] = \mu_x + \frac{\text{cov}(X, P_x)}{\text{var}(P_x)} (P_x - E[P_x]) \]
\[ = \frac{b\sigma_x^2}{\overline{b^2(\sigma_x^2 + \sigma_z^2) + d^2\sigma_z^2}} (P_x - a) \]
where we have used the fact that \( X \) and \( U \) have zero-means. Moreover,
\[ \text{var}(X|P_x) = \sigma_x^2 - \frac{b^2\sigma_x^4}{\overline{b^2(\sigma_x^2 + \sigma_z^2) + d^2\sigma_z^2}} \]
\[ = \sigma_x^2 \left( \frac{b^2\sigma_x^2 + d^2\sigma_z^2}{b^2(\sigma_x^2 + \sigma_z^2) + d^2\sigma_z^2} \right) \]

Using our previous results on optimal demands for CARA agents, we have that the informed will trade
\[ \theta_I = \frac{E[X|Y] - P_x}{\tau \text{var}(X|Y)}; \]
whereas the uninformed will trade
\[ \theta_U = \frac{E[X|P_x] - P_x}{\tau \text{var}(X|P_x)}; \]

At this point let us note that the demand from the informed agents is linear in \( Y \) and \( P_x \), and that of the uninformed is linear in \( P_x \).

Market clearing requires that
\[ \lambda \theta_I + (1 - \lambda) \theta_U = Z \]
which can be written as
\[ \lambda \frac{E[X|Y]}{\tau \text{var}(X|Y)} - Z = -(1 - \lambda) \frac{E[X|P_x]}{\tau \text{var}(X|P_x)} + P_x \left( \frac{\lambda}{\tau \text{var}(X|Y)} + \frac{1 - \lambda}{\tau \text{var}(X|P_x)} \right) \]

The above equilibrium condition must all hold for all realizations of \( Y, X, Z \), so we can impose rational expectations by “matching coefficients”: the RHS can be written as
\[ K(a + bY - dZ) \]
for some constant \( K \), whereas the LHS is simply
\[ \frac{\lambda}{\tau \sigma_z^2} Y - Z + f(a) \]
for some function \( f(a) \). Matching coefficients we have that \( a = 0 \), \( b/d = \frac{\lambda}{\tau \sigma_z^2} \) and \( dK = 1 \), where some tedious calculations allow us to solve for \( d \) in closed-form (I let you do this in the homework).
This completes the solution to the equilibrium asset prices under asymmetric information. Probably the most important aspect of the equilibrium is the relative price coefficients $b/d$, which measure how much information is revealed by prices, note that we can write:

$$\text{var}(X|P_x)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_e^2 + (d/b)^2 \sigma_z^2}$$

Since $b/d = \lambda/(\tau \sigma_e^2)$, we immediately see that: (1) the more informed agents the more informative the stock price is; (2) lower quality of information (higher $\sigma_e^2$) also implies less information revealed by prices; (3) higher risk-aversion implies less information revealed by prices (since agents trade less aggressively); and (4) the lower the noise in per capita supply $\sigma_z^2$ the more informative prices are.

We shall close the model by studying the decision to become informed, i.e. endogenizing the fraction of informed traders $\lambda$. The following result is useful. Given an (arbitrary) signal $Y_i$, the ex-ante (prior to observing the signal) expected utility for an agent is\(^4\)

$$\mathbb{E}[u(W_i)] = \mathbb{E}[\exp(-\tau \theta_i(X - P_x))] = \sqrt{\frac{\text{var}(X|Y_i, P_x)}{\text{var}(X - P_x)}}.$$ 

Basically there are two terms: one is signal dependent, $\text{var}(X|Y_i, P_x)$ and measures how good the signal is (i.e. how much it reduces the uncertainty about $X$); the second is signal independent, $\text{var}(X - P_x)$, and measures the “quality” of prices, i.e. how well they approximate the underlying asset value.

Since $Y_i$ at this point was arbitrary, this of course covers the case of uninformed agents as a special case. Their ex-ante expected utility can be written as

$$\mathbb{E}[u(W_i)] = \mathbb{E}[\exp(-\tau \theta(X - P_x))] = \sqrt{\frac{\text{var}(X|P_x)}{\text{var}(X - P_x)}}.$$ 

If the cost of acquiring a signal is $c$, one can equate the expected utilities for informed and uninformed, net of this cost, to find the endogenously determined number of informed agents. It is given by the solution to

$$e^{\tau c} = \sqrt{\frac{\text{var}(X|Y)^{-1}}{\text{var}(X|P_x)^{-1}}}.$$ 

The REE provides a few key insights on which the following literature has built upon.

1. Prices play two roles: they clear markets and they convey information.

2. Partially revealing prices are necessary in order to have information acquisition activities, i.e. noise is necessary for information to be produced in the first place.

\(^4\)One can use (1) to obtain the expression below.
2.2 Prices as aggregators

In our previous lecture we studied how prices *revealed* information about the risky asset. In that model of Grossman and Stiglitz (1980) information for all agents was the same, they all get to observe the same piece of information \( Y = X + \epsilon \). Now we will look at how prices *aggregate* information, i.e. at the case where agents have diverse pieces of information which stock prices “blends together.” These ideas where initially developed independently in Hellwig (1980) and Verrecchia (1982).

