Problem 1: [12.5 points] Suppose that you have observations \{(y_{i,t}, X_{i,t}) : i = 1, \ldots, n \text{ and } t = 1, \ldots, T\} on two random variables \(y\) and \(X\). You model the relationship between \(y\) and \(X\) as:

\[ y_{i,t} = \beta X_{i,t} + u_{i,t} \quad (1) \]
\[ u_{i,t} = \gamma_i + \epsilon_{i,t} \quad (2) \]

for \(i = 1, \ldots, n\) and \(t = 1, \ldots, T\), where \(u_{i,t}\) is the unobserved error in the model. Assume that \(\text{Cov}(X_{i,t}, \epsilon_{i,t}) = 0\). Now suppose that \(X_{i,t}\) is a dummy variable. You can think of this along the line of Problem 2, i.e. \(y_{i,t}\) can be the rate of robbery (per 100,000 people) in the \(i\)-th state in time period \(t\) and \(X_{i,t}\) is 1 if the \(i\)-th state in time period \(t\) had the "shall carry" law in effect and 0 otherwise.

If you suspect that \(\text{Cov}(X_{i,t}, \gamma_i) \neq 0\), you would naturally want to estimate \(\beta\) from (1) using fixed effect estimator.

(A): [2.5 points] One such estimator is obtained by running OLS on the first-differenced data:

\[ \hat{\beta}_{FE} = \frac{\sum_{i=1}^{n} \sum_{t=1}^{T-1} \Delta y_{i,t} \Delta X_{i,t}}{\sum_{i=1}^{n} \sum_{t=1}^{T-1} (\Delta X_{i,t})^2} \quad (3) \]

where \(\Delta y_{i,t} = y_{i,t+1} - y_{i,t}\) and \(\Delta X_{i,t} = X_{i,t+1} - X_{i,t}\). Suppose that you have \(n = 2\) and \(T = 3\). Both states started without the shall carry law. State 1 established the shall carry law in period 2 and continued with it. State 2 established the shall carry law in period 3. For State 1 the robbery rates are 200, 250, 290 in periods 1, 2 and 3 respectively. For State 2 the robbery rates are 30 and 60 in periods 1 and 2 respectively. If you know that \(\hat{\beta}_{FE} = 50\) then find out what is the robbery rate of State 2 in period 3.

(B): [2.5 points] Suppose that you now have 10 additional years (i.e. 13 years in total) of observations of robbery rates in the two states. State 1: 300, 0, 1000, 40, 20, 500, 90, 1000, 1000 and 1000 whereas for State 2: 70, 80, 90, 100, 100, 110, 120, 130, 140 and 150 respectively. Both states kept the shall carry law. What revision are you going to do to the estimated \(\beta\) obtained by using the formula (3)?

(C): [2.5 points] Define \(\Delta_2 = y_{i,t+2} - y_{i,t}\) and \(\Delta_2 X_{i,t} = X_{i,t+2} - X_{i,t}\). These is a measure of change in \(y\) and \(X\) in two periods. Note that this is also a way of removing unobserved fixed effect \(\gamma_i\). Define an alternative FE estimator:

\[ \bar{\beta}_{FE} = \frac{\sum_{i=1}^{n} \sum_{t=1}^{T-2} \Delta_2 y_{i,t} \Delta_2 X_{i,t}}{\sum_{i=1}^{n} \sum_{t=1}^{T-2} (\Delta_2 X_{i,t})^2} \quad (4) \]

Suppose that you have \(n = 2\) and \(T = 3\). Both states started without the state carry law.

\(^1\)Just to remind you, \(\gamma_i\) and \(\epsilon_{i,t}\) are also unobserved.
State 1 established the shall carry law in period 2 and continued with it. State 2 established
the shall carry law in period 3. For State 1 the robbery rates are 200, 250, 290 in periods
1, 2 and 3 respectively. For State 2 the robbery rates are 30 and 60 for periods 1 and 2
respectively. Based on your answer in part (A) find out $\tilde{\beta}_{FE}$ using (4).

(D): [2.5 points] Suppose that you now have 10 additional years of observations of robbery
rate in the two states. State 1: 300, 0, 1000, 40, 20, 500, 90, 1000, 1000 and 1000 whereas
for State 2: 70, 80, 90, 100, 100, 110, 120, 130, 140 and 150 respectively. Both states kept
the shall carry law. What revision are you going to do to the estimated $\beta$ obtained by using
the formula (4).

(E): [2.5 points] Suppose that $\text{Cov}(X_{i,t}, \gamma_i) = 0$ and $V(\gamma_i) = \sigma_{ii}$. So you can estimate $\beta$
from (1) efficiently using the random effects estimator – i.e. by using GLS or FGLS. For this
purpose, it is very important to have an idea of the covariance structure of the errors $u_{i,t}$.
Assume that $\text{Cov}(\epsilon_{i,t}, \gamma_i) = \sigma_{ei}$. Suppose you have $n = 2$ and $T = 2$. Find out the variances
and covariances of $u_{i,t}$ for $i = 1, 2$ and $t = 1, 2$. Please consult page 21 (just below equation
5.12) while answering this question.

Problem 2: [10.5 points] The following questions are based on the data set Guns.dta. We
want to find the effect of incarc (incarceration rate) and shall carry (shall carry law)
on the three different crime variables murder, robbery and violent controlling for the other
variables in the data set.

(A): [2.5 points] Estimate the effects using pooled-OLS estimator, random effect estimator
and fixed effect estimator.

(B): [2.5 points] Perform tests to find out which estimator is the most appropriate one
given the data. While performing the tests do not forget to mention the specialities (consis-
tency, efficiency, etc) of each of these 3 different types estimators.

(C): [2.5 points] Now interact shall carry and incarc. Does the shall carry law be-
come more effective if combined with more incarceration?

(D): [3 points] Consider the 2 states that have the highest and the lowest average per-
capita income (avginc) (over all the time periods). Plot the observed and predicted crime
rates for these 2 states over all the possible time periods. So I want 6 graphs – graphs 1, 2, 3
plotting the observed and the predicted robbery, murder and violent crime rates respectively
for the state with highest average per-capita income income; graphs 4, 5, 6 doing the same
for the state with the lowest average per-capita income.