International Monetary Economics

Requirements: term paper due last day of class, final exam.

Balance of Payments is record of all transactions (not just payments) between residents of home country and other countries in given time period [flow]. Appears as foreign sector in various national accounts: GDP, Flow of funds, commodity flows [input-output]. Pure trade theory assumes commodity trade is balanced (except in growth and capital movements literature). Monetary theory does not assume balanced trade.

Residency rules: IMF - center of individual’s interest. US – live in country for 6 ½ months. Applies to individuals and corporations and subsidiaries or branches (agencies abroad treated as part of domestic corporation).

Timing of transactions for goods and services can be on basis of orders, payments, or deliveries (when goods move across borders). Some countries have all three (Japan) or two (Sweden, Denmark). Transit time lag between export of goods and import leads to “world trade imbalance” [~2 ½ % of world imports in 2000-2002 = $157 billion], also due to valuation problems and recording problems, especially in investment income. Fluctuates with dock strikes, wars, etc.

Financial claims recorded at date of claim.

<table>
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<th>Currency Contract Period</th>
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<td>Production lag</td>
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Currency Contract Period (McKinnon, p. 150) – importer bears risk of exchange rate change unless hedged forward.

Pass-through Period - importer has time to adjust prices, but quantities don’t adjust yet.

Quantity-adjustment period – quantities demanded and supplied adjust, but domestic prices and costs not yet fully adjusted to exchange rate change. Real exchange rate $R = \frac{SP^*}{P}$ changes.

Purchasing Power Parity domestic prices and costs change to offset exchange rate, so real exchange rate unchanged.
Trade Finance as source of capital flows: \( TF_t = (E_t - R_t) - (I_t - P_t) = (E_t - I_t) - (R_t - P_t) \)

So \( E_t - I_t = TF_t + (R_t - P_t) \). Changes in payment times lead to changes in capital flows – leads and lags. Importer usually has flexibility in time of payment.

Measurement Problems: Exports \textit{f}as or \textit{f}ob, Imports \textit{f}as, \textit{f}ob, or \textit{c}if. \textit{Transfer} prices. Over- or under-valued exchange rates. Smuggling. Currency denomination of trade (Grassman: 2/3 exporter, 1/3, importer).

Concepts of Deficit or Surplus:

Current Balance \( CB = X^{gk} - M^{gk} - Tr = \Delta NFA (= Y - A = S - I) = \Delta LTK + \Delta STK + \Delta Res \)

Basic Balance \( BB = CB - \Delta LTK = \Delta STK + \Delta Res = \Delta STNFA = \Delta NMF + \Delta MF \)

Non-Monetary Balance \( NMB = CB - \Delta LTK - \Delta NMF = \Delta MF = \Delta NFA^b \)

Official Settlements Balance \( OSB = CB - \Delta LTK - \Delta STK = \Delta Res = \Delta NFA^{cb} \)

Relation of BoP to National Accounts:

\( GNP = GDP + r*NFA + w*NLA \)

\( GDP = Q = C_d + I_d + G_d + X = C - C_m + I - I_m + G - G_m + X = (C + I + G) + X - M = A + B \)

\( Y = Q + F = C + I + G + X - M + F = C + S^g + T - R \)

\( I + X - M + F + R = S^g + T - G \)

\( I + NFI = S^g + S^G \)

Relation to Monetary Sectors:

\( \Delta NFA^b + \Delta DC^b = \Delta M2, \quad \Delta NFA^{cb} + \Delta DC^{cb} = \Delta M0 (= \text{Curr + Bank Reserves}) \)
Intertemporal Trade and the Current Account Balance, Obstfeld & Rogoff Ch. 1
Small, open economy, single tradable good, two periods, given world interest rate $r$.

*Individuals with perishable endowments $(y_1^i, y_2^i)$ to maximize*

\[ U_1^i = u(c_1^i) + \beta u(c_2^i), \quad 0 < \beta < 1. \quad (1) \]

\[ c_1^i + \frac{c_2^i}{1 + r} = y_1^i + \frac{y_2^i}{1 + r}. \quad (2) \]

\[ u'(c_1^i) = (1 + r)\beta u'(c_2^i), \]

Assume *identical* individuals. Let population = 1, then aggregate $C = c_1, Y = y_1$.

If $\beta = 1/(1+r)$, then equilibrium $C_1 = C_2$.

\[ CA_1 = B_{t+1} - B_t = Y_t + r B_t - C_t, \]

\[ CA_1 = Y_1 - C_1 = B_2 \]

\[ CA_2 = Y_2 + r B_2 - C_2 = -B_2 \]

*Autarky real interest rate:*

\[ \frac{\beta u'(Y_2)}{u'(Y_1)} = \frac{1}{1 + r^A}. \]
Investment $Y = F(K)$, $F’ > 0$, $F'' < 0$. Wealth $W_{t+1} = B_{t+1} + K_{t+1}$.

Saving $\Delta B_{t+1} + \Delta K_{t+1} = Y_t + rB_t - C_t - G_t$

Current Account $CA_t = \Delta B_{t+1} = Y_t + rB_t - C_t - G_t - I_t = S_t - I_t$

Intertemporal Budget Constraint:

$$C_1 + I_1 + \frac{C_2 + I_2}{1 + r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1 + r}, \quad (15)$$

$$\max_{C_1, I_1} u(C_1) + \beta u \left\{ (1 + r) \left[ F(K_1) - C_1 - G_1 - I_1 \right] + F(I_1 + K_1) - G_2 + I_1 + K_1 \right\}. \quad (16)$$

$F'(K_2) = r,$

$$C_2 = F \left[ K_1 + F(K_1) - C_1 \right] + K_1 + F(K_1) - C_1. \quad (18)$$

if $C_2 = 0$, $C_1 = K_1 + F(K_1)$

if $C_1 = 0$, $C_2 = F(K_1 + F(K_1)) + K_1 + F(K_1)$
Forward Market Arbitrage:

\[ S = \text{Spot \ €/$, } F = 3 \text{ mos. Forward \ €/$,} \]

\[ R, R^* = 3 \text{ mos. Interest rate in $, €} \]

\[ $1 = \text{\€ } S \rightarrow (1+R^*)S \rightarrow (1+R^*)S/F = \text{\$ } (1+R) \]

\[ S/F - 1 = (1+R)/(1+R^*) - 1 \]

\[ (S-F)/F \approx R - R^* \text{ or } s - f = R - R^* \]

Spot and Forward Foreign Exchange Markets (see Hallwood & MacDonald)
let \( i \) = home interest rate, \( j \) = foreign interest rate, \( s \) = log spot price of foreign currency, \( f \) = log forward price of foreign currency, \( p \) = log domestic price, \( p_i \) = log foreign price

\[ \alpha := 5 \quad \beta := 2 \quad \gamma := .93 \quad f := 1 \quad i := .05 \quad j := .06 \quad \varepsilon := .5 \quad p := 2 \quad \pi := 1 \]

Forward Market Equilibrium:

\[ \alpha \cdot (i - j + s - f) = \gamma \cdot (f - \varepsilon) \]

\[ f := -1, -.9, 1.5 \]

CIP: \( s + i - j = 0.92 \)
CIP holds if \( \alpha \) tends to infinity
\( \varepsilon = 0.5 \)
Risk premium \( = f - \varepsilon \)
is inversely proportional to \( \gamma \)

\[ f = \frac{(-\alpha \cdot j + \alpha \cdot i + \alpha \cdot s + \gamma \cdot \varepsilon)}{\alpha + \gamma} \]

\[ \left[ (-j + i + s) \cdot \alpha + \gamma \cdot \varepsilon \right] \]

\[ \frac{(\alpha + \gamma)}{} = 0.8 \]
Spot Market Equilibrium:

\[
\left(\frac{\gamma}{\alpha + \gamma}\right)(i - j + s - \varepsilon) = \beta \cdot (p - \pi - s)
\]

Arbitrageurs' spot sales = Hedgers' purchases for (net) imports

\[
s := -1, -0.9, \ldots, 2
\]

PPP exchange rate \( s = p - \pi = 1 \)

UIP:

\[
\gamma = 0.51
\]

PPP:

\[
p - \pi = 1
\]

\[
\frac{\gamma}{\alpha + \gamma} = 0.286
\]

\[
\beta = 2
\]

\[
s = \frac{(p - \pi) \cdot (\alpha + \gamma) \cdot \beta + \gamma \cdot (j - i + \varepsilon)}{(\alpha + \gamma) \cdot \beta + \gamma} = 0.939
\]
Keynesian Elasticities Model of Balance of Payments

Pegged exchange rate $\pi$, 2 goods (exportable, importable), prices fixed in producer market (Kenen in Handbook V.2)

Exportable $x_1$, $p_1$ fixed, $p^*_1 = p_1/\pi$, $c_1$, $c^*_1$, $g_1$

Import good $x_2$, $p^*_2$ fixed, $p_2 = \pi p^*_2$, $c_2$, $c^*_2$, $g_2$

Domestic consumption

\[ C = p_1 c_1 + \pi p^*_2 c_2 = Y - T - S = Y - T - s(r, Y - T) \]

Foreign consumption

\[ C^* = (p_1/\pi)c^*_1 + p^*_2 c^*_2 = Y^* - T^* - S^* = Y^* - T^* - s^*(r^*, Y^* - T^*) \]

Market Equilibrium

\[
\begin{align*}
\text{exportable} & \quad c_1 + c^*_1 + g_1 = x_1 \\
\text{importable} & \quad c_2 + c^*_2 + g^*_2 = x_2
\end{align*}
\]

with fixed prices, let $p_1 = 1$, $p^*_2 = 1$

consumption share functions

\[
\begin{align*}
    c_i &= f_i(p_1, \pi p^*_2, C), \\
    c^*_i &= f^*_i(p_1/\pi, p^*_2, C^*)
\end{align*}
\]

so equilibrium conditions are

\[
\begin{align*}
    f_1(1, \pi, C) + f^*_1(1/\pi, 1, C^*) + g_1 &= Y \\
    f_2(1, \pi, C) + f^*_2(1/\pi, 1, C^*) + g^*_2 &= Y^*
\end{align*}
\]

Marshall-Lerner –Robinson Condition for Elasticity Adjustment

*Given* $Y$, $Y^*$, prices $p_1$, $p^*_2$, assume $d\pi$ (Note $B = S^p + S^g = S + T - g_1$)

\[
B = p_1 c^*_1 \pi p^*_2 c_2 = f^*_1\left(\frac{1}{\pi}, 1, C^*\right) - \pi f^*_2(1, \pi, C)
\]
let $e_{11}^* = -f_{11}^{*1/\pi} c_{11}$ and $e_{22}^* = -f_{22}^{\pi} c_{22}$ Take derivative with respect to $\pi$:

$$
 dB = -f_{11}^{*} \frac{d\pi}{\pi^2} - \pi f_{22} d\pi - c_{22} d\pi = \left( - f_{11}^{*} \frac{1}{\pi} c_{11}^* \right) \frac{d\pi}{\pi} + \left( - f_{22}^{\pi} \right) \pi c_{22} \frac{d\pi}{\pi} - \pi c_{22} \frac{d\pi}{\pi}
$$

$$
= \left( c_{11}^* e_{11}^* + \pi c_{22} e_{22} - \pi c_{22} \right) \frac{d\pi}{\pi} = c_{11}^* (e_{11}^* + e_{22} - 1) \frac{d\pi}{\pi}
$$

if initial $B = 0$ ($c_{11}^* = \pi c_{22}$)

Finite Supply Elasticities (Bruce & Purvis, Handbook V2, Ch. 16-1.2)

<table>
<thead>
<tr>
<th>Imports</th>
<th>Exports</th>
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<tbody>
<tr>
<td>$I_m^d (P_m) = I_m^s (P_m / e)$</td>
<td>$X^d (P_x / e) = X^s (P_x)$</td>
</tr>
<tr>
<td>$- \eta \hat{P}_m = \epsilon^f (\hat{P}_m - \hat{e})$</td>
<td>$- \eta^f (\hat{P}_x - \hat{e}) = \epsilon \hat{P}_x$</td>
</tr>
<tr>
<td>$\hat{P}_m = \frac{\epsilon^f}{\eta + \epsilon^f}$</td>
<td>$\hat{P}_x = \frac{\eta^f}{\eta + \epsilon}$</td>
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Effect on BoP in domestic currency $NX = P_x X - P_m I_m$

$$
 dNX = P_x X (\hat{X} + \hat{P}_x) - P_m I_m (\hat{I}_m + \hat{P}_m)
$$

$$
= Z \left[ (1 + \epsilon) \hat{P}_x - (1 - \eta) \hat{P}_m \right] = Z \left[ (1 + \epsilon) \frac{\eta^f}{\eta^f + \epsilon} - (1 - \eta) \frac{\epsilon^f}{\eta + \epsilon^f} \right]
$$

$$
= Z \left[ \frac{\eta^f \eta (1 + \epsilon^f + \epsilon) + \epsilon \epsilon^f \eta^f + \eta - 1}{\eta^f + \eta (\epsilon^f + \epsilon)} \right] \hat{e}
$$
Effect on terms of trade $t = P_x/P_m$

$$\hat{t} / e = \frac{\hat{P}_x / \hat{P}_m}{\eta_f^{\prime} / \eta + \varepsilon^{\prime} / \eta + \varepsilon^f} = \frac{\eta \eta_f^{\prime} - \varepsilon \varepsilon^f}{(\eta^{\prime} + \varepsilon)(\eta + \varepsilon^f)}$$

Armington Model (*IMF Staff Papers* 1969) derivation of trade share matrix. Importers $j=1,\ldots,n$, Exporters $i=1,\ldots,n$

$$X_j = \left[ b_{11} X_{11}^{-\rho_j} + \cdots + b_{1n} X_{1n}^{-\rho_j} \right]^{-1/\rho_j} \text{ CES } \sigma_j = \frac{1}{1 + \rho_j}$$

$U_j = U_j(X_j, X_0) = U(f(X_{1i}, \ldots, X_{1n}), X_0)$ separable utility

$X_j = X_j(Y_j, P_j, P_0)$ demand for tradable good

$$\frac{dP_j}{P_j} = \sum_i S_{ij} \frac{dP_{ij}}{P_{ij}} \text{ from aggregation theory}$$

$$\text{Min} \sum_i P_{ij} X_{ij} - \lambda (X_j - \bar{X}_j) \text{ where } \bar{X}_j = X_j(Y_j, P_j, P_0)$$

$$P_{ij} = \lambda \frac{\partial X_j}{\partial X_{ij}} \text{ and } \lambda = P_j \Rightarrow \frac{X_{ij}}{X_j} = b_{ij}^{\sigma_j} \left( \frac{P_{ij}}{P_j} \right)^{-\sigma_j}$$
Absorption Approach

Assume less than full employment, fixed prices and interest rates, monetary effect of deficit or surplus sterilized, pegged exchange rate.

\[ f_1(1, \pi, C) + f^*_1(1/\pi, 1, C^*) + g_1 = Y \]
\[ f_2(1, \pi, C) + f^*_2(1/\pi, 1, C^*) + g^*_2 = Y^* \]
\[ 1-m_1(1-s_y) = 1-(1-m_2)(1-s_y) = m_2(1-s_y) + s_y = m + s_y \]

\[ \begin{bmatrix} s_y + m & -m^* \\ -m & s_y^* + m^* \end{bmatrix} \begin{bmatrix} dY \\ dY^* \end{bmatrix} = \begin{bmatrix} f_{12} - f_{11}^* \\ f_{22} - f_{21}^* \end{bmatrix} d\pi = \begin{bmatrix} c_i e_{12} + c_i^* e_{11}^* \\ -c_i e_{12} - c_i^* e_{11}^* \end{bmatrix} d\pi / \pi \]

\[ d\pi / \pi = c_1^* \begin{bmatrix} e_{11} + e_{11}^* -1 \\ -(e_{11} + e_{11}^* -1) \end{bmatrix} d\pi / \pi = \begin{bmatrix} e_\pi \\ -e_\pi \end{bmatrix} d\pi / \pi \]

\[ \Delta = s_y s_y^* + m s_y^* + m^* s_y \]

\[ \begin{bmatrix} dY \\ dY^* \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} s_y^* + m^* & m^* \\ m & s_y + m \end{bmatrix} \begin{bmatrix} e_\pi \\ e_\pi \end{bmatrix} d\pi / \pi = \frac{1}{\Delta} \begin{bmatrix} s_y^* \\ -s_y \end{bmatrix} e_\pi d\pi / \pi \]

\[ dB = m^* dY^* - m dY + e_\pi \frac{d\pi}{\pi} = \left( -s_y m^* - m s_y^* + 1 \right) e_\pi \frac{d\pi}{\pi} = \frac{s_y s_y^*}{\Delta} e_\pi \frac{d\pi}{\pi} \]
Laursen-Metzler Effect of change in Terms of Trade on Absorption

(Dornbusch, p. 79)

real income \( Z = PY/Q \), price index \( Q = P^\beta (eP^*)^{1-\beta} \) so

\[
Q/P = (eP^*/P)^{1-\beta} = p^{1-\beta}
\]

assume real absorption \( V = V(Z) \) where \( V = PE/Q \)
then expenditure in terms of *domestic* goods

\[
E = (Q/P)V(Z) = p^{1-\beta}V(Yp^{1-\beta}) = E(p, Y)
\]

\[
\frac{d \log E}{d \log p} = (1 - \beta) - \varepsilon (1 - \beta) = (1 - \varepsilon)(1 - \beta) > 0
\]

where \( \varepsilon = \frac{d \log E}{d \log Z} < 1 \)

If \( E = E(Y) \), spend at \( A' \), since \( Y \) in terms of domestic goods is constant at \( E \). In this case \( V \) falls by \(-Mdp\).

If real expenditure \( V \) is to remain constant, income must be raised by \( Mdp \) to offset the terms of trade loss.