Again, we sill start by specifying a large economy where a fraction \( \lambda \) of agents have private information, whereas a fraction \( 1 - \lambda \) is uninformed. The per-capita supply of the risky asset is \( Z \), which is assumed to be random, namely \( Z \sim \mathcal{N}(0, \sigma^2_z) \). There is a risky asset with payoff \( X \sim \mathcal{N}(0, \sigma^2_x) \). The random variables \( Z \) and \( X \) are uncorrelated.

The informed agents get signals of the form \( Y_i = X + \epsilon_i \), where \( \epsilon_i \sim \mathcal{N}(0, \sigma^2_\epsilon) \) and the \( \epsilon_i \)'s are independent. Note how all agents get the i.i.d. signals about the risky asset, with the same quality, in contrast to the model of Grossman and Stiglitz (1980), where all agents get the same signal. The uninformed agents get to observe price, but otherwise do not have any information about the risky asset payoff. Agents can also invest in a risk-free security, whose return we normalize to 1. All agents are assumed to have CARA preferences with risk aversion parameter \( \tau \). Since initial wealth is irrelevant, we assume that all agents have zero initial wealth.

In order to solve for equilibrium asset prices in this economy, we go back to our five step approach specified in our previous notes. We first conjecture a functional form for the price function. Now we will guess that price will be of the form

\[
P_x = bX - dZ;
\]

for some real numbers \( b, d \). The intuition for this conjecture will become clear later. For now note that instead of \( Y = X + \epsilon \) we have \( X \) itself in the price function.

We now derive beliefs for the agents as a function of the price coefficients. We first note that now prices is an informative signal both for the informed agents and for the uninformed, i.e. all agents will use price to update their beliefs about the risky asset.

Some (tedious) calculations show that

\[
\mathbb{E}[X|Y_i, P_x] = \left( \frac{1}{\sigma^2_\epsilon} Y_i + \frac{b}{d^2 \sigma^2_z} P_x \right) \text{var}(X|Y_i, P_x)
\]

\[
\text{var}(X|Y_i, P_x)^{-1} = \frac{1}{\sigma^2_x} + \frac{1}{\sigma^2_\epsilon} + \left( \frac{b}{d} \right)^2 \frac{1}{\sigma^2_z}
\]

Moreover

\[
\mathbb{E}[X|P_x] = \left( \frac{b}{d^2 \sigma^2_z} P_x \right) \text{var}(X|P_x)
\]

\[
\text{var}(X|P_x)^{-1} = \frac{1}{\sigma^2_x} + \left( \frac{b}{d} \right)^2 \frac{1}{\sigma^2_z}
\]
This completes step 2 of our REE recipe. Note how in this model the informed agents update their beliefs on $X$ by using a weighted average of their signal $Y_i$ and price $P_x$, in contrast to the model of Grossman and Stiglitz (1980), where the informed did not learn anything from price.

The optimal trading strategies for the agents are given as usual by

$$\theta_i = \frac{E[X|Y_i, P_x] - P_x}{\tau \text{var}(X|Y_i, P_x)};$$

for the informed, whereas for the uninformed we get

$$\theta_j = \frac{E[X|P_x] - P_x}{\tau \text{var}(X|P_x)};$$

Now we aggregate the demands in order to impose market clearing. Note that we can write the optimal trading strategy for an informed agent as

$$\theta_i = \frac{1}{\tau \sigma^2} Y_i + P_x \frac{b}{\tau d^2 \sigma^2} - \frac{P_x}{\tau \text{var}(X|Y_i, P_x)}.$$

The aggregate demand of the informed agents is

$$\int_0^\lambda \frac{E[X|Y_i, P_x] - P_x}{\tau \text{var}(X|Y_i, P_x)} = \frac{\lambda}{\tau \sigma^2} X + \lambda P_x \frac{b}{\tau d^2 \sigma^2} - \frac{\lambda P_x}{\tau \text{var}(X|Y_i, P_x)}.$$

Note how in the aggregation of the informed agents trading strategies the $\epsilon_i$'s disappear. The intuition for this comes from writing the aggregate demand as a summation:

$$\int_0^\lambda \theta_i d_i \approx \frac{1}{I} \sum_{i=1}^n \kappa(X + \epsilon_i) + \frac{1}{I} \sum_{i=1}^n f(P_x)$$

where $\kappa = 1/(\tau \sigma^2)$, $f(P_x)$ is some linear function of $P_x$, $I$ is the total number of agents in the economy, and $n$ is the number of informed. Then it should become apparent that by the law of large numbers the $\epsilon_i$'s disappear from the price function.

