Problem 1: [2 + 2 + 2 + (2 + 2) = 10 points] Suppose that $\pi_0 = \pi^*$ and $u_0 = u^*$ are two constants. Consider the following model:

$$\pi_t = \alpha \pi_{t-1} + \beta u_{t-1} + \epsilon_t \text{ where } \epsilon_t \sim \text{i.i.d.} (0, \sigma_{\epsilon})$$

for $t = 1, \ldots, T$. Answer the following questions:

(A) What is the effect of $u_{t-2}$ on $\pi_t$?

(B) Show that the model in equation (1) can be expressed as

$$\pi_t = \alpha^2 \pi_{t-2} + (\beta u_{t-1} + \beta \alpha u_{t-2}) + (\epsilon_t + \alpha \epsilon_{t-1}).$$

Relate this to your answer in (A).

(C) Show that for any $k = 1, 2, \ldots, t$ the model in equation (1) can be written as

$$\pi_t = \alpha^k \pi_{t-k} + \beta \sum_{j=1}^{k} \alpha^{j-1} u_{t-j} + \sum_{j=1}^{k} \alpha^{j-1} \epsilon_{t-j}.$$

What is the effect of $u_{t-k}$ on $\pi_t$? With the help of a path-diagram (arrows from one variable to another, as shown in the class), justify the answer that you just got.

(D) Suppose you run a regression of $\pi_t$ on $\pi_{t-2}$, $u_{t-1}$ and $u_{t-2}$. The regressors have true coefficients say $\delta_0, \delta_1$ and $\delta_2$. If the model in equation (1) is correct, what restrictions does that impose on $\delta_0, \delta_1$ and $\delta_2$? Again, if the model in equation (1) is correct, what problems do you expect in testing of hypotheses if you use the command `reg $\pi L2.\pi L.u L2.u` for the above regression?

Problem 2: [5 points] Consider the data set `phillips_curve.dta`. Using the notation in Problem 1, let $\pi$ denote inflation and $u$ denote unemployment rate.
(A) Run the regression of $\pi_t$ on $\pi_{t-1}$ and $u_{t-1}$.

(B) Run the regression of $\pi_t$ on $\pi_{t-2}$, $u_{t-1}$ and $u_{t-2}$.

(C) Run the regression of $\pi_t$ on $\pi_{t-3}$, $u_{t-1}$, $u_{t-2}$ and $u_{t-3}$.

(D) Run the regression of $\pi_t$ on $\pi_{t-4}$, $u_{t-1}$, $u_{t-2}$, $u_{t-3}$ and $u_{t-4}$.

Which regression do you expect to give the best fit for the data based on $R^2$? Verify your answer with the results obtained. Compare the 4 different regressions based on BIC and recommend the best one.

**Problem 3:** [5 points] Consider the data set terrorism.dta. Examine the pattern of different modes of terrorism – bombings, taking hostages and assassinations. Study how the different modes (and the lags) affect each other. Is there any noticeable difference in any pattern since 9/11? [This is kind of an open question without any guidance from me. Use your creativity and see if you can find anything interesting.]