In general, with \( E = (Q/P)V(Z) \), choose an intermediate point. Spending in terms of domestic goods rises (or saving falls).
Non-Traded Goods and Purchasing Power Parity

Law of One Price  
\[ P_t = e P_t^* \]

Equilibrium in goods markets  
\[ \gamma = P_n/P_t \]

\[ P = \alpha P_t + (1 - \alpha) P_n = [\alpha + (1 - \alpha)\gamma]P_t = 0P_t \]

\[ P^* = 0^* P_t^* \]

Absolute PPP  
\[ e = P_t/P_t^* = (P/P^*)(0/0^*) \]

given \( M = PL(r,y) \)

\[ e = (M/M^*)(L/L^*)(0/0^*) \]

Relative PPP  
\[ \hat{e} = \left( \hat{M} - \hat{M}^* \right) + \left( \hat{L} - \hat{L}^* \right) + \left( \hat{\theta} - \hat{\theta}^* \right) \]

Capital and Labor in Non-Traded Goods Model (O&R, sec. 4.2)

Assume capital internationally mobile at rate \( r \)

\[ Y_T = A_T F(K_T, L_T), \quad Y_N = A_N G(K_N, L_N) \quad (1) \]

\[ A_T f'(k_T) = r \quad k_T = f^{-1}(r / A_T) \]

\[ A_T \left[ f(k_T) - f'(k_T)k_T \right] = \omega \]

\[ \omega(r, A_T) = A_T f[k_T(r, A_T)] - rk_T(r, A_T). \]

MPL:

\[ pA_N \left[ g(k_N) - g'(k_N)k_N \right] = \omega(r, A_T), \]

\[ \frac{dp}{dk_N} < 0 \]

MPK:

\[ pA_N g'(k_N) = r \quad \frac{dp}{dk_N} > 0 \]
Effect of productivity shifts on $p$:

$$A_T f(k_T) = rk_T + w,$$

$$\frac{dA_T}{A_T} + \frac{rk_T}{A_T f(k_T)} \frac{dk_T}{k_T} = \frac{rk_T}{A_T f(k_T)} \frac{dk_T}{k_T} + \frac{w}{A_T f(k_T)} \frac{dw}{w},$$

$$\hat{A}_T = \mu_{LT} \hat{w}.$$

$$\hat{p} + \hat{A}_N = \mu_{LN} \hat{w}.$$

$$\hat{p} = \frac{\mu_{LN}}{\mu_{LT}} \hat{A}_T - \hat{A}_N$$

Traded Goods – Non-Traded Goods Model

Small country case, if $\varepsilon^f$, $\eta^f$ go to $\infty$, then $dNX/de = Z(\eta + \varepsilon)$

and if $P_x = P_m = P$, then exportables and importables are a composite good $T$ and $dNX/de = Z^T(\eta^T + \varepsilon^T)$
Labor Market Equilibrium: \[ L_T \left( \frac{w}{p_T} \right) + L_N \left( \frac{w}{p_N} \right) = L \] or \[ MPL_N = \frac{w}{p_N} \]

\[
\frac{p_T}{p_N} MPL_T = \frac{p_T}{p_N} \frac{w}{p_T} = \frac{w}{p_N}
\]

Goods Market: Expenditure \( E = D_N + pD_T \)

Internal Balance:
\[
D_N \left( \frac{p_T}{p_N}, E \right) = Y_N \left( \frac{p_T}{p_N} \right)
\]
\[
d \frac{p_T}{p_N} = \frac{D_{NE}}{Y_{p} - D_{p}} < 0
\]

External Balance:
\[
D_T \left( \frac{p_T}{p_N}, E \right) = Y_T \left( \frac{p_T}{p_N} \right)
\]
\[
d \frac{p_T}{p_N} = \frac{D_{TE}}{Y_{Tp} - D_{Tp}} > 0
\]
Three sectors, Manufacturing ($m$), Services ($s$), and Energy ($e$)

\[ L = L_m \left( \frac{w}{p_m} \right) + L_s \left( \frac{w}{p_s} \right) + L_e \left( \frac{w}{p_e} \right) \]

\[ MPL_m = \frac{w}{p_m} = \left\{ \frac{p_s}{p_m} \frac{w}{p_s} = \frac{p_s}{p_m} MPL_s \right\} = \]

\[ \left\{ \frac{p_e}{p_m} \frac{w}{p_e} = \frac{p_e}{p_m} MPL_e \right\} \]

Fig. 1. Effect of the boom on the labour market.

Goods Market:
Resource Movement Effect due to upward shift in $L_E$ and $L_T$, because of rise in $MPL_E$.

Assume $0 = \frac{\partial D_s}{\partial E}$ so demand goes from a to f, but supply to b at same p. p must rise, $L_s$ shifts up to $L_s'$, raising real wage and reducing $L_m$.

Spending Effect Assume $L_e = 0$. No direct effect on labor market, but production point vertically above a. Income-consumption line $On$ takes demand to c, requiring rise in $p$, again raising $L_s$. So increase in real wage again reduces $L_m$. 

Monetary Model of BoP Adjustment

Expenditure Approach \( Y = E + T, \quad Y = Y_f \)

\[
E = e(Y, r, \frac{M + B}{P}), \quad E_Y > 0, \quad E_r < 0, \quad E_{\frac{M+B}{P}} > 0
\]

\( P = SP^* \quad \text{all goods are traded, pegged exchange rate, so } P \text{ fixed} \)

\( M = SR + B_g + J \quad \text{exchange reserves + gov’t. securities + fiat money} \)

\[
\dot{M} = SR = SP^* T = P \left( Y - E \left( Y, r, \frac{M + B}{P} \right) \right)
\]

\[
L \left( Y, r + \pi^e, \frac{M + B}{P} \right) = \frac{M}{P}
\]

Hoarding Function:

\[
\dot{M} = P \left( L \left( Y, r + \pi^e, \frac{M + B}{P} \right) - \frac{M}{P} \right)
\]

\[
\frac{\partial \dot{M}}{\partial M} = -PE_{\frac{M+B}{P}} < 0
\]

Walras’ Law: \((PL - M) + P(E - Y) = 0\)

Allow for variable \( Y \) with sticky wages or prices.

\[
Y^s \left( \frac{P}{W} \right), \quad \dot{W} = \delta \left( \frac{W}{\bar{W}} - W \right) \quad \text{with } Y^s \left( \frac{P}{\bar{W}} \right) = Y_f
\]

Endogenous monetary policy via sterilization with

\[
\dot{J} = -\phi SR \quad \text{so } \dot{M} = SR + \dot{J} = (1 - \phi) SR
\]

Capital Mobility implies 3 markets: goods, assets, money

\[
Y - E = T \quad \text{excess supply of goods}
\]

\[
A - A^D = -SF \quad \text{excess supply of assets} \quad A = B + SF
\]

\[
M - M^D = -\dot{M} \quad \text{excess supply of money}
\]

\[
T - SF - \dot{M} = 0
\]

\[
T = \dot{A} + \dot{M}
\]
Non-traded Goods and Money (Dornbusch, Ch. 7, Mundell, Ch. 8)

Pegged Exchange Rate, money supply as adjustment mechanism

Traded Goods \( D_T = \frac{(1-\gamma)VH}{P_T} = Y_T \left( \frac{P_T}{P_N} \right) \)

Nontraded Goods \( D_N = \frac{\gamma VH}{P_N} = Y_N \left( \frac{P_T}{P_N} \right) \)

Money \( PY = VH \quad P = \gamma P_N + (1-\gamma)P_T \)

Adjustment (Mundell): \( \dot{P}_N = f \left( D_N - Y_N \right) \)
\( \dot{H} = g \left( Y_T - D_T \right) \)

Adjustment (Dornbusch): \( \dot{H} = Y_T - D_T \)

Flexible Exchange Rate (Mundell, Appendix)

Traded Goods \( D_T = \frac{(1-\gamma)VH}{eP^*_T} = Y_T \left( \frac{P_T}{P_N} \right) \)

Nontraded Goods \( D_N = \frac{\gamma VH}{P_N} = Y_N \left( \frac{P_T}{P_N} \right) \)

Money \( PY = VH \)
**Polak Monetary Model**

\[ Y = \nu M_o \quad Y = \text{nominal income}, \quad M_o = M2 \text{ money stock} \]

\[ M = mY \quad M = \text{imports} \]

\[ \Delta M_o = B + \Delta D = \Delta F + \Delta D \]

\[ \Delta F = \bar{X} - M + \bar{K} \]

*Example, Ghana (Franco, *J. Dev. Studies* 15(2), 1979, “Domestic Credit and the Balance of Payments in Ghana”)*

\[ Y = 3.93 M_o \]

\[ M = 231 + .15 Y + .81 \Delta F_{-1} \]

\[ M_o = D + F \]

\[ \Delta F = \bar{X} - M + \bar{K} \]

\[ \Delta Y = -\frac{\Delta F}{.15} = 4(\Delta D + \Delta F^') \]

\[ -\Delta F = .60(\Delta D + \Delta F^') \]

\[ \Delta F = \frac{-.60}{1.60} \Delta D = -.37\Delta D, \Delta Y = \frac{-.37}{-.15} \Delta D = 2.47 \Delta D \]

\[ \Delta M_o = (1 - .37) \Delta D = .63 \Delta D \]

Santaella “Stylized Facts Before IMF Adjustment” *IMF Staff Papers*

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### Monetary Approach to the Balance of Payments (Mundell, Ch. 8)

Assume full employment, single tradable good, perfect capital mobility

\[
D + R = k(i^*)P\bar{Y}
\]

\[
\hat{d} + \hat{r} = \Delta D \frac{M}{M} + \Delta R \frac{M}{M} = \hat{p}^* + \hat{e} + \hat{y}
\]

**Fixed Rate**  
\[
\hat{r} = \hat{p}^* + \hat{y} - \hat{d}
\]

**Floating Rate**  
\[
\hat{e} = \hat{d} - \hat{p}^* - \hat{y}
\]

**Exchange Market Pressure**  
\[
\hat{r} - \hat{e} = \hat{p}^* + \hat{y} - \hat{d}
\]

**Effect of Devaluation**  
\[
\hat{r} = \hat{p}^* + \hat{y} + \hat{e} - \hat{d}
\]

**Sterilization**  
\[
\hat{d} = \alpha - \phi \hat{r}
\]

**Conolly-Taylor (Economica 1979)** 17 LDC’s and 10 DC’s

LDC's  
\[
\hat{r} = .28\hat{e} - .74\hat{d}, R^2 = .69
\]
\[
R^2 = .90
\]

DC's  
\[
\hat{r} = .27\hat{e} - 1.27\hat{d}, R^2 = .90
\]
\[
R^2 = .90
\]
Cagan Monetary Model of Exchange Rate (O&R, 8.2.7, H&M, 9.3)

\[ m_t - p_t = -\eta i_{t+1} + \phi y_t \]

\[ p_t = e_t + p_t^* \]

\[ i_{t+1} = i_{t+1}^* + E_i e_{t+1} - e_t \]

\[ e_t = p_t - p_t^* = m_t - \phi y_t + \eta i_{t+1} - p_t^* \]

\[ e_t = (m_t - \phi y_t + \eta i_{t+1}^* - p_t^*) + \eta (E_i e_{t+1} - e_t) \]

\[ e_t = f_t + \eta (E_i e_{t+1} - e_t) \]

\[ e_t = \frac{1}{1+\eta} f_t + \frac{\eta}{1+\eta} E_i e_{t+1} = \frac{1}{1+\eta} f_t + \frac{\eta}{1+\eta} E_t \left( \frac{1}{1+\eta} f_{t+1} + \frac{\eta}{1+\eta} E_t e_{t+2} \right) = ... \]

\[ e_t = \frac{1}{1+\eta} \sum_{s=0}^{\infty} \left( \frac{\eta}{1+\eta} \right)^s E_t f_{t+s} \]
Fleming-Mundell Model

\[ Y = A(r, Y) + B(\pi, Y) + G \]
\[ A_r < 0, 1 - A_y = s_Y, B_y = -m \]
\[ \frac{M}{p} = L(r, Y) \]
\[ \Delta R = B(\pi, Y) + F(r, r^*) \]

\[ IS : (s_Y + m)dY = A_r dr, \frac{dr}{dY} = \frac{A_r}{s_Y + m} < 0 \]
\[ LM : L_r dr + L_Y dY = 0, \frac{dr}{dY} = -\frac{L_Y}{L_r} > 0 \]
\[ FB : -mdY + F_r dr = 0, \frac{dr}{dY} = \frac{m}{F_r} > 0 \]

Internal Balance: \[ Y = Y_f \]
External Balance: \[ \Delta R = 0 \]

Policy Dilemma for expenditure policy in regions I and III.

Mundell solution: use fiscal & monetary policy in opposite directions

Region I: monetary expansion, fiscal contraction
Region III: monetary contraction, fiscal expansion
Differential effects of $r$ and $G$ on $Y$:

$$ \Delta_r dr + dG = (s_Y + m) dY, \quad \frac{dr}{dG} = -\frac{1}{A_r} > 0 $$

$$ -mdY + F_r dr = -\frac{m}{s_Y + m} \left[ A_r dr + dG \right] + F_r dr = 0 $$

$$ \frac{dr}{dG} = \frac{m}{(s_Y + m) F_r - mA_r} $$

$$ = \frac{1}{\left( \frac{s_Y}{m} + 1 \right) F_r - A_r} > 0 \text{ but } < -\frac{1}{A_r} $$

Slope of FB vs. LM: \[ \frac{m}{F_r} > 0 \text{ or } < -\frac{L_r}{L_r} \text{ as } F_r \leq \infty \]

Case of perfect capital mobility: $r = r^*$. Fiscal policy impotent with flexible rate. Monetary policy impotent with pegged rate.
Effect of Devaluation (Kenen) (assume \( mdY \) is small enough not to matter for \( Y^* \))

\[
s(r, Y - T) = B(Y^*, Y, \pi) + g - T
\]

\[
\frac{M}{p} = L(r, Y)
\]

\[
\dot{R} = B(Y^*, Y, \pi) + F(r, r^*)
\]

\[
\begin{bmatrix}
    s + m & s_r & -m^* \\
    L_y & L_r & 0 \\
    0 & 0 & s^* + m^*
\end{bmatrix}
\begin{bmatrix}
    dY \\
    dr \\
    dY^*
\end{bmatrix}
= \begin{bmatrix}
    e_\pi \\
    0 \\
    -e_\pi
\end{bmatrix}
d\pi / \pi
\]

\[
\text{Det} = (s^* + m^*)\left(L_r(s + m) - L_y s_r\right) < 0
\]

\[
\frac{dY}{d\pi} = \frac{1}{\text{Det}} \begin{bmatrix}
    e_\pi & s_r & -m^* \\
    0 & L_r & 0 \\
    -e_\pi & 0 & s^* + m^*
\end{bmatrix} = \frac{s^*}{s^* + m^*} \frac{1}{s + m - L_y s_r / L_r} e_\pi > 0
\]

\[
\frac{dr}{d\pi} = \frac{1}{\text{Det}} \begin{bmatrix}
    s + m & e_\pi & -m^* \\
    L_y & 0 & 0 \\
    0 & -e_\pi & s^* + m^*
\end{bmatrix} = -\frac{s^*}{s^* + m^*} \frac{L_y / L_r}{s + m - L_y s_r / L_r} e_\pi > 0
\]

\[
\frac{dY^*}{d\pi} = -\frac{1}{s^* + m^*} e_\pi < 0
\]

\[
\frac{d\dot{R}}{d\pi} = m^* \frac{dY^*}{d\pi} - m \frac{dY}{d\pi} + e_\pi + F_r \frac{dr}{d\pi}
\]

\[
= \frac{s^*}{s^* + m^*} - \frac{s - L_y s_r / L_r - F_r L_y / L_r}{s + m - L_y s_r / L_r} e_\pi > 0
\]

\[-F_r \frac{L_y}{L_r} > \text{ or } < m \text{ as slope LM > or < slope FB}\]
Effects of Variable Prices

1. Supply Curve $p(Y)$ shifts LM left as $p$ rises, FB shifts up as $\pi p^*/p$ falls.

2. If $M / \bar{p}$ with $\bar{p} = p^\beta \left(\pi p^*\right)^{1-\beta}$ then LM shifts left even more.

This assumes fixed money wage, so $Y^s(p/w)$ increases with $p$.

3. Assume instead fixed real wage

$$\omega = \frac{w}{\bar{p}} = \frac{w}{p^\beta \left(\pi p^*\right)^{1-\beta}}$$

then product real wage $\frac{w}{p} = \omega \bar{p} = \omega \left(\frac{\pi p^*}{p}\right)^{1-\beta}$

increases with devaluation and $Y$ falls, contractionary devaluation.

4. Flexible real wage

$$N^D \left(\frac{w}{p}\right) = N^S \left(\frac{w}{\bar{p}}\right) = N^S \left(\frac{w}{p} \frac{p}{\bar{p}}\right) \rightarrow Y^s = Y^s \left(\frac{\pi p^*}{p}\right)$$

5. Note if real wage is flexible and interest rate is fixed by policy or perfect capital mobility, then $Y^D \left(\frac{\pi p^*}{p}\right) = Y^S \left(\frac{\pi p^*}{p}\right)$ and $p \sim \pi$
6. So far, we assume exchange rate expectations don’t matter. But if
\[ F = F \left( r, r^* + \frac{\pi^e - \pi}{\pi} \right) \] and \( F \to \infty \), then if \( d\pi^e = \sigma d\pi \) with \( \sigma < 1 \)

Then \( dr = (\sigma - 1)d\pi < 0 \) at least in the short run.

Assignment Problem

“Swan” Diagram

Internal Balance: \( Y = Y_f \)

\[(s + m)dY = e_\pi d\pi + dG, \quad \frac{d\pi}{dG} = -\frac{1}{e_\pi} < 0\]

External Balance: \( \Delta R = 0 \)

\[dB = -mdY + e_\pi d\pi = -m \left[ \frac{e_\pi}{s + m} d\pi + \frac{dG}{s + m} \right] + e_\pi d\pi = 0\]

\[se_\pi d\pi = mdG, \quad \frac{d\pi}{dG} = \frac{m}{se_\pi} > 0\]

If External Balance line steeper, \( m > s \), assign exchange rate to internal balance, fiscal to external.