The aggregate demand of the uninformed agents is

$$\int_0^{1-\lambda} \frac{E[X|P_x] - P_x}{\tau \text{var}(X|P_x)} d_i = (1 - \lambda) P_x \frac{b}{\tau d^2 \sigma^2} - (1 - \lambda) \frac{P_x}{\tau \text{var}(X|P_x)}.$$

Market clearing (aggregate demand equals aggregate supply) then reduces to

$$\frac{\lambda}{\tau \sigma^2} X + P_x \frac{b}{\tau d^2 \sigma^2} - \frac{\lambda P_x}{\tau \text{var}(X|Y_i, P_x)} - (1 - \lambda) \frac{P_x}{\tau \text{var}(X|P_x)} = Z.$$

The above equilibrium condition must all hold for all realizations of $Y, X, Z$, so we can impose rational expectations by “matching coefficients.” You should check that the solution
for $b$ and $d$ are given by

\[
\frac{b}{d} = \frac{\lambda}{\tau \sigma_z^2}
\]

\[
d = \frac{1 + \frac{\lambda}{\tau \sigma_z^2}}{\frac{\lambda}{\tau \sigma_z^2} + 1 + \frac{\lambda (\tau \sigma_z^2)^2}{\sigma_x^2}} \left( 1 + \frac{\lambda (\tau \sigma_z^2)^2}{\sigma_x^2} \right)
\]

Note how this equilibrium price has similar qualitative properties with respect to the one in Grossman and Stiglitz (1980). It is straightforward to check that in equilibrium

\[
\text{var}(X|P_x)^{-1} = \frac{1}{\sigma_x^2} + \left( \frac{\lambda}{\tau \sigma_z^2} \right)^2 \frac{1}{\sigma_z^2}
\]

Again, fixing $\lambda$, prices are more informative: (1) the lower the risk-aversion of the agents $\tau$; (2) the lower the signal variance $\sigma_z^2$; (3) the higher the fraction of informed agents $\lambda$; (4) the lower the variance of aggregate supply $Z$.

The main conclusion of the papers by Hellwig (1980) and Verrecchia (1982) is that prices not only reveal information, but they also aggregate diverse pieces of information possessed by informed agents.

2.3 Applications

After developing the main ideas around competitive rational expectations models, we will apply them to study (1) sunshine trading, along the lines of Admati and Pfleiderer (1991); and (2) information sales, as in Admati and Pfleiderer (1986).

2.3.1 Sunshine trading

By sunshine trading the Finance community refers to the practice of pre-announcing trades prior to markets opening. This was hotly debated right after the 1987 crash, primarily because it was a trading practice of portfolio insurers (in particular the firm LOR), who were partially blamed for the crash itself.

We will only analyze sections 1 and 2 of the paper by Admati and Pfleiderer (1991) on this issue. The analysis in this paper starts by specifying a model that is very similar to that in Hellwig (1980), with the exception that instead of having a random aggregate supply of the risky asset ($Z$ in our previous notation), this aggregate supply is normalized to zero, and the authors introduce noise through the presence of liquidity traders. As it turns out, these liquidity traders are at the heart of the sunshine trading debate, since traders pushing for allowing sunshine trading were believed to trade based for exogeneous liquidity reasons, not based on private information.

In particular, consider a “continuum economy” (many agents) where all traders, which we will refer to as speculators, have CARA preferences with risk-aversion parameter $\tau$, and let $\tau = 1$. There is a stock with final payoff $X$, and we further assume $X \sim \mathcal{N}(0, 1)$. Each of the
informed agents gets a signal $Y_i = X + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$ and the $\epsilon_i$’s are independent.

The new twist in the model is the introduction of liquidity traders. In particular, the authors assume that there are demands for the risky asset $A$ and $N$, where $A \sim \mathcal{N}(0, a)$ and $N \sim \mathcal{N}(0, n)$, with both $A$ and $N$ uncorrelated with $X$. At this point the liquidity traders $A$ and $N$ are the same thing. Later we will allow the $A$ traders to announce their trades prior to opening the stock market, whereas the $N$ traders will not be able to do so. Market clearing for the stock will then require

$$\int_0^1 \theta_i di + A + N = 0;$$

where $\theta_i$ is the trading strategy of speculator $i$.

By looking at this market clearing condition, it is apparent that letting $Z = -(A + N)$ we are back to a special case of the Hellwig (1980) model discussed in our last lecture. Effectively, these liquidity traders play the same role as the (negative) supply of the risky stock in the Hellwig (1980) model.

The equilibrium price can easily checked to be of the form:

$$P_x = bX + d(A + N);$$

with

$$\frac{b}{d} = \frac{1}{\sigma^2}; \quad d = \frac{\sigma^2(1 + \sigma^2(a + n))}{1 + \sigma^2(1 + \sigma^2)(a + n)}.$$

You should be able to reproduce the above formulae from those in our previous set of notes: do it!

The question that Admati and Pfleiderer (1991) were after in this paper was the effects of releasing information on $A$ prior to trade, i.e. sunshine trading by a subset of the liquidity traders. Suppose that $A$ is known at the time of trading. We can solve for the REE price function using our usual 5 step procedure, and find that it is given by the following expression

$$P_x = d_A A + bX + d_N N;$$

where

$$\frac{b}{d_N} = \frac{1}{\sigma^2}; \quad d_N = \frac{\sigma^2(1 + n\sigma^2)}{1 + n\sigma^2(1 + \sigma^2)}$$

and

$$d_A = \frac{n\sigma^4}{1 + n\sigma^2(1 + \sigma^2)}.$$

The above analysis has simply been an application of our previous work. Now let’s ask interesting questions.

