Problem 1. [Points = 3] In September, Chapel Hill’s daily high temperature has a mean of 81 degree F and a standard deviation of 10 degree F. What is the mean, standard deviation and variance in terms of Celsius?

Problem 2. [Points = 3] In a given population of two-earner male/female couples, male earnings have a mean of 40,000 USD per year and a standard deviation of 12,000 USD. Female earnings have a mean of 45,000 USD per year and a standard deviation of 18,000 USD. The correlation between male and female earnings for a couple is 0.80. Let $C$ denote the combined earnings for a randomly selected couple.

(a) What is the mean of $C$?

(b) What is the covariance between male and female earnings?

(c) What is the standard deviation of $C$?

Problem 3. [Points = 3] $X$ and $Y$ are discrete random variables with the following joint distribution:

<table>
<thead>
<tr>
<th>Joint Probability Distribution</th>
<th>Value of $Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14</td>
</tr>
<tr>
<td>Value of $X$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>8</td>
</tr>
</tbody>
</table>

That is, $Pr[X = 1, Y = 14] = 0.02$, and so forth.

(a) Calculate the probability distribution, mean, and variance of $Y$.

(b) Calculate the probability distribution, mean, and variance of $Y$ given $X = 8$.

(c) Calculate the covariance and correlation between $X$ and $Y$.

Problem 4. [Points = 4] Let $X$ and $Z$ be independently distributed standard normal random variables, and let $Y = X^2 + Z$. 

(a) Show that $E[Y|X] = X^2$.

(b) Show that $E[Y] = 1$.

(c) Show that $E[XY] = 0$. (Hint: Use the fact that the odd moments of a standard normal random variable are all zero.)

(d) Show that $Cov(X,Y) = 0$.

**Problem 5. [Points = 7]** Grades on a standardized test are known to have a mean of 1000 for students in the United States. The test is administered to 453 randomly selected students in Florida. In this sample, the mean is 1013 and the standard deviation is 108.

(a) Construct a 95% confidence interval for the average test score for Florida students.

(b) Is there statistically significant evidence that Florida students perform differently than other students in the United States?

(c) Another 503 students are selected at random from Florida. They are given a three-hour preparation course before the test is administered. Their average test score is 1019 with a standard deviation of 95.

(i) Construct a 95% confidence interval for the change in average test score associated with the prep course.

(ii) Is there a statistically significant evidence that the prep course helped?

(d) The original 453 students are given the prep course and then asked to take the test a second time. The average change in their test scores is 9 points and the standard deviation of the change is 60 points.

(i) Construct a 95% confidence interval for the change in average test scores.

(ii) Is there statistically significant evidence that students will perform better on their second attempt after taking the prep course?
(iii) Students may have performed better in their second attempt because of the prep course or because they gained test-taking experience in their first attempt. Describe an experiment that would quantify these two effects.