If Internal Balance line steeper, \( m < s \), assign exchange rate to external balance, fiscal policy to internal.
Gylfason & Risager “Does Devaluation Improve the Current account?” (EER 1984)

\[ GDP : y' = e + x - z - \frac{E \nu}{P} \]
\[ GNP : y = y' - \frac{E \nu}{P} \]

trade balance \( b' = y' - e \) current account \( b = y - e \)

expenditure \( e = \left[ \frac{yP}{\Pi}, \frac{M + A - \tau P \nu D}{\Pi} \right], \quad \Pi = P^{1-\nu} E^\nu \)

\( \hat{\epsilon} = \alpha \left( \hat{y} + \hat{P} - \hat{\Pi} \right) - (1 - \alpha) \left( \tau \phi \hat{E} + \hat{\Pi} \right) \)

\( \tau \) = private share of debt, \( \phi \) = debt/wealth

if initial trade deficit is \( \omega x \), then \( e = \rho y, \quad \rho = 1 + \frac{\omega x}{y} > 1 \)

exports \( \hat{x} = \eta \left( \hat{E} - \hat{P} \right) \)

final goods imports \( \hat{z} = \hat{e} + \hat{\Pi} - \hat{P} - \delta \left( \hat{E} - \hat{P} \right) = \hat{e} + \left( \psi - \delta \right) \left( \hat{E} - \hat{P} \right) \)

intermediate imports \( \hat{n} + \hat{E} - \hat{P} = \hat{y} + \left( 1 - \sigma \right) \left( \hat{E} - \hat{W} \right) \)

Production \( q = q \left( l, n \right) \) labor, imported input \( n \)

\( \hat{q} = \left( 1 - \theta_n \right) \hat{l} + \theta_n \hat{n} \) if \( \theta_1 + \theta_n = 1 \)

intermediate imports \( \hat{n} = \hat{q} - \left( 1 - \theta_n \right) \sigma \left( \hat{E} - \hat{W} \right) \) since \( \sigma = \frac{\hat{n} - \hat{l}}{\hat{E} - \hat{W}} \)

consider case \( \mu = 0 \) \( y = q - \frac{E \nu}{P} n \) and \( \hat{y} = \frac{1}{1-\theta_n} \hat{q} - \frac{\theta_n}{1-\theta_n} \left( \hat{E} - \hat{P} + \hat{n} \right) \)

\( \hat{P} = \left( 1 - \theta_n \right) \hat{W} + \theta_n \hat{E} \) \[ \text{[or] \quad \frac{\theta_1}{\theta} \hat{W} + \frac{\theta_n}{\theta} \hat{E} + \frac{1 - \theta}{\theta} \hat{q} \text{ if } \theta < 1 \]}

\( \text{let } \mu = \frac{E \nu \theta D^*}{P y} \) if \( \mu = 0 \) and \( \theta = 1 \), then \( \hat{y} = \hat{q} - \theta_n \left( 1 - \sigma \right) \left( \hat{E} - \hat{W} \right) \)
Effect of devaluation on income:

(case \( \theta = 1, \phi = 0, \mu = \frac{E^r D^s}{P_y} = 0 \))

substitute for \( \hat{e}, \hat{x}, \hat{z}, \hat{n} \) from above in

\[
\hat{y} = \rho \hat{e} + \left[ \lambda + \frac{\theta_n}{1 - \theta_n} \right] \left\{ \frac{\hat{x}}{1 + \omega} - (1 - \beta) \hat{z} - \beta \left( \hat{E} - \hat{P} + \hat{n} \right) \right\}
\]

\[
\lambda = \frac{z}{\gamma} \cdot \frac{\theta_n}{1 - \theta_n} = \frac{En}{P_y}
\]

\[
\hat{y} = -a_1 \hat{P} + a_2 \hat{E} + a_3 \hat{W}
\]

so if \( \theta = 1, \hat{W} = 0, \hat{P} = \theta_n \hat{E} \)

\[
\hat{y} = (a_2 - \theta_n a_1) \hat{E}
\]

where \( a_2 = \frac{1}{\Delta} \left[ \left( \lambda + \frac{\theta_n}{1 - \theta_n} \right) \Omega - \rho \psi (1 - \Gamma (1 - \beta)) \right] > 0, \)

\[
\Omega = \frac{\eta}{1 + \omega} + (1 - \beta)(\delta - \psi) - \beta \text{ [Marshall-Lerner } \eta + \delta] \]

\( \Delta = 1 - \alpha + \alpha \lambda \text{ [s+m]} \)
Effect of devaluation on trade balance (y'-e) or current account (y-e)

\[ db = dy - de = y(\hat{y} - \rho \hat{e}) \]

\[ \frac{db}{y} = \hat{y} - \rho \hat{e} = (1 - \alpha \rho) \hat{y} + \rho (1 - \alpha - \psi) \hat{P} + \rho \left[ (1 - \alpha) \phi + \psi \right] \hat{E} \]

\[ \frac{db}{y} = (1 - \alpha \rho) \frac{\hat{y}}{\hat{E}} + \rho (1 - \alpha) \frac{\hat{P}}{\hat{E}} + \rho \psi \left( 1 - \frac{\hat{P}}{\hat{E}} \right) + \rho (1 - \alpha) \phi > 0 \]

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<th>wealth effect</th>
<th>terms of trade effect</th>
<th>debt effect</th>
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Overall effect positive if \( \hat{y} > 0 \) and \( 0 < \frac{\hat{P}}{\hat{E}} < 1 \) terms of trade worsen

Empirical estimates show \( \hat{y} < 0 \) often but other effects make \( db > 0 \)

Increase in wages shifts up AS curve, more likely that \( y \) falls and total improves.

Example Brazil:

\[ \sigma = .3, \eta = .4, \delta = 1.7, \alpha = .7, \theta_n = .05, \theta_i = .39, \theta = .44, \]
\[ \lambda = .03, \beta = .42, \mu = .04, \varepsilon = .33, \phi = .10, \omega = .50 \]
\[ \hat{E} = 10\% \rightarrow \hat{y} = -1.6\% \]

income effect: \( (1 - \alpha \rho) \frac{\hat{y}}{\hat{E}} = -0.4 \), wealth effect: \( \rho (1 - \alpha) \frac{\hat{P}}{\hat{E}} = 0 \),

terms of trade effect: \( \rho \psi \left( 1 - \frac{\hat{P}}{\hat{E}} \right) = 0.3 \), debt effect: \( \rho (1 - \alpha) \phi = 0.3 \),

total \( \frac{db}{y} = 0.2\% \)
Intertemporal Model – Obstfeld & Rogoff

Two regions, two periods, one good, one bond

\[ Y_t + Y_t^* = C_t + C_t^*. \quad S_t + S_t^* = 0. \quad S_1 + S_1^* = 0. \]

Slope of Savings Schedule: Maximize

\[ U_i^j = u(c_i^j) + \beta u(c_2^j), \quad c_i^j + \frac{c_2^j}{1 + r} = y_i^j + \frac{y_2^j}{1 + r}. \]

subject to

yields first order condition

\[ \frac{\beta u'(c_2^j)}{u'(c_1^j)} = \frac{1}{1 + r}. \]

Take logs and differentiate to get

\[
\begin{align*}
\text{d} \log (1 + r) &= \frac{u''(C_1)}{u'(C_1)} \text{d} C_1 - \frac{u''(C_2)}{u'(C_2)} \text{d} C_2 \\
&= \frac{C_1 u''(C_1)}{u'(C_1)} \text{d} \log C_1 - \frac{C_2 u''(C_2)}{u'(C_2)} \text{d} \log C_2. \\
\end{align*}
\]

since

\[ \sigma (C) = -\frac{u'(C)}{C u''(C)}. \]

we get

\[ \text{d} \log \left( \frac{C_2}{C_1} \right) = \sigma \text{d} \log (1 + r). \]

so that curvature of indifference curve depends on elasticity of intertemporal substitution. Slope of savings function from response of \( C_1 \) to \( r \). Differentiate
\begin{align*}
    u'(C_1) &= (1 + r) \beta u' \left[ (1 + r)(Y_1 - C_1) + Y_2 \right]. \quad \text{to get} \\
    \frac{dC_1}{dr} &= \frac{\beta u'(C_2) + \beta (1 + r)u''(C_2)(Y_1 - C_1)}{u''(C_1) + \beta (1 + r)^2u''(C_2)}.
\end{align*}

If \( \sigma > 0 \), then

\[ u(C) = \frac{C^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}, \quad \sigma > 0. \]

so from

\[ \left[ u''(C_1) + (1 + r)^2 \beta u''(C_2) \right] dC_1 = \left[ \beta u'(C_2) + \beta (1 + r)u''(C_2)(Y_1 - C_1) \right] dr \]

divide both sides by \(-u'(C_2)/C_2\) and use \( \sigma \) to get

\[ \left[ -\frac{C_2u''(C_1)}{u'(C_2)} + \frac{(1 + r)^2 \beta}{\sigma} \right] dC_1 = \left[ -\beta C_2 + \frac{\beta (1 + r)}{\sigma} (Y_1 - C_1) \right] dr \]

Since \( \frac{C_2u''(C_1)}{u'(C_2)} = \frac{C_2}{C_1} \frac{C_1u''(C_1)}{u'(C_1)} (1 + r) \beta = -\frac{C_2}{C_1} \frac{(1 + r) \beta}{\sigma} \)

we find

\[ \frac{dC_1}{dr} = \frac{(1 + r)(Y_1 - C_1) - \sigma C_2}{(1 + r)^2 + (1 + r) \frac{C_2}{C_1}} = \frac{Y_1 - C_1 - \sigma C_2 / (1 + r)}{(1 + r) + C_2 / C_1} > \text{ or } < 0 \]

The substitution effect reduces consumption and raises saving, while the income or terms of trade effect \( Y_1 - C_1 \) is positive if saving is positive, negative if it is negative.

Given isoelastic utility,

\[ u(C), \ u'(C) = c^{-1/\sigma}, \quad \text{so Euler condition is} \]

\[ C_1^{-1/\sigma} = (1 + r) \beta C_2^{-1/\sigma}, \quad \text{from budget constraint} \]

\[ C_1 = \frac{1}{1 + (1 + r)^{\sigma-1}} \beta^\sigma \left( Y_1 + \frac{Y_2}{1 + r} \right) \]
Substitution effect > or < income effect as $\sigma$ > or < 1.

Metzler Diagram for Investment and Savings

$Y = AF(K), Y^* = A^*F^*(K^*)$

$I$ from $A_2F'(K_1 + I_1) = r$

$I^*$ from $A_2^*F^*(K_1^* + I_1^*) = r$

$S^*, I^*$
Increase in $\beta$ shifts $S$ left, raises $r$; $I$ and $I^*$ fall.

Increase current productivity of capital $A_1$, shifts $S$ right, $I$ and $I^*$ rise.

Increase future productivity of capital $A_2$, shifts $I$ right, shifts $S$ left.

Suppose initial CA=0, then new CA<0 so CA*>0. Effect on world investment $I+I^*$ depends on relative shift of $I$ and $S$. From $A_2F'=r$,

vertical shift in $I+I^*$ is

$dr / r = \hat{A}_2$, so $dr = rC$ O&R show $S+S^*$ shifts up by $dr = \frac{1+r}{1+\alpha/r} \hat{A}_2 > r\hat{A}_2$

since $\alpha<1$. Net effect on $I+I^*$ is negative.
Multiperiod Model (O&R, Ch. 2)

\[ U_t = \sum_{s=t}^{t+T} \beta^{s-t} u(C_s) \]

subject to

\[ CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t - G_t - I_t \quad \text{or} \]

\[ (1 + r)B_t = C_t + G_t + I_t - Y_t + B_{t+1} \]

iterating,

\[ (1 + r)B_t = C_t + G_t + I_t - Y_t \]

\[ + \frac{C_{t+1} + G_{t+1} + I_{t+1} - Y_{t+1}}{1 + r} \]

\[ + \frac{B_{t+2}}{1 + r} \]

so

\[ \sum_{s=t}^{t+T} \left( \frac{1}{1 + r} \right)^{s-t} (C_s + I_s) + \left( \frac{1}{1 + r} \right)^T B_{t+T+1} \]

\[ = (1 + r)B_t + \sum_{s=t}^{t+T} \left( \frac{1}{1 + r} \right)^{s-t} (Y_s - G_s). \]

First order conditions:

\[ A_{s+1} F'(K_{s+1}) = r. \]

Terminal

\[ B_{t+T+1} = 0 \]

Infinite Horizon as \( T \to \infty \)
\[
\lim_{T \to \infty} \left( \frac{1}{1 + r} \right)^T B_{t+1} = 0
\]

From

\[
TB_s = Y_s - C_s - G_s - I_s \quad \text{so} \quad (1 + r)B_t = -\sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} TB_s
\]

Suppose steady growth in output \( Y_{s+1} = (1 + g)Y_s \) if \( B/Y \) is constant \( B_{s+1} = (1 + g)B_s \)

So \( B_{s+1} - B_s = gB_s = rB_s + TB_s \) and therefore

\[
\frac{TB_s}{Y_s} = -\frac{(r - g)B_s}{Y_s}
\]

is consistent with a stable debt/GDP ratio. Higher surplus if \( r > g \).

Permanent level of \( X_t \) such that

\[
\sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} X_s = \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} \tilde{X}_s
\]

or

\[
\tilde{X}_t = \frac{r}{1 + r} \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} X_s
\]

If \( \beta(1+r) = 1, C_s \) is constant. O&R show that \( C_t = rB_t + (\tilde{Y}_t - \tilde{G}_t - \tilde{I}_t) \) and so

\[CA_t = B_{t+1} - B_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) \]

\( B_t \) acts as a buffer stock.

If \( \beta(1+r) \neq 1 \) but utility is isoelastic, O&R show that

\[
C_t = \frac{r + \vartheta}{1 + r} \left[ (1 + r)B_t + \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} (Y_s - G_s - I_s) \right]
\]

where \( \vartheta = 1 - (1+r)^\sigma \beta^\sigma \)

So if \( \beta > \frac{1}{1 + r} \) \( C_t \) is increasing over time (decreasing if \( \beta < \frac{1}{1 + r} \)).
Stochastic Model (2.3)

\[ U_t = \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\} \]

with same budget constraint, first order condition is

\[ \mathbb{E}_t \left\{ u'(C_s) \right\} = (1 + r) \beta \mathbb{E}_t \left\{ u'(C_{s+1}) \right\} \]

Quadratic utility \( u(C) = C - \frac{a_0}{2} C^2 \), \( a_0 > 0 \).

If \( \beta(1+r) = 1 \), then \( u'(C) = 1 - a_0 C \).

So

\[ C_t = \frac{r}{1 + r} \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} \mathbb{E}_t \left\{ Y_s - G_s - I_s \right\} \]

Factors affecting the Current Account (O&R, 2.3.5)

Net Output \( Z \equiv Y - G - I \) gives \( CA_t = Z_t - E_t \tilde{Z}_t \)

Where

\[ E_t \tilde{Z}_t = \frac{r}{1 + r} \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} E_t Z_s = \frac{r}{1 + r} \left( \frac{1}{1 - \frac{1}{1 + r} L^{-1}} \right) E_t Z_s \]

\[ = \frac{r}{1 + r} \left[ 1 + \frac{r}{1 + r} L^{-1} + \frac{r}{(1+ r)^2} L^{-2} + \cdots \right] E_t Z_s \]

\[ \therefore Z_t - E_t \tilde{Z}_t = E_t \left[ 1 - \frac{r}{1 + r} \left( 1 - \frac{1}{1 + r} L^{-1} \right)^{-1} \right] Z_t = E_t \frac{\frac{1}{1 + r} \left( 1 - L^{-1} \right)}{1 - \frac{1}{1 + r} L^{-1}} Z_t \]

since \( 1 - \frac{r}{1 + r} L^{-1} = \frac{1 - \frac{r}{1 + r} L^{-1} - \frac{1}{1 + r} L^{-1}}{1 - \frac{1}{1 + r} L^{-1}} = \frac{\frac{1}{1 + r} \left( 1 - L^{-1} \right)}{1 - \frac{1}{1 + r} L^{-1}} \) so

\[ Z_t - E_t \tilde{Z}_t = -\frac{1}{1 + r} E_t \frac{\Delta Z_{t+1}}{1 - \frac{1}{1 + r} L^{-1}} \]
\[ CA_t = - \sum_{s=t+1}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} E_t \Delta Z_s \]  

Current account depends on future changes in output minus absorption \( Z \).

Forecast output changes from

\[
\begin{bmatrix}
\Delta Z_s \\
CA_s
\end{bmatrix} =
\begin{bmatrix}
\psi_{11} & \psi_{12} \\
\psi_{21} & \psi_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta Z_{s-1} \\
CA_{s-1}
\end{bmatrix} +
\begin{bmatrix}
\epsilon_{1s} \\
\epsilon_{2s}
\end{bmatrix}
\]  

(44)

So

\[
E_t \begin{bmatrix}
\Delta Z_s \\
CA_s
\end{bmatrix} =
\begin{bmatrix}
\psi_{11} & \psi_{12} \\
\psi_{21} & \psi_{22}
\end{bmatrix}^{s-t} \begin{bmatrix}
\Delta Z_t \\
CA_t
\end{bmatrix}
\]

Premultiply by vector \( \begin{bmatrix} 0 & 1 \end{bmatrix} \) to get

\[
E_t \Delta Z_s =
\begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
\psi_{11} & \psi_{12} \\
\psi_{21} & \psi_{22}
\end{bmatrix}^{s-t} \begin{bmatrix}
\Delta Z_t \\
CA_t
\end{bmatrix}
\]

Then the predicted current account is

\[
\hat{CA}_t = -\begin{bmatrix}
1 & 0
\end{bmatrix} \left( \frac{1}{1 + r} \psi \right) \left( I - \frac{1}{1 + r} \psi \right)^{-1} \begin{bmatrix}
\Delta Z_t \\
CA_t
\end{bmatrix}
\]

\[
\equiv \begin{bmatrix}
\Phi_{\Delta Z} & \Phi_{CA}
\end{bmatrix} \begin{bmatrix}
\Delta Z_t \\
CA_t
\end{bmatrix}
\]  

(45)

For example, for Belgium,

\[
\begin{bmatrix}
\Delta Z_t \\
CA_t
\end{bmatrix} =
\begin{bmatrix}
0.20 & -0.09 \\
-0.03 & 0.83
\end{bmatrix}
\begin{bmatrix}
\Delta Z_{t-1} \\
CA_{t-1}
\end{bmatrix} +
\begin{bmatrix}
\epsilon_{1t} \\
\epsilon_{2t}
\end{bmatrix}
\]

and the hypothesis \( \begin{bmatrix}
\hat{\Phi}_{\Delta Z} & \hat{\Phi}_{CA}
\end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix} \) is rejected.
To include additional factors such as changes in the real interest rate, the terms of trade, or the real exchange rate, one must include additional goods (importable and non-traded). See Bergin & Sheffrin (2000) and Goodger.

For example, Goodger derives

$$CA_t = -E_t \left[ \sum_{s=1}^{\infty} \rho^s \left( \Delta Z_{t+s} - \gamma r_{t+s} - a(1-\gamma)\Delta q_{t+s} - (1-a-b)(1-\gamma)\Delta p_{t+s} \right) \right]$$

Where $q$ is the price of non-traded goods and $p$ is the price of importables. Unfortunately, these generalizations do not appear to add a lot of explanatory power. More successfully, Ventura (2002) adds the concepts of risk premia and adjustment costs for investment.

Open Economy RBC Models

Razin

Investment behavior

$$Y_t = A_t K_t^\alpha, A_t = \rho A_{t-1}, Z_t = I_t (1 + \frac{\rho}{2} \frac{K_t}{K_t})$$

$$I_t = \lambda I_{t-1} + (b \frac{\Delta K_t}{K_t-1}) \Delta A_t$$ if $\rho = 1$

or $\Delta I_t = (\lambda - 1) \Delta I_{t-1} + (b \frac{\Delta K_t}{K_t-1}) \Delta A_t$

if $\rho = 0, \Delta I_t = (\lambda - 1) \Delta I_{t-1}$

so positive correlation of $\Delta I$ with country-specific $\Delta A$ if and only if $\rho > 0$ (productivity shocks are persistent, not transitory). Global productivity shock will raise world interest rate, have less effect on investment.
Consumption behavior

Max $E_t \sum_{i=1}^{\infty} \delta^i u(C_{t+i})$, $u(C) = hC - \frac{1}{2} C^2$
subject to $C_t + F_t = Y_t + RF_{t-1}$, F=foreign assets
if $\delta = 1/R$, $C_t = (\frac{R}{R-1}) W_t$, $W_t = E_t \sum_{i=0}^{\infty} R^{-i} Y_{t+i} + RF_{t-1}$

Conclusions: if $\rho = 1$, effect of $\Delta A$ on $\Delta C$ is larger than effect on $\Delta Y$ because increase in permanent income $W$ is larger than increase in current income $Y$. But not in case $\rho = 0$. Current account affected negatively by permanent productivity shock, positively by transitory productivity shock (since output rises, but not $C$ or $I$).

Empirical evidence: high levels of persistence in $Y, C, I$. Common factors explain significant fraction (20%-40%) of cross-country variation in $Y, C, I$. Volatility of current account and output low for industrial countries, NIC’s. cor(TOT,TB) $> 0$ for most countries.