1. Would the $A$ traders like to announce their trades? Their costs of trading (negative of their profits) are given by $E[-A(X - P_x)]$, or $da$ when they do not announce their trades, and $d_A a$ when they do. Since $d_A < d$ we immediately conclude that the announcers are better off by revealing their trading intentions, i.e. sunshine trading.
2. What is the effect of this announcement on the $N$ traders? Their costs of trading are, as for the $A$ traders, given by $dn$ when there is no sunshine trading, whereas they are $d_Nn$ when there is sunshine trading. Since $d_N > d$ these liquidity traders are worse off.

Also note that the total liquidity traders costs are $d(n + a)$ without sunshine trading, and $d_Aa + d_Nn$ with sunshine trading. One can check that $d_Aa + d_Nn < d(n + a)$, so in aggregate liquidity traders are better off if sunshine trading is allowed.

3. What is the effect of sunshine trading on price informativeness? Using our pricing equations, we can see that without preannouncement the equilibrium price satisfies

$$\text{var}(X|P_x)^{-1} = 1 + \frac{1}{\sigma_x^2(a + n)};$$

whereas with sunshine trading we have

$$\text{var}(X|P_x)^{-1} = 1 + \frac{1}{\sigma_X^2n}.$$ 

Therefore we immediately conclude that sunshine trading will increase price informativeness. Note that this is very intuitive: by announcing their trades the $A$ traders are taking noise out of the equilibrium price, so price becomes more informative.

4. Would the speculators support the move to sunshine trading? The calculations here are more messy (see pages 473-474), but the basic idea is to compare the expected utility of these traders, given by

$$-\sqrt{\frac{\text{var}(X|Y_i, P_x)}{\text{var}(X - P_x)}},$$

under the two regimes. Admati and Pfleiderer (1991) show that speculators are worse off with sunshine trading.

### 2.3.2 Information sales

We consider in this section a special case of the problem discussed in Admati and Pfleiderer (1986), where an agent endowed with a signal $Y = X + \epsilon$ is deciding how to sell this information to the general public. For simplicity, it is assume that this agent will not trade on his own account.

The population of potential clients is given by a continuum of traders on $[0, 1]$, all with CARA preferences with risk-aversion $\tau$. The monopolist seller is planning to sell a “newsletter” in which he will tell his clients about his signal $Y$. All other assumptions and notation from previous models apply: in particular there is a risky asset with payoff $X$, which has aggregate supply $Z$.

Given that he decides to sell this signal to a fraction $\lambda$ of the traders, the price that he will
set for the newsletter \( c \) will be given by the indifference condition

\[
-\sqrt{\frac{\text{var}(X|P_x)}{\text{var}(X - P_x)}} = -c^{\tau c} \sqrt{\frac{\text{var}(X|Y, P_x)}{\text{var}(X - P_x)}}
\]

or simplifying

\[
c = \frac{1}{2\tau} \log \left( \frac{\text{var}(X|P_x)}{\text{var}(X|Y, P_x)} \right)
\]

Fixing the signal that he will sell, he would decide on the circulating size of his newsletter by maximizing \( \lambda c \), or

\[
\max \lambda \frac{\lambda}{2\tau} \log \left( \frac{\text{var}(X|P_x)}{\text{var}(X|Y, P_x)} \right)
\]

Now note that I have not talked about equilibrium prices \( P_x \) yet. The reason is that these prices are given in the lecture on Grossman and Stiglitz (1980)! Revising our notes we see that

\[
\text{var}(X|P_x)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma^2} + \frac{1}{\tau \sigma^2 / \lambda^2 \sigma_x^2}
\]

and

\[
\text{var}(X|Y, P_x)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma^2}
\]

From these expressions one can easily calculate the optimal circulation size for the newsletter \( \lambda^* \).

Admati and Pfleiderer further explore the possibility of adding noise to the signal of the monopolist. Literally, instead of printing copies of the newsletter telling agents what \( Y \) is, he could print newsletters that contained \( Y + \eta \), for some random variable \( \eta \sim \mathcal{N}(0, s) \) (photocopied noise), or newsletters with information \( Y + \eta_i \), for some i.i.d. random variables \( \eta_i \sim \mathcal{N}(0, s) \) (personalized noise). We shall explore this later possibility in the homework. We outline here the analysis of selling newsletters with photocopied noise.