Two-Country RBC Model (Backus, Kehoe, Kydland, AER 1994)

2 countries, 2 goods, consumers maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{it}, 1 - n_{it})$$

$U(c, 1 - n) = [c^{\mu}(1 - n)^{1-\mu}]^{\gamma} / \gamma,$

each country produces its own good, Home $a$ Foreign $b$, using $k$ and $n$.

$a_{1t} + a_{2t} = y_{1t} = z_{1t} F(k_{1t}, n_{1t})$
$b_{1t} + b_{2t} = y_{2t} = z_{2t} F(k_{2t}, n_{2t})$ with $F(k, n) = k^\theta n^{1-\theta}$

consumption is a composite commodity $G(a, b) = [\omega_1 a^{-\rho} + \omega_2 b^{-\rho}]^{-1/\rho}$
\[ c_{1t} + x_{1t} + g_{1t} = G(a_{1t}, b_{1t}) \]
\[ c_{2t} + x_{2t} + g_{2t} = G(b_{2t}, a_{2t}) \]

Investment has a one-period \((J=1)\) lag in \(k_{i,t+1} = (1 - \delta)k_{i,t} + s_{i,t-J+1}\)

Shocks \(z_t, g_t\) are AR(1). Homogeneity of \(G(a,b)\) implies that

\[ c_{1t} + x_{1t} + g_{1t} = q_{1t}a_{1t} + q_{2t}b_{1t} \]

where \(q_{1t}, q_{2t}\) are prices, so output

\[ y_{1t} = a_{1t} + a_{2t} \]

is

\[ y_{1t} = (c_{1t} + x_{1t} + g_{1t}) / q_{1t} + (a_{2t} - p_{t}b_{1t}) \]

= absorption + net exports. \(p_{t} = q_{2t} / q_{1t}\) is the terms of trade.

Define \(nx_t = (a_{2t} - p_{t}b_{1t}) / y_{1t}\) as the net export ratio to GDP.

Calibration: \(\beta = .99, \mu = .34, \gamma = -1, \theta = .36, \delta = .025, \sigma = \frac{1}{1 + \rho} = 1.5\)

Import shares = .15, autocorrelation of shocks = .91. Results:

<table>
<thead>
<tr>
<th>Economy</th>
<th>Standard deviation (percent)</th>
<th>Autocorrelation</th>
<th>Correlation (nx, y)</th>
<th>Correlation (nx, p)</th>
<th>Correlation (y, p)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nx</td>
<td>y</td>
<td>p</td>
<td>nx</td>
<td>y</td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.30</td>
<td>1.38</td>
<td>0.48</td>
<td>0.61</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.18)</td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Median</td>
<td>1.06</td>
<td>1.53</td>
<td>2.92</td>
<td>0.71</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Absorption varies more than output, because of investment, \(cov(nx, y) < 0\).

Since \(cov(y, p) > 0\), this implies \(cov(nx, p) < 0\).
Observe J-curve, regardless of $\sigma$. In this model Marshall-Lerner condition always satisfied, but $\text{cov}(nx,p) > 0$ as $\sigma > \sigma^*$.
Feldstein-Horioka Puzzle (EJ, 1980)

\[
\frac{Se}{GNP} = \text{external saving rate; } \frac{Sd}{GNP} = \text{domestic saving rate; } \frac{I}{GNP} = \text{investment rate}
\]

(3.1) \[ \frac{I}{GNP} = \alpha + \beta \frac{Sd}{GNP} \]  \quad \text{Feldstein-Horioka equation}

(3.2) \[ \frac{I}{GNP} = \frac{Sd}{GNP} + \frac{Se}{GNP} \]  \quad \text{(identity)}

Replacing equation (3.1) into the identity:

(3.3) \[ \alpha + \beta \frac{Sd}{GNP} = \frac{Sd}{GNP} + \frac{Se}{GNP} \]

Reordering the terms:

(3.4) \[ \frac{Se}{GNP} = \alpha + (\beta - 1) \frac{Sd}{GNP} \]  \quad \text{(substitutability equation)}

or,

(3.5) \[ \frac{Se}{GNP} = \alpha + \gamma \frac{Sd}{GNP} \]  \quad \text{(substitutability equation, where } \gamma = \beta - 1 \).

Feldstein and Horioka estimate (3.1) across 15 industrial countries 1960-74, find \( \beta = .88 (.07) \), not significantly different from unity. In first difference form, \( \beta = 1.04 \). Instrumenting for simultaneity bias with dependency ratios makes no difference. Feldstein & Bachetta estimate

<table>
<thead>
<tr>
<th>60’s</th>
<th>70’s</th>
<th>80’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>.91</td>
<td>.86</td>
<td>.79</td>
</tr>
</tbody>
</table>

Implication: capital mobility low in 60’s, increasing over time. But Dooley et al (IMFSP, 1987) estimate \( \beta \) lower for developing countries than for industrial countries.

O&R (3.4) show that for OLG economy with population growth \( n \) and technical progress \( g \), \[ \frac{\bar{T}}{Y} = \frac{\alpha}{r} (n + g + ng) \], \[ \frac{\bar{S}}{Y} \approx \frac{\beta(1-\alpha)}{1+\beta} (n + g + ng) \]

Sachsida and Caetano argue that \( \beta = 1 \) implies only that \( \gamma = 0 \) in the substitutability equation.
Kraay and Ventura (QJE, 2000) argue that F&H results come from costs of adjustment and risk premia. Simple case is foreign loans $x < 1$ with return $\rho$ and domestic capital with return equal to flow of production less “operating costs” related to investment. Flow of production is

$$\pi dt + \sigma d\omega$$

where $\omega$ is a Wiener process

$$E[d\omega] = 0, E[d\omega]^2 = dt$$

Operating costs are

$$\alpha = \lambda \frac{1}{k} \frac{dk}{dt}$$

$\lambda$, effectively a negative externality.

Consumers maximize

$$E \int_0^\infty \ln(c) e^{-\delta t} dt$$

subject to

$$da = [\left(\pi - \alpha\right)(1-x) + \rho x]a - c]dt + (1-x)a\sigma d\omega$$

$$c^* = \delta a, x^* = 1 - \max \left\{ \frac{\pi - \alpha - \rho}{\sigma^2}, 0 \right\}$$

$$(1-x)a = k + dk$$

demand = supply of capital

Interior Solution

$$\frac{k}{a} = (1-x) = \frac{\pi - \alpha - \rho}{\sigma^2}$$

implies

$$\sigma \left( \frac{k}{a} \right) = \frac{\pi - \alpha - \rho}{\sigma} \quad \text{and} \quad \sigma^2 \left( \frac{k}{a} \right)^2 = \left( \frac{\pi - \alpha - \rho}{\sigma^2} \right)^2$$

$$\left( \pi - \alpha \right) \left( \frac{\pi - \alpha - \rho}{\sigma^2} \right) + \rho \left( 1 - \frac{\pi - \alpha - \rho}{\sigma^2} \right) = \left( \frac{\pi - \alpha - \rho}{\sigma^2} \right)^2 + \rho$$
\[
\frac{da}{a} = \left[ \frac{(\pi - \alpha - \rho)^2}{\sigma^2} + \rho - \delta \right] \cdot dt + \frac{\pi - \alpha - \rho}{\sigma} d\omega
\]

\[
\lambda \frac{1}{k} \frac{dk}{dt} = \alpha = \pi - \rho - \sigma^2 \left( \frac{k}{a} \right) \text{ or solution}
\]

\[
\frac{dk}{k} = \lambda^{-1} \left( \pi - \rho - \sigma^2 \frac{k}{a} \right) dt
\]

\[
\frac{da}{a} = \left[ \sigma^2 \left( \frac{k}{a} \right)^2 + \rho - \delta \right] dt + \frac{k}{\sigma} d\omega
\]

let \( S = \frac{1}{a} \cdot \frac{da}{dt}, CA = \frac{1}{a} \cdot \frac{d(x \cdot a)}{dt} \) then \( CA = x \cdot S + \frac{dx}{dt} \) is a rule that allocates part of saving to foreign assets in proportion to the existing share \( x \) and requires another component due to portfolio rebalancing.
The standard intertemporal model says that $K/L$ ratio depends only on technology and world interest rate $r$, independently of wealth. Excess wealth is invested in foreign assets. Windfall effects only affect saving in foreign assets: $CA = \Delta F = \Delta W = S$. Test via regression

$CA_t = \alpha + \beta S_t + u_t$. Should find $\beta = 1$, but Ventura & Kraay find $\beta = 0.214$ for 21 industrial countries 1966-97. This is equivalent to F&H.

Figure 3: Choosing the country portfolio to maximize return
Figure 5: Savings and the Current Account

- **Pooled Regression**
  - Equation: $y = 0.2141x - 0.0582$
  - $R^2 = 0.1169$

- **Between Regression**
  - Equation: $y = 0.2212x - 0.0695$
  - $R^2 = 0.194$

- **Within Regression**
  - Equation: $y = 0.2077x + 6.7E-10$
  - $R^2 = 0.0695$
But Kraay and Ventura’s rule applies if there is a risk premium on investment and if diminishing returns on investment are low.

Now changes in wealth lead to changes in $K$ to keep $K/W$ constant (points A & C). Test via $CA_{it} = \alpha + \beta X_{it} S_{it} + u_{it}$ Result $\beta = .939$. 
Figure 10: The New Rule and the Current Account

Pooled Regression

\[ y = 0.9367x - 0.0023 \]
\[ R^2 = 0.3015 \]

Between Regression

\[ y = 1.05x - 0.0011 \]
\[ R^2 = 0.8164 \]

Within Regression

\[ y = 0.4636x - 0.0003 \]
\[ R^2 = 0.0261 \]
Exchange Rate Dynamics

Monetary Approach – Frenkel, Bilson, Mussa, Barro

Assume flexible prices, full employment

\[ m - p = \phi y - \lambda r \]
\[ m^* - p^* = \phi y^* - \lambda r^* \]
\[ r - r^* = E \hat{e} = \pi - \pi^* \quad \text{UIP & PPP in short run} \]
\[ e = p - p^* = m - m^* - \phi (y - y^*) + \lambda (\pi - \pi^*) \]

Forward solution (see Frenkel & Mussa, Handbook, Vol. 2, Ch. 14, sec. 4.2)

\[ m_t - p_t = -\lambda r_{t+1} + \phi y_t, \quad m^*_t - p^*_t = -\lambda r^*_{t+1} + \phi y^*_t \]
\[ p_t = e_t + p^*_t \quad \text{PPP} \]
\[ r_{t+1} = r^*_{t+1} + E_t e_{t+1} - e_t \quad \text{UIP} \]
\[ e_t = p_t - p^*_t = m_t - m^*_t - \phi (y_t - y^*_t) + \lambda (r_{t+1} - r^*_{t+1}) \]
\[ e_t = (m_t - m^*_t) - \phi (y_t - y^*_t) + \lambda (E_t e_{t+1} - e_t) \]
\[ e_t = f_t + \lambda (E_t e_{t+1} - e_t) \]
\[ e_t = \frac{1}{1 + \lambda} f_t + \frac{\lambda}{1 + \lambda} E_t e_{t+1} = \frac{1}{1 + \lambda} f_t + \frac{\lambda}{1 + \lambda} E_t \left( \frac{1}{1 + \lambda} f_{t+1} + \frac{\lambda}{1 + \lambda} E_{t+1} e_{t+2} \right) = ... \]
\[ e_t = \frac{1}{1 + \lambda} \sum_{s=0}^{\infty} \left( \frac{\lambda}{1 + \lambda} \right)^s E_t f_{t+s} \]
Sticky Price Model (Dornbusch – Frankel)

\( m - p = \phi y - \lambda r \)

\( r - r^* = E\hat{e} \) Rational Expectations and UIP

\( \dot{p} = \pi \ln(D/Y) = \pi[\delta(e - p) + \gamma y - \sigma r - \bar{y}] \)

in long run, \( r = r^*, \bar{p} = m + \lambda r^* - \phi y, m = \bar{p} = \bar{e}, \bar{r} = r^* \)

in short run, \( E\hat{e} = \frac{1}{\lambda}(p - \bar{p}) \) because \( p - \bar{p} = \lambda(r - r^*) \)

\( \dot{p} = \pi[\delta(e - \bar{e}) - \delta(p - \bar{p}) - \sigma(r - r^*)] \)

\( \dot{p} = \pi[\delta(e - \bar{e}) - (\delta + \frac{\sigma}{\lambda})(p - \bar{p})] \)

Rational Expectations solution:

\( E\hat{e} = \theta(\bar{e} - e) \) so \( \theta(\bar{e} - e) = \frac{1}{\lambda}(p - \bar{p}) \)

\( \dot{p} = \theta(\bar{p} - p) = \pi[\delta(e - \bar{e}) - (\delta + \frac{\sigma}{\lambda})(p - \bar{p})] \)

\[ \dot{p} = \pi \left[ \frac{\delta}{\lambda \theta} + \delta + \frac{\sigma}{\lambda} \right](\bar{p} - p) \]

\[ \theta = \pi \left[ \frac{\delta + \sigma \theta}{\lambda \theta} + \delta \right] \text{ or } \theta^2 = \left( \frac{\pi \sigma}{\lambda} + \pi \delta \right) \theta + \frac{\pi \delta}{\lambda} \]

Overshooting

\[ \frac{de}{dm} = \frac{d\bar{e}}{dm} + \frac{1}{\lambda \theta} \frac{d\bar{p}}{dm} = 1 + \frac{1}{\lambda \theta} > 1 \text{ from } e - \bar{e} = \frac{1}{\lambda \theta}(\bar{p} - p) \]

Notice output is still assumed to be at full employment, but demand can differ from supply \((D < Y)\).
**Buiter-Miller model (EER, 1982)**

\[ m - p = ky - \lambda r \quad \text{LM} \]
\[ y = -\gamma(r - Dp) + \delta(e - p) \quad \text{IS} \]
\[ Dp = \phi y + \pi \quad \text{Phillips curve} \]
\[ \pi = Dm \quad \text{Expected inflation} \quad \text{[alternatives} \quad \pi = \xi(p - \pi) \text{or} \quad \pi = \dot{p}] \]
\[ De = r - r^* \quad \text{UIP} \]
\[ \ell = m - p \quad \text{real balances} \]
\[ c = e - p \quad \text{real exchange rate} \]

**Solution**

\[ r = \frac{k}{\lambda} y - \frac{1}{\lambda} \ell \quad \text{substitute in IS} \]

\[ y = -\gamma \left[ \frac{k}{\lambda} y - \frac{1}{\lambda} \ell - \phi y - Dm \right] + \delta c \]
\[ \left[ 1 + \frac{\gamma}{\lambda} - \gamma \phi \right] y = \frac{\ell}{\lambda} + \gamma Dm + \delta c \]
\[ y = \frac{\gamma}{\lambda + \gamma(k - \lambda \phi)} \ell + \frac{\gamma \lambda}{\lambda + \gamma(k - \lambda \phi)} Dm + \frac{\delta \lambda}{\lambda + \gamma(k - \lambda \phi)} c \]

\[ D\ell = Dm - Dp = -\phi y = \frac{-1}{\lambda + \gamma(k - \lambda \phi)} \left[ \gamma \lambda \ell + \gamma \lambda \phi Dm + \delta \lambda c \right] \]

\[ Dc = De - Dp = r - r^* - \phi y - Dm = \frac{-1}{\lambda + \gamma(k - \lambda \phi)} \left[ \ell - \delta(k - \lambda \phi) c + \lambda Dm \right] - r^* \]

\[ \begin{bmatrix} D\ell \\ Dc \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \phi \gamma & \delta \lambda \phi \\ 1 & -\delta(k - \phi \lambda) \end{bmatrix} \begin{bmatrix} \ell \\ c \end{bmatrix} + \frac{1}{\Delta} \begin{bmatrix} \gamma \lambda \phi & 0 \\ \lambda & \lambda + \gamma(k - \lambda \phi) \end{bmatrix} \begin{bmatrix} Dm \\ r^* \end{bmatrix} \]

\[ \Delta = \gamma(\lambda \phi - k) - \lambda < 0 \]

\[ |A| = \frac{-\phi \gamma \delta(k - \phi \lambda) - \phi \delta \lambda}{\Delta^2} = \frac{\phi \delta \Delta}{\Delta} = \frac{\phi \delta}{\Delta} < 0 \quad \text{if} \quad \Delta < 0 \]
\[ D\ell = 0 \Rightarrow \ell + \frac{\delta\lambda}{\gamma} c + \lambda Dm = 0 \Rightarrow dc / d\ell = -\gamma / \delta\lambda < 0 \]
\[ Dc = 0 \Rightarrow \ell - \delta(k - \lambda\phi)c + \lambda Dm + [\lambda + \gamma(k - \lambda\phi)]r^* = 0 \Rightarrow dc / d\ell = 1 / \delta(k - \lambda\phi) > 0 \]
\[
\begin{bmatrix}
\hat{c} \\
\hat{\ell}
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{\gamma}{\delta} \\
-\lambda & -\lambda
\end{bmatrix}
\begin{bmatrix}
Dm \\
r^*
\end{bmatrix}
\]

Suppose a fall in money growth \((d\mu < 0)\). Then \(d\hat{\ell} = -\lambda d\mu > 0\).
Calculating the initial jump in the real exchange rate $c$:

\[
\begin{bmatrix}
D\ell' \\
Dc'
\end{bmatrix} = A \begin{bmatrix}
\ell' \\
c'
\end{bmatrix} = B\Lambda B^{-1} \begin{bmatrix}
\ell' \\
c'
\end{bmatrix},
\]

$B = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}$, $AB = B\Lambda$

Define canonical variables

\[
\begin{bmatrix}
s \\
u
\end{bmatrix} = B^{-1} \begin{bmatrix}
\ell' \\
c'
\end{bmatrix}, \quad \text{so} \quad \begin{bmatrix}
Ds \\
Du
\end{bmatrix} = \begin{bmatrix}
\Lambda_1 & 0 \\
0 & \Lambda_2
\end{bmatrix} \begin{bmatrix}
s \\
u
\end{bmatrix}
\]

choose $c'(0)$ so that $u(0) = 0$ on saddle path

\[
\begin{bmatrix}
\ell' \\
c'
\end{bmatrix} = B \begin{bmatrix}
s \\
u
\end{bmatrix} = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix} \begin{bmatrix}
s \\
u
\end{bmatrix}
\]