If the newsletter had instead of the signal \( Y \) the signal \( Y + \eta \), the previous analysis follows as stated, with \( s + \sigma^2 \) replacing \( \sigma^2 \). The monopolist problem thereby reduces to

\[
\max_{s, \lambda} \frac{\lambda}{2\tau} \log \left( \frac{\text{var}(X|P_x)}{\text{var}(X|Y, P_x)} \right)
\]

where now

\[
\text{var}(X|P_x)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma^2} + \frac{1}{(\tau \sigma^2 + s) / \lambda^2 \sigma_x^2}
\]

and

\[
\text{var}(X|Y, P_x)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma^2}
\]

Consider the case where \( \sigma^2 \) is 0, i.e. the seller of information has perfect information on

\footnote{Note that there is no easy expression in general for \( \lambda^* \).}
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X. It is clear that in this case he would like to add some noise to his signal, since otherwise
prices would be perfectly revealing. Some simple calculations show that, fixing \( \lambda \), the optimal
noise added is \( s = \lambda / (\sigma_z \tau) \). This further reduces the monopolist problem to

\[
\max_y y \log \left( 1 + \frac{1}{y(2 + y)} \right)
\]

where I have used the change of variable \( y = \lambda / (\tau \sigma_z) \). If we let \( y^* \) denote the optimum of the
above optimization problem (such an optimum always exists, although no closed-form solution
can be provided, it is \( y^* \approx 0.6514 \)), then we learn that the seller of information will ration its
newsletter (\( \lambda < 1 \)) if and only if \( y^* \tau \sigma_z < 1 \). In plain English, restricting the circulation of the
newsletter will be optimal in markets where the amount of noise (as measured by \( \sigma_z \)) is small,
or agents are risk-tolerant (low \( \tau \)).

3 Strategic market order models

The models studied this far were competitive in the sense that investors were “small.” This
simply meant that when traders were making investment decisions, they took prices as given.
Although this is a good approximation for some markets, this ignores the fact that sometimes
a single trade can actually move markets. This is the topic that we visit now, in particular
in the context of the paper by Kyle (1985). After laying out the basic ideas, we discuss two
simple extensions, and then one application from the literature, that of Fishman and Hagerty

3.1 Foundations

We start by the basic bare-bones version of insider trading studied in the first two sections of
Kyle (1985). The model elements should be familiar by now:

- There are two assets available for trade: a risky asset with final payoff \( X \sim N(\mu_x, \sigma_x^2) \), and
  a riskless asset with gross risk-free rate normalized to 1.
- There is a single risk-neutral insider who knows the value of \( X \). We denote his trading
  strategy by \( \theta \).
- There noise-traders who submit orders which we denote by \( Z \sim N(0, \sigma_z^2) \).
- There is a market-maker that sets prices after observing total order flow \( \omega = Z + \theta \). In
  particular he sets \( P_x = E[X|\omega] \).

Note that the insider is risk-neutral, in contrast to the competitive REE studied previously.
This is for analytical convenience, although we lose the ability to study risk-aversion effects.
The noise-traders play a similar role to the per capita aggregate supply (or liquidity traders)
that we discussed previously.

In order to solve the model, we start with the insider’s problem. Let the insider conjecture
that the price function is of the form \( P_x = \gamma + \lambda \omega \), for some constants \( \gamma \) and \( \lambda \). His optimal
investment problem is then

$$\max_{\theta} \ E[\pi|X] = \ E[\theta(X - P_x)|X] = \theta(X - \gamma - \lambda \theta)$$

since $E[Z|X] = 0$. The optimal trading strategy for the insider is

$$\theta^* = \frac{X - \gamma}{2\lambda}.$$ 

Now consider the market maker’s problem. Let him conjecture that the insider is trading according to $\theta = \alpha + \beta X$. Then his pricing rule becomes

$$P_x = \ E[X|\omega] = \ E[X|\alpha + \beta X + Z]$$

$$= \mu_x + \frac{\text{cov}(X, \omega)}{\text{var}(\omega)}(\omega - \ E[\omega])$$

$$= \mu_x + \frac{\beta \sigma_x^2}{\beta^2 \sigma_x^2 + \sigma_z^2}(\omega - \ E[\omega])$$

Imposing rational expectations, we get four equations for the conjectured coefficients $\alpha$, $\beta$, $\lambda$ and $\gamma$. They are

$$\lambda = \frac{\beta \sigma_x^2}{\beta^2 \sigma_x^2 + \sigma_z^2}; \quad \beta = \frac{1}{2\lambda};$$

$$\gamma = \mu_x; \quad \alpha = -\mu_x \beta.$$ 

Some simple manipulations yield the equilibrium conditions

$$\lambda = \frac{1}{2} \frac{\sigma_x}{\sigma_z}; \quad \beta = \frac{\sigma_z}{\sigma_x}.$$ 

The variable $\lambda$ measures the impact in price of a one unit trade. The larger $\lambda$ is, the higher the price impact, or the less liquid a market is. The literature usually talks about $\lambda^{-1}$ as measuring market depth, i.e. how liquid a particular market is: the higher $\lambda^{-1}$ is the more liquid the market. Note how liquidity critically depends on $\sigma_z$: if there were no noise traders then the market would “dry up,” meaning trades will move prices a lot. But also note how the insider scales back his trading strategy as $\sigma_z$ becomes small. As a matter of fact, it is straightforward to show that

$$\text{var}(X|P_x) = \frac{\sigma_x^2}{2};$$

for any $\sigma_z$. This is a special property of the Kyle (1985) model: prices reveal $1/2$ of the information of the insider.

Finally, let’s compute the expected profits for the insider. They are given by

$$\ E[\theta^*(X - P_x)] = \frac{1}{2} \sigma_x \sigma_z.$$
The expected profits are increasing in $\sigma_z$ (the amount of noise traders), as well as the volatility of $X$.

We consider next a couple of examples that illustrate the mechanics of solving for equilibria in this class of models.

**Example 1.** Consider now the case where the informed agent gets a noisy signal $Y = X + \epsilon$.

We can solve the model again by (1) going over the insider’s problem given some conjecture on $P_x$; (2) going over the market maker’s price setting decision.

Suppose $P_x = \gamma + \lambda \omega$. Then the insider’s problem is

$$\max_\theta \ E \left[ \theta (X - P_x) \right] = \theta \mathbb{E} [X | Y] - \theta \gamma - \lambda \theta^2;$$

where $\mathbb{E} [X | Y] = \sigma_x^2 Y / (\sigma_x^2 + \sigma_z^2)$. The optimal trading strategy is therefore

$$\theta^* = \frac{1}{2\lambda \sigma_x^2 + \sigma_z^2} Y - \frac{\gamma}{2\lambda}.$$

Let the market maker conjecture that $\theta = \alpha + \beta Y$. Then

$$P_x = \mathbb{E} [X | \omega] = \mu_x + \frac{\beta \sigma_x^2}{\beta^2(\sigma_x^2 + \sigma_z^2) + \sigma_z^2} (\omega - \mathbb{E} [\omega]).$$

Imposing rational expectations and solving for the price coefficients we get

$$\lambda^{-1} = \frac{2\sigma_z}{\sigma_z^2} \sqrt{\sigma_x^2 + \sigma_z^2}, \quad \beta = \frac{\sigma_z}{(\sigma_x^2 + \sigma_z^2)^{1/2}}.$$

Note how the market becomes deeper as $\sigma_z^2$ increases, and how the insider puts less weight on his signal as $\sigma_z^2$ increases. □

**Example 2.** Now consider the case where there are two insiders, each endowed with perfect information about the payoff of the risky asset. Order flow is $\omega = \theta_1 + \theta_2 + Z$.

Let agent 1 conjecture that the pricing function is of the form $P_x = \gamma + \lambda \omega$. Moreover, let him conjecture that agent 2’s trading strategy is of the form $\theta_2 = \beta_2 X$. Then his optimal trading strategy solves

$$\max_{\theta_1} \ \theta_1 X - \theta_1 (\gamma + \lambda (\theta_1 + \beta_2 X))$$

i.e.

$$\theta_1^* = \frac{X - \gamma}{2\lambda} - \frac{1}{2} \beta_2 X.$$

Similarly we have that

$$\theta_2^* = \frac{X - \gamma}{2\lambda} - \frac{1}{2} \beta_1 X.$$

Since $\theta_1 = \theta_2$ due to symmetry, note the above imply that

$$\theta^* = \frac{X - \gamma}{3\lambda}.$$
The pricing problem for the market maker is given by

\[ P_x = \mathbb{E}[X|\omega] = \mu_x + \frac{2\beta \sigma_x^2}{4\beta^2 \sigma_x^2 + \sigma_z^2}(\omega - \mathbb{E}[\omega]). \]

Matching coefficients we get

\[ \lambda^{-1} = \left( \frac{3}{\sqrt{2}} \right) \frac{\sigma_z}{\sigma_x}; \quad \beta = \frac{\sigma_z}{\sqrt{2}\sigma_x}. \]

Note how price informativeness has gone up as a new trader was added to the market, and how traders react less aggressively to their private information. □

### 3.2 Insider trading regulation

The paper by Fishman and Hagerty (1992) discusses the implications of insider regulation for price informativeness. The first argument, given in section 2 of the paper, is with regards to information acquisition activities by market professionals with (or without) the presence of an insider that is endowed with a precise signal on the firm’s prospects. The introduction to this paper is a really nice review of the issues at play in the insider trading debate.

The model is a la Kyle (1985). There are \( m \) “market professionals” who can observe a signal of the form \( Y_i = X + \eta_i \), where \( \eta_i \sim \mathcal{N}(0,1/h_\eta) \), at a cost \( c \). There is also an insider, who observes \( Y = X + \epsilon_i \), with \( \epsilon_i \sim \mathcal{N}(0,1/h_\epsilon) \). Further assume that \( X \sim \mathcal{N}(0,1/h_x) \).

Consider two market structures: (i) one with \( m_N \) market professionals where the insider is not allowed to trade; (ii) one with \( m_I \) market professionals when the insider is allowed to trade. Note that in principle the number of market professionals can be different in each case, since the profits to these investors will differ whether the insider is allowed to trade or not. If \( \pi(m) \) denotes the profits for the market professionals when there are \( m \) of them, the equilibrium number of traders (ignoring integer constraints) is given by \( \pi(m) = c \). Equilibrium expressions are provided in the paper (p. 108).