$\ell'(0) = B_{11}s(0) + B_{12}u(0) = B_{11}s(0)$

$c'(0) = B_{21}s(0) + B_{22}u(0) = B_{21}s(0)$

so $c'(0) = B_{21}B_{11}^{-1}\ell'(0)$

Suppose Rational Expectations for prices as well as exchange rate:

\[
\pi = Dp \quad \text{or} \quad \phi \to \infty \quad \text{then} \quad y = 0 \quad \text{so from IS curve}
\]

\[
r - Dp = \frac{\delta}{\gamma} (e - p)
\]

\[
De = r - r^* = \frac{\delta}{\gamma} (e - p) + Dp - r^*
\]

\[
Dc \equiv De - Dp = \frac{\delta}{\gamma} c - r^* \quad \text{with a positive root}
\]

\[
r = -\frac{1}{\lambda} \ell \quad \text{so} \quad D\ell \equiv Dm - Dp = Dm + r - Dp - r
\]

\[
D\ell = Dm + \frac{\delta}{\gamma} (e - p) + \frac{1}{\lambda} \ell = Dm + \frac{\delta}{\gamma} c + \frac{1}{\lambda} \ell
\]

both roots positive. System totally unstable, but jumps to new equilibrium.
Portfolio Model (Black & Salemi, *JIE* 1988)

\[ f = \alpha (r^* + De - r) \] portfolio allocation

\[ Df = \eta (e + p^* - p) \] current account

\[ \hat{f} = \alpha (r^* - r) \]

\[ \hat{e} = p - p^* \]

\[
\begin{bmatrix}
Df \\
De
\end{bmatrix} = \begin{bmatrix}
0 & \eta \\
\alpha^{-1} & 0
\end{bmatrix} \begin{bmatrix}
f - \hat{f} \\
e - \hat{e}
\end{bmatrix}
\]

\[
|A - \lambda I| = \begin{vmatrix}
-\lambda & \eta \\
\alpha^{-1} & -\lambda
\end{vmatrix} = \lambda^2 - \alpha^{-1}\eta = 0
\]

\[ \lambda = \pm \sqrt{\alpha^{-1}\eta},
B = \begin{bmatrix}
\frac{1}{2} & \frac{1}{2} \\
-\frac{1}{2\sqrt{\alpha\eta}} & \frac{1}{2\sqrt{\alpha\eta}}
\end{bmatrix} = \begin{bmatrix}
1 & -\sqrt{\alpha\eta} \end{bmatrix}^{-1}
\]

\[ BAB^{-1} = \begin{bmatrix}
-\sqrt{\alpha^{-1}\eta} & 0 \\
0 & \sqrt{\alpha^{-1}\eta}
\end{bmatrix}
\]

Saddle path: \((e_0 - \hat{e}) = \left(-\frac{1}{2\sqrt{\alpha\eta}}\right)\left(\frac{1}{2}\right)^{-1} \left(f_0 - \hat{f}\right)\)

\[
\frac{e_0 - \hat{e}}{f_0 - \hat{f}} = -\frac{1}{\sqrt{\alpha\eta}}
\]

Assume \(df_0 < 0\)

---

**Phase Diagram**

- **Red Line**: \(DF = 0\)
- **Dotted Blue Line**: SS
- **Dotted Pink Line**: UU

---

**Graphs**

- **Graph 1**: Plot of \(e\) vs. \(f\)
- **Graph 2**: Plot of \(e_t\) and \(f_t\) vs. \(t\)
Black & Salemi derive \( f = \frac{1}{\rho \sigma^2} \left( r^* - r + De \right) \) so \( \alpha = \frac{1}{\rho \sigma^2} \) where \( \rho \) is the coefficient of risk aversion and \( \sigma^2 \) is the conditional variance of the exchange rate. An increase in the volatility of interest rates will raise \( \sigma^2 \) and thus lower \( \alpha \), thereby causing the slope of the saddle path to be steeper and further raising the volatility of the exchange rate.

Alternatively, increased central bank intervention to reduce \( \sigma^2 \) will raise \( \alpha \) and flatten the slope of the saddle path, further dampening exchange rate fluctuations. This is the “Harrod” effect.

Empirical Tests:

Frankel (AER, 1979) test of extended Dornbusch “sticky price” model

<table>
<thead>
<tr>
<th>Technique</th>
<th>Constant</th>
<th>( m - m_t )</th>
<th>( y - y^* )</th>
<th>( r - r^* )</th>
<th>( \pi - \pi^* )</th>
<th>( R^2 )</th>
<th>D.W.</th>
<th>( \rho )</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OLS</strong></td>
<td>1.33</td>
<td>(.10)</td>
<td>-.72</td>
<td>-1.55</td>
<td>28.65</td>
<td>.80</td>
<td>.76</td>
<td>.98</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.17)</td>
<td>(.22)</td>
<td>(1.94)</td>
<td>(2.70)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>CORC</strong></td>
<td>.80</td>
<td>(.19)</td>
<td>-.33</td>
<td>-2.59</td>
<td>7.72</td>
<td>.91</td>
<td>.98</td>
<td></td>
<td>43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.25)</td>
<td>(.20)</td>
<td>(1.96)</td>
<td>(4.47)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>INST</strong></td>
<td>1.39</td>
<td>(.08)</td>
<td>-.54</td>
<td>-4.75</td>
<td>27.42</td>
<td>1.00</td>
<td></td>
<td></td>
<td>42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.14)</td>
<td>(.18)</td>
<td>(1.69)</td>
<td>(2.26)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FAIR</strong></td>
<td>1.39</td>
<td>(.12)</td>
<td>-.52</td>
<td>-5.40</td>
<td>29.40</td>
<td>.46</td>
<td></td>
<td></td>
<td>41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.21)</td>
<td>(.22)</td>
<td>(2.04)</td>
<td>(3.33)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors are shown in parentheses.
Definitions: Dependent Variable (log of) Mark/Dollar Rate.
\( m - m_t = \log \text{of German } M_t / \text{U.S. } M_t \)
\( y - y^* = \log \text{of German production}/\text{U.S. production} \)
\( r - r^* = \text{Short-term German-U.S. interest differential} \)
\( (r - r^*)_{t-1} = \text{Short-term German-U.S. interest differential lagged} \)
\( \pi - \pi^* = \text{Expected German-U.S. inflation differential, proxied by long-term government bond differential} \)

Meese and Rogoff (AER, 1983) show that the monetary models cannot beat a random walk. Cointegration tests by MacDonald and Taylor (JIMF, 1994) find a long run dollar-mark cointegrating relationship of

\( s_t = (m_t - m_{t*}) - (y_t - y_{t*}) + 0.049i_t - 0.05i_{t*} \) with an error-correction adjustment mechanism that beats a random walk.
Black and Salemi (*JIE*, 1988) estimate the portfolio model in the form
\[\Delta f_t = \eta (e_t + p_t^* - p_t) \quad \text{and} \quad f_t = \alpha (r_t^* + E_t e_{t+1} - e_t - r_t)\] and find that the model explains the dollar-mark exchange rate over the period 1965-83, allowing for differences in monetary policy behavior. The estimated \(\eta = .038\) and \(\alpha = \rho \sigma^2\) varies over different monetary policy regimes, with \(\rho = 4.08\).

MacDonald and Marsh (*RESTAT*, 1997) test a portfolio-type equation of the form
\[\alpha (s_t - p_t + p_t^*) = \mu (i_t^* + \Delta s_{t+1}^* - i_t)\] and find cointegrating relationships of the exchange rate with both relative prices and interest differentials for the dollar-mark, dollar-pound, and dollar-yen, again beating a random walk over all but the shortest horizons.
Given \( s_t = \frac{1}{1+\alpha} \sum_{i=0}^{\infty} \left( \frac{\alpha}{1+\alpha} \right)^i E_t f_{t+i} \) with \( f_t = \epsilon_t \) a random walk without drift, the fundamental solution for the exchange rate is \( s_t^* = \epsilon_t \). Assume that market beliefs about the exchange rate depend on chartists with expectations \( E_{\epsilon,t}(\Delta s_{t+1}) = \beta \sum_{i=1}^{T} \alpha_i \Delta s_{t-i} \) that extrapolate recent changes and fundamentalists with expectations that return to equilibrium \( E_{f,t}(\Delta s_{t+1}) = \theta(s_{t-1} - s_{t-1}^*) \) with \( \theta < 0 \) if \( s_{t-1} - s_{t-1}^* > C \) and \( \theta > 0 \) if \( s_{t-1} - s_{t-1}^* < -C \) where \( C \) is the level of transactions costs beyond which fundamentalists will act on their beliefs [so \( E_{f,t}(\Delta s_{t+1}) = 0 \) if \( |s_{t-1} - s_{t-1}^*| < C \)]. Combining these with weights of 0.5 on each type and setting \( T = 1 \) and \( C = 0 \) with \( s_t^* = 0 \), De Grauwe finds \( s_t = \left( 1 + \frac{\beta}{2} \right) s_{t-1} - \frac{\beta}{2} s_{t-2} + \frac{\theta}{2} s_{t-1}^2 \) which gives a logistic function for the exchange rate.

![logistic curve for speed of adjustment: 0.3 beta=5](image)

Near the long-run equilibrium (zero), the extrapolative elements dominate and the exchange rate moves upward. Eventually, the
fundamentalist factor becomes more important and the curve slopes downward. Higher speeds of adjustment $\theta$ cause the fundamentalists to take over sooner. If shocks are added to the fundamental equilibrium and transactions costs are considered, both rates will vary over time, but the market rate can deviate for long periods from fundamental equilibrium.

Figure 8b
Simple Chaotic Exchange Rate Model (De Grauwe, et al, 1993)

Chartists: \( E_{ct}(S_{t+1}) / S_{t-1} = (S_{t-1} / S_{t-2})^{y} (S_{t-3} / S_{t-2})^{y} \)
Fundamentalists \( E_{ft}(S_{t+1}) / S_{t-1} = (S^{*} / S_{t-1})^{a} \)

Weighting on Chartists \( m_{t} = 1/(1 + \beta(S_{t-1} - S^{*})^{2}) \)

\[
\begin{pmatrix}
    s_{t} \\
    m_{t}
\end{pmatrix}
= 
\begin{pmatrix}
    \left[ b \left[ 1 + \gamma \cdot m_{t} - \alpha \cdot (1 - m_{t}) \right] \right] (s_{t-2}) - 2 \cdot b \cdot \gamma \cdot m_{t} (s_{t-3}) \cdot b \cdot \gamma \cdot m_{t} \\
    1 \\
    1 + \beta \cdot (s_{t-1} - 1)^{2}
\end{pmatrix}
\]

\[ s_{t} \]
\[ 0 \quad 200 \quad 400 \]
\[ 0.8 \quad 1.2 \quad 1.4 \]

\[ s_{t-1} \]
\[ 0.8 \quad 1 \quad 1.2 \quad 1.4 \]

\[ m_{t} \]
\[ 0 \quad 0.5 \quad 1 \]
\[ 0.8 \quad 1.2 \quad 1.4 \]
Model with Money and Prices

\[ M_t = Y_t^\alpha P_t (1 + r_t)^{-c} \]

\[ E_t(S_{t+1}) / S_t = (1 + r_t) / (1 + r_{t+1}) \]

\[ P_t / P_{t-1} = \left( \frac{S_t P_{t+1}}{P_t} \right)^k \]

**Expectations as above**

\[
\begin{pmatrix}
  s_t \\
  m_t \\
  p_t
\end{pmatrix} :=
\begin{bmatrix}
  (s_{t-1})^{b[1 + \gamma \cdot m_{t-1} - \alpha \cdot (1 - m_t)]} & (s_{t-2})^{b \cdot \gamma \cdot m_t} & (s_{t-3})^{b \cdot \gamma \cdot m_t} \\
  1 & (1 + k)^{c - k} & k \\
  1 & 1 + \beta (s_{t-1} - 1)^2 & 1 \\
  k & (s_{t})^{1 + k} \cdot (p_{t-1})^{1 + k} & 1
\end{bmatrix}
\]

\[
\begin{align*}
  s_t & \quad p_t \\
  0.8 & \quad 1.0 & \quad 1.2 & \quad 1.4 \\
  0 & \quad 100 & \quad 200
\end{align*}
\]

\[
\begin{align*}
  s_t & \quad p_t \\
  0.8 & \quad 1.0 & \quad 1.2 & \quad 1.4 \\
  0.96 & \quad 1.02 & \quad 1.04 & \quad 0.8
\end{align*}
\]

\[
\begin{align*}
  s_t & \quad s_{t-1} \\
  0.8 & \quad 1.2 & \quad 1.4 \\
  0.8 & \quad 1.0 & \quad 1.2 & \quad 1.4
\end{align*}
\]
Equilibrium Models of Exchange Rate Behavior

Money in the Utility Function (O&R, 8.3)

\[ U_t = \sum_{s=t}^{\infty} \beta^{s-t} u \left( C_s, \frac{M_s}{P_t} \right), \quad (35) \]

if \( u(C_s, L_s) \) and \( L_s = L(M_s/P_s C_s) \) due to time required to change assets if \( M/P \) is low. Subject to

\[ B_{t+1} + \frac{M_t}{P_t} = (1 + r) B_t + \frac{M_{t-1}}{P_t} + Y_t - C_t - T_t, \quad (34) \]

substitute into \( U \)

\[ U_t = \sum_{s=t}^{\infty} \beta^{s-t} u \left[ -B_{s+1} - \frac{M_s}{P_s} + (1 + r) B_s + \frac{M_{s-1}}{P_s} + Y_s - T_s, \frac{M_s}{P_s} \right] \]

first order conditions

\[ u_C \left( C_t, \frac{M_t}{P_t} \right) = (1 + r) \beta u_C \left( C_{t+1}, \frac{M_{t+1}}{P_{t+1}} \right), \quad (35) \]

\[ \frac{1}{P_t} u_C \left( C_t, \frac{M_t}{P_t} \right) = \frac{1}{P_t} u_{M/P} \left( C_t, \frac{M_t}{P_t} \right) + \frac{1}{P_{t+1}} \beta u_C \left( C_{t+1}, \frac{M_{t+1}}{P_{t+1}} \right) \quad (36) \]

Substitute from (35) into (36)

\[ \left( \frac{1}{P_t} - \frac{1}{(1 + r)P_{t+1}} \right) u_C = \frac{1}{P_t} u_{M/p} \]

where \( 1 + i_{t+1} = (1 + r_{t+1})(1 + \pi_{t+1}) \)

\[ \frac{u_M}{u_C} = 1 - \frac{P_t}{P_{t+1}} + \frac{1}{1 + i_{t+1}} = \frac{i_{t+1}}{1 + i_{t+1}} \approx i_{t+1} \]

59
If we assume \( u(C, \frac{M}{P}) = \left[ C^\gamma (M / P)^{1-\gamma} \right]^{\frac{1}{\gamma}} \) then \( \frac{M_t}{P_t} = \left( \frac{1-\gamma}{\gamma} \right) \left( 1 + \frac{1}{i_{t+1}} \right) C_t \)

Cash in Advance Model (O&R, 8.3.6)

\[
U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \tag{57}
\]

S. t.

\[
B_{t+1} + \frac{M_t}{P_t} = (1 + r) B_t + \frac{M_{t-1}}{P_t} + Y_t - C_t - T_t
\]

\( M_{t-1} \geq P_t C_t. \) Assuming equality if \( r > 0, \)

\[
B_{t+1} = (1 + r) B_t + Y_t - T_t - \frac{P_{t+1}}{P_t} C_{t+1}. \tag{59}
\]

since \( \frac{M_t}{P_t} = \frac{P_{t+1}}{P_t} C_{t+1} \) Substitute from (59) into (57) and maximize over \( B_s \)

\[
U_t = \sum_{s=t}^{\infty} \beta^{s-t} u \left( \frac{P_{s-1}}{P_s} \left( (1 + r) B_{s-1} - B_s + Y_{s-1} - T_{s-1} \right) \right)
\]

First order conditions

\[
\frac{P_{s-1}}{P_s} u'(C_s) = (1 + r) \frac{P_s}{P_{s+1}} \beta u'(C_{s+1}).
\]

or

\[
\frac{u'(C_s)}{1 + i_s} = (1 + r) \beta \frac{u'(C_{s+1})}{1 + i_{s+1}} \text{ where } 1 + i_{s+1} = (1 + r_{s+1}) \left( 1 + \frac{P_{s+1}}{P_s} \right)
\]

Money demand is

\[
\frac{M_{t-1}}{P_t} = C_t.
\]
Stochastic General Equilibrium Model with \(N\) countries (O&R, 8.7)

Maximize

\[
U_t^n = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ u(C_s^n) + v \left( \frac{M_s^n}{P_s^n} \right) \right], \quad n = 1, \ldots, N
\]

(91)

\[
\sum_{n=1}^{N} C_t^n = Y_t = \sum_{n=1}^{N} Y_t^n \text{ endowments}
\]

The first order condition for the single good:

\[
\frac{1}{P_t} u'(C_t) = (1 + i_{t+1}) \beta E_t \left\{ \frac{1}{P_{t+1}} u'(C_{t+1}) \right\}
\]

or

\[
u'(C_t) = \beta E_t \left\{ (1 + i_{t+1}) \frac{P_t}{P_{t+1}} u'(C_{t+1}) \right\}, \quad (95)
\]

Suppose there is a riskless bond that pays \(1 + r_{t+1}\). Then

\[
u'(C_t) = (1 + r_{t+1}) \beta E_t \{u'(C_{t+1})\}. \quad (96)
\]

But \((1 + r_{t+1}) E_t \{u'(C_{t+1})\} = E_t \left\{ (1 + i_{t+1}) \frac{P_t}{P_{t+1}} u'(C_{t+1}) \right\}\) does not imply

\[
1 + r_{t+1} = (1 + i_{t+1}) \frac{P_t}{P_{t+1}} \quad (97)
\]

since the components of \(\frac{P_t}{P_{t+1}} u'(C_{t+1})\) are random and correlated with each other.
The first order condition for money is

\[
\frac{1}{P_t} u'(C_t) = \frac{1}{P_t} v' \left( \frac{M_t}{P_t} \right) + \beta E_t \left\{ \frac{1}{P_{t+1}} u'(C_{t+1}) \right\}
\]

(98)

So

\[
\frac{v' \left( \frac{M_t}{P_t} \right)}{u'(C_t)} = \frac{i_{t+1}}{1 + i_{t+1}}.
\]

Given Purchasing Power Parity, we have

\[P^n = \varepsilon^{nm} P^m\]

Given identical utility functions, consumption is shared proportionally, so

\[C^n_t = x^n Y^n_t\]

In the Cash in Advance case

\[P^n_t = M^n_t / Y^n_t\] or velocity = unity and \[\varepsilon^{nm}_t = \frac{P^n_t}{P^m_t} = \frac{M^n_t}{M^m_t} \cdot \frac{Y^n_t}{Y^m_t}\] with PPP
Maximize

\[ U_i = E_i \sum_{s=1}^{\infty} \beta^{s-i} u(C_s, C_s^*) \text{ subject to } M_i \geq P_i C_i, \quad M_i^* \geq P_i^* C_i^* \]

Stochastic Per capita Endowments \((2Y, 0), (0, 2Y^*), M, M^*\)

Pooled Equilibrium \(C = Y, C^* = Y^*\). If \(r > 0, M = P Y, M^* = P^* Y^*\)

\[ \frac{P}{\mathcal{E} P^*} = \frac{u_C(Y, Y^*)}{u_C^*(Y, Y^*)} \quad \text{and} \quad \mathcal{E} = \frac{P}{P^*}, \quad \frac{u_C^*}{u_C} = \frac{M / Y}{M^* / Y^*} \cdot \frac{u_C^*}{u_C} \]

\[ \frac{d \ln \mathcal{E}}{d \ln Y} = -1 + \frac{d \ln u_C^*}{d \ln Y} - \frac{d \ln u_C}{d \ln Y} = -1 + \gamma \text{ if } U = \frac{C^{1-\gamma}}{1-\gamma} + \frac{C^{1-\delta}}{1-\delta} \]

\[ \frac{d \ln P}{d \ln Y} = -1 \text{ from } P = \frac{M}{Y} \text{ so } 0 > \frac{d \ln \mathcal{E}}{d \ln Y} > \frac{d \ln P}{d \ln Y} \]

based on supply shocks only. Stockman & Dellas (JIE, 1989) add demand shocks and show that \( \text{Var}(d \ln \mathcal{E}) > \text{Var}(d \ln P) \) is possible. Lucas

Model timing: asset trades occur after current state of economy is known. So \(M = P Y\). Svensson (JPE, 1985) assumes asset trades occur before state is known, giving rise to a precautionary demand for money.

Grilli & Roubini (JIE, 1992) develop a model that assumes cash must also be held in advance for asset transactions. Two countries, two goods, two bonds, two moneys, three-member household. One receives endowment, sells for cash to be used next period, second takes cash and buys goods, third takes remaining cash and buys assets \((B_1^i, B_2^i)\) Cash in advance constraints for each country \(i = 1, 2\)
\[ N^1_{it} \geq P^1_t C^1_{it}, N^2_{it} \geq P^2_t C^2_{it} \]
\[ (M^1_{it} - N^1_{it}) + s_i (M^2_{it} - N^2_{it}) \geq q^1_i B^1_{it} + s_i q^2_i B^2_{it} \]

let \( Z^j_{it} = M^j_{it} - N^j_{it} \) be cash held for asset transactions

then \( P^j_i = (M^j_i - Z^j_i) / y^j_i \) and

\[ s = \frac{P^1}{P^2} \cdot \frac{U_2}{U_1} \cdot \frac{q^1}{q^2} = \frac{M^1 - Z^1}{M^2 - Z^2} \cdot \frac{y^2}{y^1} \cdot \frac{U_2}{U_1} \cdot \frac{q^1}{q^2} \]

Therefore fluctuations in asset prices \( q \) affect exchange rate even if \( M \) is constant (and therefore prices don’t change).
Portfolio Approach to Exchange Rates


Partial equilibrium model of asset markets: 4 assets domestic money and bonds $M, B$; foreign money and bonds $N, F$, held by domestic (non-*) or foreign (*) residents.

Private wealth $W = M + B + E(N + F)$, $W^* = (M^* + B^*) / E + N^* + F^*$ allocated as functions of $(r_i, X, W)$ where $X$ is income

$W = m(\cdot) + n(\cdot) + b(\cdot) + f(\cdot)$, $EW^* = m^*(\cdot) + n^*(\cdot) + b^*(\cdot) + f^*(\cdot)$

Asset stocks $\tilde{M} = M + M^*$, $\tilde{N} = N + N^*$, $\tilde{B} = B + B^*$, $\tilde{F} = F + F^*$

adding up conditions $m_k + n_k + b_k + f_k = 0$, $m_w + n_w + b_w + f_w = 1$

Simplify by assuming no currency substitution $M^* = N = 0$

Money demands independent of wealth and foreign return $i^*$

$W = M + B + EF$, $W^* = B^* / E + N^* + F^*$

$m(i, PX) = \tilde{M}$, $n(i^*, EP^*Y) = E\tilde{N}$

$b(i, i^* + \varepsilon, PX, W) + b^*(i - \varepsilon, i^*, EW^*) = \tilde{B}$

$f(i, i^* + \varepsilon, W) + f^*(i - \varepsilon, i^*, EP^*Y, EW^*) = E\tilde{F}$

Short-run equilibrium, given asset stocks, incomes $PX, P^*Y$, expected change in exchange rate $\varepsilon$. Because of adding –up conditions, only three of four equilibrium conditions is independent (Tobin’s Law)
Above BB, excess demand for B, i falls; to right of FF, excess demand for F, \(i^*\) falls. FF steeper than BB: if not, excess supply in all markets. Depreciation of exchange rate (rise in \(E\)) shifts BB and FF down. Open market operation \(d\tilde{M} + d\tilde{B} = 0\) shifts MM and BB down, MM by more, since \(-m_i < b_i + b_i^*\). Depreciation then shifts BB and FF to new equilibrium with lower \(i\).

Effects of intervention on the exchange rate. Slope of FF curve:

\[
\frac{dE}{di} = \frac{(f_i + f_i^*)/\tilde{F}}{1 - f_w \frac{F}{\tilde{F}} - f_y \frac{P^*Y}{\tilde{F}} - f_{w*} \frac{N^* + F^*}{\tilde{F}}} < 0
\]

BB steeper than FF because \(|b_i + b_i^*| > |f_i + f_i^*|\)

Open Market Operation \(dM = -dB\) MM shifts left along the FF curve.

Unsterilized Intervention: \(dM = -dF\) MM shifts left along the BB curve, largest effect on \(E\).

Sterilized Intervention: \(dB = -EdF\) BB and FF shift up, no effect on \(i\).
Stock-Flow Equilibrium

Let $w = M + B + \bar{E}F, w^* = B^* / \bar{E} + N^* + F^*, e = \ln(E), de = dE / E$, initial $E = P = P' = 1$

At equilibrium $\bar{e} = 0, \, d\bar{e} = \bar{e}$. Then $\tilde{b}_c de + \tilde{b}_s di + \tilde{b}_d \bar{e} + \tilde{b}_w dw = dB$

$$\tilde{b}_c e = -\frac{\tilde{b}_i}{m_i} dB + dB - \tilde{b}_s de - \tilde{b}_w dw$$

$$\bar{e} = -\frac{\tilde{b}_c}{\tilde{b}_s} de - \frac{\tilde{b}_w}{\tilde{b}_s} dw - \frac{1}{\tilde{b}_s} \left( \frac{\tilde{b}_i}{m_i} + 1 \right) dM \quad \text{if} \, dB = -dM$$

$$\bar{e} = \bar{e}_c de + \bar{e}_w dw - \bar{e}_{OMO} dM \quad \text{Asset Stock Equilibrium} \quad \frac{de}{dw} = -\frac{\bar{e}_w}{\bar{e}_e} < 0$$

Goods market Equilibrium with sticky (fixed) prices $\dot{w} = \eta de$

Transfer of wealth shifts $w$ up or down. OMO shifts Asset Stock Equilibrium up or down, shifting the saddle path SS.

Flexible price case: assume local goods preferred, so wealth transfer $dw$ raises spending on local goods, deteriorates trade balance

$$\dot{w} = \alpha de - \beta dw \quad \text{so} \quad \frac{de}{dw} = \frac{\beta}{\alpha} > 0 \quad \text{so the} \quad \dot{w} = 0 \quad \text{curve slopes upward.}$$
Net Foreign Assets Version of Model (Black & Salemi)

\[ W = M + B + SF, \quad W^* = N^* + F^* + B^* / S \]
\[ \tilde{B} = B + B^*, \quad \tilde{F} = F + F^* \]

Net Private Wealth \( V = W - M - \tilde{B} = SF - B^*, \quad V^* = W^* - N^* - \tilde{F} = B^* / S - F \)

\[ B = b \left( i, i^* + \Delta s^e \right) W, \quad b_1 > 0, b_2 < 0, \quad SF = f \left( i, i^* + \Delta s^e \right) W, \quad f_1 < 0, f_2 > 0 \]
\[ B^* / S = b^* \left( i - \Delta s^e, i^* \right) W^*, \quad b_1^* > 0, b_2^* < 0, \quad F^* = f^* \left( i - \Delta s^e, i^* \right) W^*, \quad f_1^* < 0, f_2^* > 0 \]
\[ V = SF - B^* = f \left( i, i^* + \Delta s^e \right) W - b^* \left( i - \Delta s^e, i^* \right) SW^* \]

Assume \( f = -b^* = \alpha = 1 / \rho \sigma_s^2 \). Then \( v = \frac{V}{W + SW^*} = \alpha \left( i^* + \Delta s^e - i \right) \)
\[ \Delta v = \eta \left( s + p^* - p \right) \]

Empirical Tests of Portfolio Model (Frankel, JIMF, 1982)

Portfolio shares \( x_i = \left[ x_i^{DM}, x_i^F, x_i^F, x_i^{Fr}, x_i^{Fr}, x_i^{C^S} \right], \sum_{i=1}^{S} x_i^j + x_i^s = 1 \)

Real rates of return \( r_{t+1} = \left[ r_{t+1}^{DM}, r_{t+1}^F, r_{t+1}^{Fr}, r_{t+1}^{Fr}, r_{t+1}^{C^S} \right], \quad r_{t+1}^j = i_{t+1}^j - \pi_{t+1}^j - \Delta s_{t+1}^j, \quad r_{t+1}^s = i_{t+1}^s - \pi_{t+1}^s \)
\[ z_{t+1} = r_{t+1} - i \cdot r_{t+1}^s \]
\[ W_{t+1} - W_t = x_t^j r_{t+1} W_t + \left( 1 - \sum x_i^j \right) r_{t+1}^s W_t \]

Maximize \( E_t \left( \frac{W_{t+1}}{W_t} \right) - \frac{\rho}{2} V_t \left( \frac{W_{t+1}}{W_t} \right) = x_t' E_t z_{t+1} - \frac{\rho}{2} x_t' E_t z_{t+1} E_t x_t + x_t' E_t z_{t+1} x_t = x_t' E_t z_{t+1} - \frac{\rho}{2} x_t' \Omega x_t \)

First order condition \( \rho \Omega x_t = E_t z_{t+1}, \quad \text{or} \quad x_t = (\rho \Omega)^{-1} E_t z_{t+1} \)

ex ante real risk premium \( E_t z_{t+1} = \rho \Omega x_t, \quad \text{[UIP if } \rho = 0 \text{ or } \Omega = 0 \] \)

Test as \( z_{t+1} = \alpha_0 + \rho \Omega x_t + z_{t+1} - E_t z_{t+1} = \alpha_0 + \rho \Omega x_t = u_{t+1} \)

Assumes rational expectations \( z_{t+1} - E_t z_{t+1} = u_{t+1}; E_t \left( u_{t+1} \right) = 0, E_t \left( u_{t+1} u_t \right) = 0 \), no error term in portfolio optimization, and constant variance-covariance matrix \( \Omega \). Results support UIP and reject portfolio model \( (\rho = 0) \). If there is an error term \( v_t \) in the portfolio choice condition, then \( x_t \)
is correlated with $v_t$, and OLS is not acceptable. Simultaneous equations approach required. See Black & Salemi for joint estimation of $x_t$ and $z_{t+1}$, which also includes current account equation explaining $\Delta x_t$.

Other generalizations include assuming that the CPI depends on consumption shares, so there is a risk-minimizing portfolio:

$$\pi^s_t = \alpha' (\pi_t - \Delta s_t) + (1 - \alpha') \pi^US_t, \quad \pi' = \left( \pi^DM, \pi^E, \pi^Y, \pi^Fr, \pi^Cg \right)$$

$$E_t z_{t+1} = \rho \Omega (x_t - \alpha)$$

where $\alpha$ is a vector of international consumption shares allocated to goods of country $i$. Or if the $\alpha_j$ vary across countries and $w_t$ is a vector of shares of world wealth, then $E_t z_{t+1} = \rho \Omega (x_t - \alpha w_t), \quad \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_5)$

Other recent studies using the portfolio model:

- MacDonald and Marsh (RESTAT, 1997) test a portfolio-type equation of the form $\alpha(s_t - p_t + p_t^*) = \mu(i_t^* + \Delta s_{t+1}^e - i_t)$, find cointegration of the exchange rate with both relative prices and interest differentials for the dollar-mark, dollar-pound, dollar-yen, beating a random walk over all but the shortest horizons.
Exchange Market Efficiency

Prices fully reflect available information. Excess returns above normal should be mean zero, uncorrelated over time.

\[ z_{j,t+1} = r_{j,t+1} - E[r_{j,t+1} | \Phi_t] \] is mean zero, uncorrelated over time.

Depends on (i) equilibrium hypothesis, (ii) information set. Weak form test use only past price information. Strong form tests use all other information.

Risk neutrality implies unbiased forward market: \( f_t = E_t(s_{t+1}) \)

Perfect capital mobility implies Covered Interest Parity:

(CIP) \( i_t - i_t^* = f_t - s_t \)

CIP & risk neutrality imply Uncovered Interest Parity:

(UIP) \( i_t - i_t^* = E_t(s_{t+1}) - s_t \)

Law of One Price implies \( \pi_t^T = \Delta s_t + \pi_t^T^* \) for traded goods.

\[ \pi_t = \alpha \pi_t^N + (1 - \alpha) \pi_t^T, \quad \pi_t^* = \alpha^* \pi_t^N + (1 - \alpha^*) \pi_t^T \]

\[ \pi_t - \pi_t^T = \alpha (\pi_t^N - \pi_t^T), \quad \pi_t^* - \pi_t^T^* = \alpha^* (\pi_t^N - \pi_t^T) \]

\[ \pi_t - \pi_t^* = \pi_t^T - \pi_t^T^* + \alpha (\pi_t^N - \pi_t^T) - \alpha^* (\pi_t^N - \pi_t^T) = \Delta s_t + \alpha (\pi_t^N - \pi_t^T) - \alpha^* (\pi_t^N - \pi_t^T) \]

\[ i_t - i_t^* = E_t(\pi_t - \pi_t^*) - \alpha (\pi_t^N - \pi_t^T) + \alpha^* (\pi_t^N - \pi_t^T) \]

\[ \rho_t = \rho_t^* - \alpha (\pi_t^N - \pi_t^T) + \alpha^* (\pi_t^N - \pi_t^T) \]

Tests of market efficiency (Levich):

1. CIP holds for offshore markets except in turbulent times, also for some onshore markets (US vs. euro, etc.)

2. Random walk approximately true for spot exchange rates, some evidence of mean reversion, auto-regressive conditional heteroscedasticity (ARCH), fat tailed distributions.

3. Filter rules show profits for Buy if \( s_t - s_t^T > k \) or sell if \( s_t^P - s_t > k \)
4. Forward bias tests:
   a. **Level tests** \( s_{t+1} = \alpha + \beta f_t + u_{t+1} \) find \( \beta = 1 \), but \( s \) and \( f \) are integrated of order 1 \([I(1)]\), so non-stationary.
   b. **Difference tests** \( \Delta s_{t+1} = \alpha + \beta (f_t - s_t) + u_{t+1} \) find \( 0 > \beta > -2 \)
   c. \( \text{Var}(\Delta s) > \text{Var}(f - s) \)

Covered Interest Parity defines the forward rate (O&R, 8.7.5.3)

\[
1 + i_{t+1} = (1 + i_{t+1}^*) \frac{F_t}{E_t}.
\]

(104)

The first order condition for the return on any asset \( m \) is

\[
u'(C_t) = \beta E_t\{ (1 + r_{t+1}^m)u'(C_{t+1}) \},
\]

so for any two assets \( n \) and \( m \),

\[
0 = E_t \left\{ (r_{t+1}^n - r_{t+1}^m) \frac{u'(C_{t+1})}{u'(C_t)} \right\}
\]

(116)

**Equivalence of real interest rate differentials and forward premia:**

\[
\frac{(1 + i_{t+1}) P_t}{P_{t+1}} - \frac{(1 + i_{t+1}^*) P_t^*}{P_{t+1}^*} = (1 + i_{t+1}) P_t \left( \frac{\Xi_t - \varepsilon_{t+1}}{P_{t+1}} \right)
\]

substitute in (116)

and factor out the first term (which is date \( t \) information), results in

\[
E_t \left( \frac{u'(C_{t+1})}{u'(C_t)} : \frac{\Xi_t - \varepsilon_{t+1}}{P_{t+1}} \right) = 0
\]

This does not imply \( \Xi_t = E_t \varepsilon_{t+1} \) unless there is (a) no uncertainty or (b) risk neutrality.
Jensen’s Inequality: both $\mathcal{F}_t = E_t \{ \varepsilon_{t+1} \}$ and $\frac{1}{\mathcal{F}_t} = E_t \left\{ \frac{1}{\varepsilon_{t+1}} \right\}$ are not possible at the same time, since

Gap depends on $\text{Var}(\varepsilon_{t+1})$

Hansen & Hodrick (1980) tested for forward bias in

$$s_{t+13}^i - f_t^i = a_i + b_{i1}(s_t^i - f_{t-13}^i) + b_{i2}(s_{t-1}^i - f_{t-14}^i) + u_t^i$$

for seven currencies using weekly data for 3-month forward premia. Rejected null hypothesis for deutschemark-dollar. When cross-currency forecast errors included, null rejected for three of five currencies.

$$s_{t+13}^i - f_t^i = a_i + \sum_{j=1}^{5} b_{ij}(s_t^i - f_{t-13}^j) + u_t^i$$

<table>
<thead>
<tr>
<th>Currency</th>
<th>Sample Period</th>
<th>N Observations</th>
<th>$\hat{\theta}_t$ Confidence</th>
<th>$\hat{b}_{i1}$ Confidence</th>
<th>$\hat{b}_{i2}$ Confidence</th>
<th>$\hat{b}_{i3}$ Confidence</th>
<th>$\hat{b}_{i4}$ Confidence</th>
<th>$\hat{b}_{i5}$ Confidence</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Canadian dollar</td>
<td>10/2/75-1/16/79</td>
<td>277</td>
<td>-.004 (.004)</td>
<td>.033 (.145)</td>
<td>-.045 (.088)</td>
<td>-.161 (.081)</td>
<td>-.095 (.070)</td>
<td>.051 (.077)</td>
<td>.20508 .434</td>
</tr>
<tr>
<td>2. Deutschemark</td>
<td>10/2/75-1/16/79</td>
<td>277</td>
<td>.003 (.011)</td>
<td>.147 (.422)</td>
<td>-.666 (.200)</td>
<td>-.224 (.227)</td>
<td>.201 (.203)</td>
<td>.563 (.226)</td>
<td>12.390 .276</td>
</tr>
<tr>
<td>3. French franc</td>
<td>10/2/75-1/16/79</td>
<td>277</td>
<td>.003 (.011)</td>
<td>.067 (.417)</td>
<td>-.358 (.236)</td>
<td>.055 (.234)</td>
<td>.174 (.201)</td>
<td>.401 (.223)</td>
<td>7.561 .176</td>
</tr>
<tr>
<td>4. U.K. pound</td>
<td>10/2/75-1/16/79</td>
<td>277</td>
<td>-.004 (.012)</td>
<td>-.650 (.453)</td>
<td>-.100 (.248)</td>
<td>-.260 (.255)</td>
<td>-.087 (.219)</td>
<td>.253 (.244)</td>
<td>3.882 .085</td>
</tr>
<tr>
<td>5. Swiss franc</td>
<td>10/2/75-1/16/79</td>
<td>277</td>
<td>.003 (.013)</td>
<td>.020 (.320)</td>
<td>.977 (.324)</td>
<td>.257 (.295)</td>
<td>.225 (.225)</td>
<td>.362 (.287)</td>
<td>13.872 .256</td>
</tr>
</tbody>
</table>

Note: See table 1. The coefficient $b_{ij}$ refers to the regression coefficient of currency $i$ on currency $j$ where the $i$ and $j$ subscripts identify the currencies as shown above.