**Example 3.** The equilibrium, fixing the number of “professional” traders \( m \) without an insider is straightforward to analyze. We conjecture that the price function is linear in order flow, \( P_x = \gamma + \lambda \omega \). Each trader maximizes

\[ \max_\theta \theta \mathbb{E}[X|Y_i] - \theta \left[ \gamma + \lambda \left( \sum_{i=1}^{m} \mathbb{E}[\theta_i|Y_i] \right) \right] \]

If they conjecture that \( \theta_i = \beta_i X \), and \( \beta_i = \beta \) for all \( i \), the optimal trade is given by

\[ \theta^* = \frac{\mathbb{E}[X|Y_i](1 - \lambda \sum_{j \neq i} \beta_j)}{2\lambda}. \]

Furthermore, the pricing problem of the market maker reduces to

\[ P_x = \frac{m \beta \sigma_x^2}{(m^2 \beta^2 \sigma_x^2 + m \beta^2 \sigma_\eta^2 + \sigma_z^2)\omega}. \]
Matching coefficients one can easily obtain an expression for market depth \( \lambda_N^{-1} \), and subsequently other equilibrium variables such as price informativeness \( \text{var}(X\mid P_x)^{-1} \).

Price informativeness in the case where there is no insider is given by

\[
\text{var}(X\mid P_x)^{-1} = h_x + \frac{m_N h_x h_x}{2h_x + h_\eta},
\]

whereas in the case with an insider we have

\[
\text{var}(X\mid P_x)^{-1} = h_x + \frac{m_I h_\eta h_x}{2h_x + h_\eta} + \frac{h_x h_\epsilon}{2h_x + h_\epsilon}.
\]

The last two terms of the last expression can be interpreted as the contribution of the market professionals’ and that of the insider’s order flow to price informativeness. If \( m_I = m_N \) it is immediate that the presence of an insider indeed makes price more informative. On the other hand, \( m_I < m_N \), that is, the presence of an insider reduces the incentives to gather information by the market professionals. There is a tradeoff which Fishman and Hagerty (1992) exploit in the discussion of section 2 of their paper. The authors show that if the number of professionals \( m \) is small, then price efficiency will be increased by allowing an insider to trade. On the other hand, when \( m \) is large an insider can be detrimental for price informativeness.

4 Sequential trade models

The following is a sketch of the papers by Glosten and Milgrom (1985) and Easley and O’Hara (1987). The main ideas have similar flavor to those already studied, although the focus is on strategic behavior (as in Kyle (1985)), and on the price formation itself. In both models a bid-ask spread arises due to the presence of asymmetric information.

4.1 A simple model of the bid-ask spread

Consider a stock whose value next period will be either \( \bar{X} \), with probability \( \eta \), or \( X \), with probability \( 1 - \eta \). There is a probability \( \mu \) that an insider will show up. If a liquidity trade shows up, he places a buy order with probability \( \gamma \), and a sell order with probability \( 1 - \gamma \). Traders can only buy or sell 1 unit, and all traders are chosen from a continuum, so they will not trade a second time (this eliminates strategic behavior across time).

The main insight from Glosten and Milgrom (1985) was to pospulate that prices in this market should be given by ask and bid prices such that conditional on the trade occurred \( A = \mathbb{E}[X\mid \text{buy}] \) and \( B = \mathbb{E}[X\mid \text{sell}] \).

In order to compute this conditional expected values, note that by Bayes’ rule

\[
P(\bar{X}\mid \text{buy}) = \frac{\eta(\mu + (1 - \mu)\gamma)}{\eta(\mu + (1 - \mu)\gamma) + (1 - \eta)(1 - \mu)\gamma}.
\]
Therefore we have that the ask price quoted by the market maker is

\[ A = \mathbb{E}[X | \text{buy}] = \bar{X}P(\bar{X} | \text{buy}) + X(1 - P(\bar{X} | \text{buy})). \]

Similar calculations apply to bid prices.

**Example 4.** Let \( \eta = \gamma = 1/2 \). Then we have that \( A = \mathbb{E}[X] + 0.5\mu(\bar{X} - X) \), and \( B = \mathbb{E}[X] + 0.5\mu(\bar{X} - X) \). The bid-ask spread is therefore \( \mu(\bar{X} - X) \), rather intuitively increasing in the probability of informed trading and also increasing in the size of the informational advantage. □

The authors further considered dynamics properties of this market. In particular, consider repeating the above game through time. We first note that the market maker will continue to update through the amount of trading that occurs in the market.

**Example 5.** Let \( \bar{X} = 20, X = 10 \) and set \( \mu = 1/2 \). In the previous example, the bid ask spread after a buy in the first period will move up, from \([12.5, 17.5]\) to \([15, 19]\). After two buys the market maker will be quoting \([17.5, 19.2]\), whereas after a buy and a sell he will be back to \([12.5, 17.5]\). □

Let \( q = P(\text{buy}|X) \) and \( p = P(\text{buy}|\bar{X}) \). After a sequence of \( b \) buy orders and \( s \) sell orders, we have that

\[ P(X|b, s) = \frac{\eta q^b(1 - q)^s}{\eta q^b(1 - q)^s + (1 - \eta)p^b(1 - p)^s}. \]

One can verify that if the insider knows that \( X = \bar{X} \), when this probability converges to 1 as we increase the number of trading periods. Therefore prices are fully revealing in some asymptotic sense.