*Marginal significance level = .1

Results, replicated many times, show that UIP does not hold.
Suppose CRRA preferences and lognormal disturbances. Then

\[ E_t \left[ \left( \frac{C_t}{C_{t+1}} \right)^\rho \left( \frac{z_t - \epsilon_{t+1}}{p_{t+1}} \right) \right] = 0 \Rightarrow \]

\[ f_t - E_t(e_{t+1}) = \frac{1}{2} \text{var}(e_{t+1}) - \text{cov}(e_{t+1}, p_{t+1}) - \rho \text{cov}(e_{t+1}, c_{t+1}) \]

which is not equal to zero even if \( \rho = 0 \) due to Jensen’s inequality for the real risk premium.
Domowitz & Hakkio (JIE, 1985) consider Lucas model with Cobb-Douglas utility to try to explain the risk premium

\[ E \left[ \sum_{t=0}^{\infty} \beta^t U(x, y) \right] \] with \( U(x, y) = A x^\alpha y^{1-\alpha} \) with random endowments \( \psi_t = (\xi_t, \eta_t) \)

divided between Home and Foreign Home: \((\xi_0, 0)\) Foreign: \((0, \eta_0)\)

subject to \( p_x \xi_t \leq M_t, p_y \eta_t \leq N_t, \) random \((M_t, N_t)\)

Equilibrium under complete markets \((x, y) = (\xi_t / 2, \eta_t / 2)\)

Marginal rate of substitution \( \tilde{p}_y(\psi_t) = \frac{U_y(\psi)}{U_x(\psi)} = \frac{(1-\alpha) A x^\alpha y^{-\alpha}}{\alpha A x^{\alpha-1} y^{1-\alpha}} = \frac{1-\alpha}{\alpha} \cdot \frac{x}{y} = \frac{1-\alpha}{\alpha} \cdot \frac{\xi_t}{\eta_t} \)

Spot exchange rate \( S_t = \tilde{p}_y(\psi_t) \cdot \frac{p_x}{p_y}, \) \( p_x = \frac{1-\alpha}{\alpha} \cdot \frac{\xi_t}{\eta_t} \cdot M_t / \xi_t, p_y = \frac{1-\alpha}{\alpha} \cdot M_t / \eta_t \)

Intertemporal marginal rate of substitution \( \tilde{Q}_t^{m} = \frac{\beta U_x(\psi_{t+1}) / p_{x_{t+1}}}{U_x(\psi_t) / p_{x_t}} = \frac{\beta \left( \frac{\eta_{t+1}}{\xi_{t+1}} \right)^{1-\alpha} / \left( \frac{M_{t+1}}{\xi_{t+1}} \right)}{\left( \frac{\eta_t}{\xi_t} \right)^{1-\alpha} / \left( \frac{M_t}{\xi_t} \right)} \)

let \( x_t = \ln \xi_t, y_t = \ln \eta_t, m_t = \ln M_t, n_t = \ln N_t, x_t = \rho x_{t-1} + u_{1t}, y_t = \rho y_{t-1} + u_{2t} \)

\( m_t = \gamma_1 m_{t-1} + u_{3t}, n_t = \gamma_2 n_{t-1} + u_{4t} \)

assume \( u_{it} \) distributed as lognormal with variances \( (h_{1t}, h_{2t}, h_{3t}, h_{4t}) \)

\( Et \ln Q_t^{m} = \exp \left\{ \ln \beta - \alpha \gamma_1 \left( 1 - \rho_1 \right) x_t - \alpha \gamma_2 \left( 1 - \rho_2 \right) y_t + \left( 1 - \gamma_1 \right) m_t + \frac{\alpha^2}{2} h_{1t+1} + \right\} \)

From \( S_t = \frac{1-\alpha}{\alpha} \cdot \frac{M_t}{N_t}, \) \( E_t(s_{t+1}) = \ln \left[ (1-\alpha) / \alpha \right] + \gamma_1 m_t - \gamma_2 n_t \)

From \( \mathbb{S}_t = S_t \cdot \frac{E_t Q_{t+1}^N}{E_t Q_{t+1}^M}, \) \( f_t = \ln \left[ (1-\alpha) / \alpha \right] + \gamma_1 m_t - \gamma_2 n_t - \frac{1}{2} \left( h_{3t+1} - h_{4t+1} \right) \)

so \( E_t(s_{t+1} - f_t) = \frac{1}{2} \left( h_{3t+1} - h_{4t+1} \right) \)

Note \( \frac{\partial (s_{t+1} - f_t)}{\partial h_3} > 0, \) which has the wrong sign!
Since Cobb-Douglas has elasticity of substitution $\sigma = 1$, there is no risk aversion. The “risk premium” arises from Jensen’s inequality because the lognormal is skewed. $E \ln z = e^{\mu + \frac{1}{2} \sigma^2}$, if $Ez = \mu = 0$, $E \ln z > 0$

Domowitz and Hakkio empirically test ARCH-in-mean:

$$(S_{t+1} - S_t) / S_t = P_t + \beta_1 (S_{t+1} - S_t) / S_t + \varepsilon_{t+1}$$

$P_t = \beta_0 + \theta h_{t+1}, \varepsilon_{t+1} \sim N(0, h_{t+1}^2)$

$h_{t+1}^2 = \alpha_0^2 + \sum_{j=1}^{4} \alpha_j^2 \varepsilon_{t+1-j}$

If D&H assume $\beta_1 = 1$, cannot reject $\theta = 0$. Reject joint hypothesis $\beta_1 = 1$, $\theta = 0$.

Dutton (JMCB, 1993) shows that in this type of model

$$S_t = \left( \frac{M_t}{N_t} \right) \left( \frac{X_{2t}}{X_{1t}} \right)^{\frac{1}{\sigma}}$$

So if $\sigma = 1$, the exchange rate is independent of the real risk in the model. Zengin (2000) simulates such a model and generates realistic risk premia if there are random deviations from purchasing power parity and $\sigma \neq 1$.

Explanations for failure of UIP (bias in forward market) (Lewis, Handbook, Vol. 3, ch. 37)

CIP $i_t - i^*_t = f_t - s_t$

excess return $er_{t+1} = i^*_t + s_{t+1} - s_t - i_t = s_{t+1} - f_t$

predicted excess return $per_t = E_t (er_{t+1}) = E_t \Delta s_{t+1} - (f_t - s_t) = E_t s_{t+1} - f_t$

ex post excess return $er_{t+1} = per_t + \varepsilon_{t+1}$, where $\varepsilon_{t+1} = s_{t+1} - E_t s_{t+1}$ is the forecast error

risk premium $rp_t = E_t^m s_{t+1} - f_t$, where the "market" expectation is $E_t^m s_{t+1}$

"market" forecast error $\eta_{t+1} = s_{t+1} - E_t^m s_{t+1}$

$er_{t+1} = s_{t+1} - f_t = rp_t + \eta_{t+1}$

rational expectations $E_t^m s_{t+1} = E_t s_{t+1}$
Fama decomposition

\[ f_t - s_{t+1} = \alpha_0 + \alpha_1(f_t - s_t) + v_{t+1} \]

\[ \Delta s_{t+1} = \beta_0 + \beta_1(f_t - s_t) + u_{t+1}, \quad \hat{\beta}_1 \approx -2! \]

\[ f_t - s_{t+1} + \Delta s_{t+1} \equiv f_t - s_t \] so \( \alpha_1 + \beta_1 = 1, \alpha_0 + \beta_0 = 0, u_{t+1} + v_{t+1} \equiv 0 \)

\[ \hat{\beta}_1 = \frac{\text{Cov}(\Delta s_{t+1}, f_t - s_t)}{\text{Var}(f_t - s_t)} = \frac{\text{Cov}(E_t \Delta s_{t+1} + \epsilon_{t+1}, f_t - s_t)}{\text{Var}(f_t - s_t)} = \frac{\text{Cov}(E_t \Delta s_{t+1}, f_t - s_t)}{\text{Var}(f_t - s_t)} \]

\[ \text{Var}(\epsilon_{t+1}) = \text{Var}(E_t \Delta s_{t+1}) + \text{Var}(f_t - s_t) - 2 \cdot \text{Cov}(E_t \Delta s_{t+1}, f_t - s_t) \]

\[ = \text{Var}(E_t \Delta s_{t+1}) + \text{Var}(f_t - s_t) - 2 \beta_1 \cdot \text{Var}(f_t - s_t) > \text{Var}(E_t \Delta s_{t+1}) \]

if \( \beta_1 < \frac{1}{2} \)

so variance of the risk premium, or the predicted excess return, is high.

Explanations:

1. Time-varying risk premium. Assume rational expectations. Then

\[ er_{t+1} = rp_t + \epsilon_{t+1} \]

and variance of the risk premium is the explanation.

Portfolio model or general equilibrium model, for example.

2. Expectational errors. If the risk premium is constant, but

expectations are not rational, \( er_{t+1} = rp_0 + \eta_{t+1} \)

Frankel & Froot (QJE, 1989)

\[ \Delta s_{t+1} = \beta_0 + \beta_1(f_t - s_t) + u_{t+1} \]

<table>
<thead>
<tr>
<th>Data set</th>
<th>Dates</th>
<th>( \hat{\beta} )</th>
<th>t: ( \beta = 0 )</th>
<th>t: ( \beta = 1 )</th>
<th>R²</th>
<th>DF</th>
<th>F test</th>
<th>( \hat{\beta} = 0, \beta = 1 )</th>
<th>F probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economist data</td>
<td>6/81-12/85</td>
<td>-0.6684 (1.0711)</td>
<td>-0.56</td>
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<td>-1.04</td>
<td>-1.91*</td>
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<tr>
<td>Econ 6-month</td>
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<td>-1.9819 (1.4445)</td>
<td>-1.27</td>
<td>-2.06**</td>
<td>0.07</td>
<td>174</td>
<td>1.47</td>
<td>0.191</td>
<td></td>
</tr>
<tr>
<td>Econ 12-month</td>
<td>6/81-12/85</td>
<td>-0.3892 (1.3753)</td>
<td>0.23</td>
<td>-0.56</td>
<td>0.29</td>
<td>149</td>
<td>3.23</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>MMS 3-month</td>
<td>11/83-1/84</td>
<td>-1.7409 (0.9781)</td>
<td>-1.77</td>
<td>-2.89***</td>
<td>0.01</td>
<td>735</td>
<td>12.48</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>MMS 3-month</td>
<td>1/83-10/84</td>
<td>-0.2540 (2.108)</td>
<td>-2.91***</td>
<td>-3.37***</td>
<td>0.00</td>
<td>183</td>
<td>12.91</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>AMEX data</td>
<td>1/76-7/85</td>
<td>-2.2107 (0.9623)</td>
<td>-2.30**</td>
<td>-3.34***</td>
<td>0.23</td>
<td>86</td>
<td>2.60</td>
<td>0.007</td>
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<tr>
<td>AMEX 5-month</td>
<td>1/76-7/85</td>
<td>-2.4181 (1.2066)</td>
<td>-1.92*</td>
<td>-2.71***</td>
<td>0.26</td>
<td>45</td>
<td>2.42</td>
<td>0.041</td>
<td></td>
</tr>
<tr>
<td>AMEX 13-month</td>
<td>1/76-7/85</td>
<td>-0.1377 (1.0549)</td>
<td>2.65**</td>
<td>-0.95***</td>
<td>0.21</td>
<td>40</td>
<td>1.56</td>
<td>0.157</td>
<td></td>
</tr>
</tbody>
</table>

Note: Method of Moments standard errors are in parentheses. *Represents significance at the 10 percent level; ** and *** represent significance at the 5 percent and 1 percent levels, respectively. Regressions aggregate over all currencies. Constant terms were estimated for each currency, but are not reported to save space.
F&F use survey data to identify expectational errors.

\[ \Delta s_{t+1} = \beta_0 + \beta_1 (f_t - s_t) + u_{t+1} \]

\[ \Delta s_{t+1} = E_i^m \Delta s_{t+1} + \eta_{t+1}, \quad r_p = E_i^m s_{t+1} - f_t = E_i^m \Delta s_{t+1} - (f_t - s_t) \]

\[ \hat{\beta}_1 = \frac{\text{Cov}(E_i^m \Delta s_{t+1}, f_t - s_t)}{\text{Var}(f_t - s_t)} = \frac{\text{Cov}(E_i^m \Delta s_{t+1}, f_t - s_t) + \text{Cov}(\eta_{t+1}, f_t - s_t)}{\text{Var}(f_t - s_t)} \]

\[ = \frac{\text{Cov}(r_p, f_t - s_t)}{\text{Var}(f_t - s_t)} + \frac{\text{Cov}(\eta_{t+1}, f_t - s_t)}{\text{Var}(f_t - s_t)} = 1 + \frac{\text{Cov}(r_p, f_t - s_t)}{\text{Var}(f_t - s_t)} + \frac{\text{Cov}(\eta_{t+1}, f_t - s_t)}{\text{Var}(f_t - s_t)} \]

\[ = 1 - b_{rp} - b_{re} \]

where

\[ b_{rp} = \frac{-\text{Cov}(r_p, f_t - s_t)}{\text{Var}(f_t - s_t)} = \frac{-\text{Cov}(r_p, E_i^m \Delta s_{t+1} - r_p)}{\text{Var}(f_t - s_t)} = \frac{\text{Var}(r_p) - \text{Cov}(r_p, E_i^m \Delta s_{t+1})}{\text{Var}(f_t - s_t)} \]

and

\[ b_{re} = \frac{-\text{Cov}(\eta_{t+1}, f_t - s_t)}{\text{Var}(f_t - s_t)} \]

### TABLE II

<table>
<thead>
<tr>
<th>Data set</th>
<th>Approximate dates</th>
<th>N</th>
<th>( b_{re} )</th>
<th>( b_{rp} )</th>
<th>( \beta )</th>
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<tbody>
<tr>
<td>Economist data</td>
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<td>525</td>
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<td>2.51</td>
<td>-0.30</td>
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<td>Econ 6-month</td>
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<td>0.19</td>
<td>0.29</td>
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<td>MMS 1-month</td>
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<td>-1.74</td>
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<tr>
<td>MMS 3-month</td>
<td>1/83-10/84</td>
<td>188</td>
<td>6.07</td>
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<td>AMEX data</td>
<td>1/76-7/85</td>
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<td>-0.03</td>
<td>-2.21</td>
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<td>AMEX 9-month</td>
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<td>3.63</td>
<td>-0.22</td>
<td>-2.42</td>
</tr>
<tr>
<td>AMEX 12-month</td>
<td>1/76-7/84</td>
<td>46</td>
<td>3.11</td>
<td>0.03</td>
<td>-2.14</td>
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</table>

Expectational errors appear to be dominant. Frankel & Froot (AER, 1987) examine the properties of expectational errors, as defined by survey expectations, and find them to include adaptive and regressive components.

Alternative explanations:

1. “chartists” and “fundamentalists” (DeGrauwe)
3. Learning effects and “peso” problem (Lewis, AER, 1989)

5. Endogenous interest rates (McCallum, *JME*, 1994)

Learning about Regime Change at time $\tau$ (Lewis, *AER*, 1989)

$$E_t(s_{t+1}) = (1 - \lambda_t)E_t(s_{t+1} | N) + \lambda_tE_t(s_{t+1} | O), \quad \lambda_t \equiv \text{probability of old regime as of time } t > \tau$$

Bayes' Rule

$$\lambda_t = \frac{\lambda_{t-1} L(\Delta s_t, \Delta s_{t-1}, \Delta s_{t-2}, \ldots \Delta s_{t+1} | O)}{(1 - \lambda_{t-1})L(\cdot | N) + \lambda_{t-1}L(\cdot | O)}$$

if change actually occurred at time $\tau$, then $p \lim \lambda_t \to 0$.

During the learning period, the forecast error is

$$s_{t+1}^N - E_t(s_{t+1} | N) = \eta_{t+1} = \left[ s_{t+1}^N - E_t(s_{t+1} | N) \right] - \lambda_t \left[ E_t(s_{t+1} | O) - E_t(s_{t+1} | N) \right]$$

let $\nabla s_{t+1} \equiv E_t(s_{t+1} | O) - E_t(s_{t+1} | N)$,

Mean $er_t = \frac{1}{T} \sum_{t=1}^{T} (s_{t+1} - E_t s_{t+1}) = -\frac{1}{T} \sum \lambda_t \nabla s_{t+1} \neq 0$

Suppose $rp_t = 0$, then $f_t - s_t = E_t \Delta s_{t+1}$ and

$$b_{re} = -\frac{\text{Cov}(\eta_{t+1}, f_t - s_t)}{\text{Var}(f_t - s_t)} = -\frac{\text{Cov}(\eta_{t+1}, E_t \Delta s_{t+1})}{\text{Var}(E_t \Delta s_{t+1})}$$

If forecasts in each regime are uncorrelated and variances are the same in each regime

$$b_{re} = -\lambda_t (1 - 2\lambda_t) > 0 \text{ if } \lambda_t > \frac{1}{2} \text{ and therefore } \beta_1 = 1 - b_{re} < 1$$

“Peso” Problem: anticipation of future change in policy

$$E_t(s_{t+1}) = (1 - \ell_t)E_t(s_{t+1} | C) + \ell_tE_t(s_{t+1} | A), \quad \ell_t \equiv \text{probability of shift to alternative } A$$

$$s_{t+1}^C - E_t(s_{t+1} | C) = \eta_{t+1} = \left[ s_{t+1}^C - E_t(s_{t+1} | C) \right] - \ell_t \left[ E_t(s_{t+1} | C) - E_t(s_{t+1} | A) \right]$$

let $\nabla s_{t+1} \equiv E_t(s_{t+1} | C) - E_t(s_{t+1} | A)$ then $\eta_{t+1} = \eta_{t+1}^C + \ell_t \nabla s_{t+1}$ as before

Engel & Hamilton (*AER*, 1990) estimate a segmented trend model, show the significance of allowing for multiple regimes, using only weak form information to identify regime changes. Regimes are different, but probability of regime change is small, so small effect on bias. Kaminsky (*AER*, 1993) adds information about central bank intervention and credibility of Fed chairmen, finds significant effects.
Endogenous interest rates (McCallum, *JME*, 1994)

UIP: $\Delta s_{t+1}^e = R_t - R_t^* + u_t \ [= f_t - s_t + u_t]$, $u_t = u_{t-1} + v_t$

Exchange rate (or interest rate) rule $R_t - R_t^* = \lambda \Delta s_t + \gamma (R_{t-1} - R_{t-1}^*) + e_t$

(*) $E_t \Delta s_{t+1} = \lambda \Delta s_t + \gamma \tilde{R}_{t-1} + e_t + u_t$, where $\tilde{R}_t \equiv R_t - R_t^*$

let $\Delta s_t = \phi_1 \tilde{R}_{t-1} + \phi_2 e_t + \phi_3 u_t$

then (**) $E_t \Delta s_{t+1} = \phi_1 \tilde{R}_t + \phi_3 u_t = \phi_1 \left( \lambda \Delta s_t + \gamma \tilde{R}_{t-1} + e_t \right) + \phi_3 u_t$

Equating (*) and (**), $\phi_1 \left[ \lambda \left( \phi_1 \tilde{R}_{t-1} + \phi_2 e_t + \phi_3 u_t \right) + \gamma \tilde{R}_{t-1} + e_t \right] + \phi_3 u_t = \lambda \left( \phi_1 \tilde{R}_{t-1} + \phi_2 e_t + \phi_3 u_t \right) + \gamma \tilde{R}_{t-1} + e_t + u_t$

Equating coefficients, $\tilde{R}_{t-1} : \phi_1^2 \lambda + \phi_1 \gamma = \lambda \phi_1 + \gamma$

$e_t : \phi_1 \lambda \phi_2 + \phi_1 = \lambda \phi_2 + 1$

$u_t : \phi_1 \lambda \phi_3 + \phi_3 = \lambda \phi_3 + 1$

From $\tilde{R}_{t-1} : \lambda \phi_1^2 + (\gamma - \lambda) \phi_1 - \gamma = 0, \phi_1 = \frac{(\lambda - \gamma) \pm \sqrt{(\gamma - \lambda)^2 + 4 \gamma \lambda}}{2 \lambda} = \begin{cases} \frac{1}{\lambda} \\
\frac{-\gamma}{\lambda} \end{cases}$

$e_t : \phi_2 = (1 - \phi_1) / \lambda (\phi_1 - 1) = - \frac{1}{\lambda}$

$u_t : \phi_3 = 1 / (1 - \lambda - \phi_1 \lambda) = \frac{(1 - \gamma - \lambda)}{1 - \gamma - \lambda}$

$\therefore \Delta s_t = -(\gamma / \lambda) \tilde{R}_{t-1} - (1 / \lambda) e_t + \frac{1}{1 - \gamma - \lambda} u_t$ and $\beta = - \frac{\gamma}{\lambda} < 0$

Meredith & Chinn (NBER WP# 6797, 1998) Long Horizon Bias [=0]

Combine UIP with Taylor Rule, Phillips curve, and IS curve.