### 4.2 Large and small orders

The above model captured several natural aspects of the trading process, although it oversimplified many others. Easley and O’Hara (1987) consider a variation of the above model which: (1) allows each period for the occurrence of an “information event,” when insiders will be present (probability \( \alpha \)); (2) the liquidity traders now can submit two types of buy and sell orders: for one unit or for two units. We note that if there is no information event then only liquidity traders are present in the market. For simplicity consider the case where liquidity traders are willing to buy or sell one or two units with equal probabilities (1/4 to each event). Lastly assume \( \bar{X} \) and \( X \) are equally likely.

In equilibrium, the insider will be either: (1) always trading 2 units (separating equilibrium), or (2) mixing between trading 1 unit, with probability \( \psi \), or trading 2 units, with probability \( 1 - \psi \) (pooling equilibrium). We discuss next the characterization of the equilibrium in the later case, and let you deal with the other case in the homework.

In this market the dealer with post 4 prices: \( a^1, a^2, b^1 \) and \( b^2 \), corresponding to ask and bid prices for 1 and 2 units. Consider the bid side of the market. Following the same logic as
in Glosten and Milgrom (1985) we have that

\[ b_1 = \delta(S_1)X + (1 - \delta(S_1))\bar{X} \]  
(4)

\[ b_2 = \delta(S_2)X + (1 - \delta(S_2))\bar{X} \]  
(5)

with \( \delta(S_1) = \mathbb{P}(X|\text{sell}_1) \), and \( \delta(S_2) = \mathbb{P}(X|\text{sell}_2) \). These conditional probabilities can be computed using Bayes’ rule. For example, one can verify that

\[ \delta(S_1) = 0.5(\alpha \mu \psi + 0.25(1 - \alpha \mu)) \]

Finally, note that \( \psi \) needs to be endogeneous. In particular, \( \psi \) is given by the indifference condition (trading 1 unit versus 2 units) \( b_1 - \bar{X} = 2(b_2 - \bar{X}) \). Also note that I am taking advantage here of the symmetry of the problem, since in principle \( \psi \) should depend on whether \( \bar{X} \) or \( X \) has occurred.

In summary, an equilibrium in the bid-side of the market is given by: (1) a bid price for small orders characterized in (4), (2) a bid price for large orders characterized in (5), (3) a mixing probability for the informed agent \( \psi \in (0, 1) \). If one can find these three numbers, where \( \psi \) needs to belong to [0, 1], then we have the pooling equilibrium described by Easley and O’Hara (1987). If such \( \psi \) does not exist, then we have a separating equilibrium. The details for the later are very similar to those in Glosten and Milgrom (1985), so I let you check them from the paper.

**Example 6.** Let \( \alpha = 1, \mu = 0.5 \) \( \bar{X} = 20 \) and \( X = 10 \). Then the equilibrium optimal bid and ask prices are \( b_1 = 13.75, b_2 = 11.875 \), and (by symmetry) \( a_1 = 16.25 \) and \( a_2 = 18.125 \). The informed agent’s mixing probability is \( \psi = 0.166 \).

If \( \alpha \) drops to 0.8, the bid-ask spread narrows to \( b_1 = 14.5 \) and \( b_2 = 12.25 \), as expected. As the bid-ask spread narrows the informed agent mixes with a lower probability, in this case \( \psi = 0.08333 \).

Eventually, as \( \alpha \) drops the equilibrium stops being a pooling one, and becomes separating. For \( \alpha = 0.6 \) for example we have \( b_1 = a_1 = 15 \), and \( b_2 = 11.13 \). The informed agent only trades 2 units and therefore there is a zero spread for 1 unit orders (since they are all from the liquidity traders). \( \square \)

The paper has tons of nice details about the implications of a dynamic version of this model for trading volume and price patterns as a function of order flow (large versus small orders). Although tedious, conceptually this analysis is a simple extention or our previous discussion in the Glosten and Milgrom (1985) setting.

### 5 Conclusion

This brief course in asymmetric information in financial markets has only introduced the main ideas in the literature. The class of models studied is a fairly rich one, and should allow you to continue working through the literature. The competitive REE models can be thought as
a particular type of large auction market, and this later literature would be the only thing that you have missed out from having such a short course. Kyle (1989) relates the standard competitive REE to the share auctions literature. It removes the price taking assumption from the discussion of Hellwig (1980). See García and Urosević (2013) for related issues. Needless to say, knowing more about auctions in general is also a good thing: many securities are sold to the public via auctions! There are also some nice connections between Kyle (1985) and Glosten and Milgrom (1985) that have recently been discovered. Dynamic versions of these models, explaining volume patterns for example, are also common in current research. It will be at least 10-20 years until we have a clear understanding of the role of asymmetric information in financial markets.
References