UIP $\Delta s_{t+1}^e = \tilde{R}_t - \eta_t$ $\tilde{R}_t \equiv R_t - R_t^*$ $\eta_t$ = shocks to risk premium

Taylor Rule $i_t - \pi_t = .5(\pi_t + y_t)$

Phillips curve $\pi_t = .6 \pi_{t-1} + .94 \pi_{t-1} + .25 y_t + .1 \Delta (s_t - p_t) + v_t$

IS curve $y_t = .1 (s_t - p_t) - .5 (i_{t-1}^e - \pi_{t-1}^e) + .1 \gamma y_{t-1} + \varepsilon_t$

where long rates are $i_t^e = \frac{1}{5} \sum_{s=1}^{5} i_{t,s,s+1}^e$, $\pi_t^e = \frac{1}{5} \sum_{s=1}^{5} \pi_{t,s,s+1}^e$

Shocks $v$ and $\varepsilon$ push $i$ up and $s$ down, given $s^e$. $\eta$ shocks push $i$, $s$ same.
Assume $\sigma_{\eta}^2 = 9.7\%, \sigma_{v}^2 = 1.3\%, \sigma_{\varepsilon}^2 = 1.1\%$ and simulate.

Estimated UIP coefficients

<table>
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<tr>
<th>Horizon</th>
<th>1 year</th>
<th>5 years</th>
<th>10 years</th>
</tr>
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<tbody>
<tr>
<td>Simulated data</td>
<td>-.5</td>
<td>.82</td>
<td>.78</td>
</tr>
<tr>
<td>Actual data</td>
<td>-.82</td>
<td>1.01</td>
<td>.71</td>
</tr>
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</table>
Sticky Price Intertemporal Model (O&R, Ch. 10)

Goods \( z \) are distributed continuously on the interval \([0,1]\), home goods \([0,n]\), foreign goods \([n,1]\). Produced by individuals in monopolistic competition. Aggregate consumption is an Armington CES function with elasticity of substitution \( \theta > 1 \). Consumers maximize

\[
U_t^j = \sum_{s=t}^{\infty} \beta^{s-t} \left[ \log C_s^j + \chi \log \frac{M_s^j}{P_s} - \frac{\kappa}{2} y_s(j)^2 \right]
\]

where

\[
C^j = \left[ \int_0^1 c^j(z)^{\frac{\theta-1}{\theta}} \, dz \right]^{\frac{\theta}{\theta-1}}
\]

subject to the constraint including money \( M \) and real bond holdings \( B \)

\[
P_t B^j_{t+1} + M_t^j = P_t (1 + r_t) B_t^j
\]

\[
+ M_{t-1}^j + p_t(j) r_t(j) - P_t C_t^j - P_t \tau_t
\]

World consumption is

\[
C^w = \int_0^n C^j \, dj + \int_n^1 C^j \, dj = nC + (1 - n)C^*(j)
\]

The first order conditions are [similarly for the foreign consumer]

\[
C_{t+1} = \beta (1 + r_{t+1}) C_t, \quad \frac{M_t}{P_t} = \chi C_t \left( \frac{1 + i_{t+1}}{i_{t+1}} \right)
\]

\[
\theta + 1 \frac{\beta}{\beta C_t} = \frac{\theta - 1}{\theta \kappa} (C_t^w)^{\frac{1}{\theta}} \frac{1}{C_t}, \quad 1 + i_{t+1} = \frac{P_{t+1}}{P_t} (1 + r_{t+1})
\]

with transversality condition

\[
\lim_{T \to \infty} R_{t,t+T} \left( B_{t+T+1} + \frac{M_{t+T}}{P_{t+T}} \right) = 0.
\]
Long-run Equilibrium:

\[ \bar{r} = \delta \equiv \frac{1 - \beta}{\beta}, \quad \bar{C} = \delta \bar{B} + \frac{\bar{p}(h)\bar{y}}{\bar{p}}, \]

\[ \bar{C}^* = -\left(\frac{n}{1 - n}\right) \delta \bar{B} + \frac{\bar{p}^*(f)\bar{y}^*}{\bar{p}^*}. \]

If initial \( \bar{B}_0 = 0 \), then \( \bar{C}_0 = \bar{C}_0^* = \bar{y}_0 = \bar{y}_0^* = \bar{C}_0^W \), and

\[ \bar{p}_0(h) / \bar{P}_0 = \bar{p}_0^*(f) / \bar{P}_0^* = 1 \]

\[ \frac{M_0}{P_0} = \frac{M_0^*}{P_0^*} = \frac{\chi (1 + \delta)}{\delta} \bar{y}_0, \quad \bar{y}_0 = \bar{y}_0^* = \left(\frac{\theta - 1}{\theta \kappa}\right)^{\frac{1}{2}}. \]

As \( \theta \to \infty, \bar{y}_0 \to \sqrt{1/k} \). Since preferences are assumed identical, \( P = \bar{P}^* \).

Deviations from long-run equilibrium (allowing goods prices to vary):

From

\[ P_t = \left\{ n p_t(h)^{1 - \theta} + (1 - n) \left[ \varepsilon_t p_t^*(f) \right]^{1 - \theta} \right\}^{1 / (1 - \theta)}, \]

\[ P_t^* = \left\{ n \left[ p_t(h) / \varepsilon_t \right]^{1 - \theta} + (1 - n) p_t^*(f)^{1 - \theta} \right\}^{1 / (1 - \theta)} \]

we find

\[ p_t = np_t(h) + (1 - n) \left[ \varepsilon_t + p_t^*(f) \right] \quad (27) \]

\[ p_t^* = n \left[ p_t(h) - \varepsilon_t \right] + (1 - n) \left[ p_t^*(f) \right] \quad (28) \]

where

\[ p_t \equiv dP_t / \bar{P}_0, \quad \varepsilon_t \equiv d\varepsilon_t / \bar{\varepsilon}_0, \quad p_t(h) = dp_t(h) / \bar{p}_0(h), \quad p_t^*(h) = dp_t^*(h) / \bar{p}^*_0(h) \]

\[ \varepsilon_t = p_t - p_t^*. \quad (29) \]

Demands for output are

\[ y_t = \theta \left[ p_t - p_t(h) \right] + c_t^W, \quad y_t^* = \theta \left[ p_t^* - p_t^*(f) \right] + c_t^W. \quad (30), (31) \]
Supplies of output are

\[(\theta + 1)y_t = -\theta c_t + c_t^w, (\theta + 1)y_t^* = -\theta c_t^* + c_t^w,\]  
\[(33), (34)\]

Linearized Euler equations are

\[c_{t+1} = c_t + \frac{\delta}{1 + \delta} r_{t+1}, \quad c_{t+1}^* = c_t^* + \frac{\delta}{1 + \delta} r_{t+1}\]  
\[(35), (36)\]

Demands for money are

\[m_t - p_t = c_t - \frac{r_{t+1}}{1 + \delta} - \frac{p_{t+1} - p_t}{\delta},\]  
\[\begin{align*}
 m_t^* - p_t^* &= c_t^* - \frac{r_{t+1}}{1 + \delta} - \frac{p_{t+1}^* - p_t^*}{\delta}.
\end{align*}\]  
\[(37), (38)\]

Subtracting \(m^*\) from \(m\) and using PPP,

\[m_t - m_t^* - e_t = c_t - c_t^* - \frac{1}{\delta} (e_{t+1} - e_t).\]  
\[(39)\]

Steady-state consumption-income identities are \((40), (41)\)

\[\bar{c} = \delta \bar{b} + \bar{p}(h) + \bar{y} - \bar{p}, \quad \bar{c}^* = -\left(\frac{n}{1 - n}\right) \delta \bar{b} + \bar{p}^*(f) + \bar{y}^* - \bar{p}^*\]

where \(\bar{c} = d\bar{C} / \bar{C}_0, \bar{b} = d\bar{B} / \bar{C}_0^w\), etc. are changes in steady-state values.

7 equations \((30-34, 40-41)\) in 7 unknowns \(c, c^*, y, y^*, p-p(h), p^*-p(f), c^w\).

Subtracting \((31)\) from \((30)\) and using PPP,

\[y_t - y_t^* = \theta [e_t + p_t^*(f) - p_t(h)].\]  
\[(42)\]

Subtracting \((34)\) from \((33)\),

\[y_t - y_t^* = -\frac{\theta}{1 + \theta} (c_t - c_t^*),\]  
\[(43)\]
Subtracting (41) from (40), as deviations in the equilibrium, using PPP,
\[
\bar{c} - \bar{c}^* = \left( \frac{1}{1 - n} \right) \delta \bar{b} + \bar{y} - \bar{y}^*
\]
\[- [\bar{e} + \bar{p}^*(f) - \bar{p}(h)]
\]
(44)

Using barred versions of (42) and (43) in (44), and eliminating \(y - y^*\),
\[
\bar{c} - \bar{c}^* = \left( \frac{1}{1 - n} \right) \left( \frac{1 + \theta}{2\theta} \right) \delta \bar{b}.
\]
(45)

Wealth transfer \(b\) leads to interest income gain \(\delta b\) which raises consumption but also lowers work effort. \(\bar{y} - \bar{y}^* = -\frac{1}{2} \left( \frac{1}{1 - n} \right) \delta \bar{b}\) so terms of trade improve

\[
\bar{p}(h) - \bar{e} - \bar{p}^*(f) = \left( \frac{1}{1 - n} \right) \left( \frac{1}{2\theta} \right) \delta \bar{b}.
\]
(46)

Adding up the supply equations (33) and (34) weighted by \(n \& 1-n\),
\[
(1 + \theta)\bar{y}_t^W = (1 - \theta)\bar{c}_t^W.
\]
Using this with \(c_t^W = y_t^W\) from (32) gives \(\bar{y}^W = \bar{c}^W = 0\). (47) so \(\delta \bar{b}\) has no effect on global \(c^W\) or \(y^W\).

Given sums \(x^W\) and differences \(x - x^*\), compute levels via
\[
x = x^W + (1-n)(x-x^*) \quad \text{and} \quad x^* = x^W - n(x-x^*). \quad \text{From (47) and (45),}
\]
\[
\bar{c} = \left( \frac{1 + \theta}{2\theta} \right) \delta \bar{b}, \quad \bar{c}^* = -\left( \frac{n}{1 - n} \right) \left( \frac{1 + \theta}{2\theta} \right) \delta \bar{b}.
\]
(48), (49)

and so on. Real variables are independent of nominal variables, because of price flexibility. Prices depend on money supply relative to demand.
\[
\bar{p} = \bar{m} - \bar{c}, \quad \bar{p}^* = \bar{m}^* - \bar{c}^*.
\]
(50), (51) Using PPP gives (52)
\[
\bar{e} = \bar{m} - \bar{m}^* - (\bar{c} - \bar{c}^*).
\]
Short-Run Equilibrium

Producer prices $p(h)$ and $p(f)$ are sticky in the short run, as monopolistic competitors price above marginal cost and adjust output instead of price. Consider an unanticipated permanent money shock

$$\bar{m} - \bar{m}^* \equiv m - m^*,$$  

where $m \equiv (M_1 - \bar{M}_0) / \bar{M}_0$. Note full adjustment to equilibrium takes place in period two, when prices are free to adjust to long-run equilibrium. Producer prices are fixed for one period, flexible in next (long-run) period, output is variable. So agents not on supply curves (33), (34). Income not equal to expenditure.

Current account surplus is

$$B_{t+1} - B_t = r_t B_t + \frac{p_t(h)y_t}{P_t} - C_t \quad \text{(54)}$$

Linearized, this is for home and foreign:

$$\bar{b} = y - c - (1 - n)e, \quad \left(\frac{-n}{1 - n}\right) \bar{b} = \bar{b}^* = y^* - c^* + n e \quad \text{(55)}$$

The new foreign asset holdings are permanent, since (long-run) outputs and consumptions are permanent.

Solving for short run changes (un-barred)

From the consumption Euler conditions,  

$$\bar{c} - \bar{c}^* = c - c^* \quad \text{(57)}$$

From (39), since $e_{t+1}$ is the same as $\bar{e}$

$$m - m^* - c = c - c^* - \frac{1}{\delta} (\bar{e} - e) \quad \text{(58)}$$

The solution (52) for $\bar{e}$ and (57) imply that

$$\bar{e} = (\bar{m} - \bar{m}^*) - (c - c^*)$$
Substituting this in (58) and using the permanence of the money shock (53)

We get

\[ e = (m - m^*) - (c - c^*) \]  \hspace{1cm} (60),

which is similar to the Lucas model result. Also \( e = \bar{e} \), so there is no over-shooting in this model. This gives a downward-sloping MM schedule between \( e \) and \( c-c^* \) with intercept \( m-m^* \). The second relationship comes from the balance of payments equations (55)-(56):

\[ \bar{b} = (1 - n) \left[ (y - y^*) - (c - c^*) - e \right]. \]  \hspace{1cm} (62)

From (42) , we substitute out \( y - y^* = \theta e \) (63). And from (45) we can substitute out \( \bar{b} = \left( \frac{1-n}{\delta} \right) \left( \frac{2\theta}{1+\theta} \right) (c - c^*) \) using (57). The result gives an upward-sloping GG schedule

\[ e = \frac{\delta(1 + \theta) + 2\theta}{\delta(\theta^2 - 1)} (c - c^*) \]  \hspace{1cm} (64)

Depreciation of the exchange rate raises home output relative to foreign, allowing a rise in \( c-c^* \).
Combining equations (60) and (64) gives the solution
\[ e = \frac{\delta (1 + \theta) + 2\theta}{\theta \delta (1 + \theta) + 2\theta} (m - m^*) < m - m^* \] (65)

so the exchange rate is not neutral even in long-run equilibrium, since \( e = \bar{e} \). The depreciation generates a surplus, which raises wealth and leads to a fall in home goods supply and an improvement in the terms of trade.

Other solutions are
\[ \bar{b} = \frac{2(1 - n)(\theta - 1)}{\delta (1 + \theta) + 2} (m - m^*) \]

and
\[ \bar{p}(h) - \bar{e} - \bar{p}^*(f) = \frac{\delta (\theta - 1)}{\theta \delta (1 + \theta) + 2\theta} (m - m^*). \]

O&R show that
\[ r = -\left(\frac{1 + \delta}{\delta}\right) m^w \quad \text{and} \quad c^w = -\frac{\delta}{1 + \delta} r \quad \text{and} \quad c^w = m^w = y^w, \]

so that global monetary expansion lowers world interest rates and expands world consumption and output, temporarily. Using this with (63) gives
\[ y = m^w + (1 - n)\theta \bar{e}. \]

By (65), a foreign monetary expansion appreciates the exchange rate and shifts world output towards foreign goods. So home output falls (since \( \theta > 1 \)), as in the Fleming-Mundell model:
\[ y = \frac{\delta (1 + \theta) + 2[n(1 - \theta) + \theta]}{\delta (1 + \theta) + 2} m \]
\[ + \frac{(1 - n)2(1 - \theta)}{\delta (1 + \theta) + 2} m^* \] (74)
Welfare Effects of Monetary Shocks

General monetary expansion raises world output and consumption and therefore welfare, since in monopolistic competition, price is above marginal cost and output is below the optimum.

Country-specific monetary expansions also raise world consumption, but have different effects on consumption and output due to exchange rate. For example, a foreign monetary expansion matched with a domestic contraction causes the exchange rate to fall (appreciate), domestic consumption, output, and foreign assets to fall. The fall in interest earnings will cause future output to rise even as future consumption falls. These effects can be aggregated in the (real part of the) welfare function

\[
\frac{dU^R}{\theta} = c - \left( \frac{\theta - 1}{\theta} \right) y + \frac{1}{\delta} \left[ \bar{c} - \left( \frac{\theta - 1}{\theta} \right) \bar{y} \right].
\]

and, surprisingly, they offset, leaving

\[
\frac{dU^R}{\theta} = \frac{\bar{c}^w}{\theta} = \frac{\bar{m}^w}{\theta}.
\]

so that monetary changes that do not affect world money supply do not affect individual country welfare.
Exchange Rate Dynamics and the Welfare Effects of Monetary Policy in a Two-Country Model with Home-Product Bias,

Frank Warnock


“Under OR’s assumption of identical tastes, consumption baskets are identical across countries and thus, not surprisingly, both relative and absolute consumption-based purchasing power parity (PPP) hold at all times. The assumption of identical preferences also precludes exchange rate overshooting: interest rates, both real and nominal, are identical across countries, and by uncovered interest parity (UIP) conditions, the nominal exchange rate jumps immediately to its long-run level.

“Allowing for a home-product bias makes the model consistent with two aspects of observed exchange rate behavior that macroeconomic models are notoriously poor at replicating: the extreme volatility of nominal exchange rates and the remarkable persistence of deviations from PPP. As home bias increases, Dornbusch (1976) type nominal exchange rate overshooting becomes more pronounced, as long as the elasticity of substitution between consumption and real balances is not unitary. Moreover, when there is home bias, nominal exchange rates are more volatile than fundamentals such as price levels and money supplies. Wealth transfers that accompany net foreign asset positions are spent disproportionately on domestically produced goods and, therefore, induce movements in the real exchange rate.

“When preferences are identical (and initial net foreign assets are zero), the standard ‘beggar-thy-neighbor’ effect of a monetary expansion is no longer present. By decomposing into two components, the shifting effect of increased world demand and the switching effect of an unexpected exchange rate depreciation, it is clear that this result is due to the assumption of identical preferences. The shifting effect is identical across countries and is not affected by the degree of home bias, but the switching effect, only apparent when there is home bias, increases Home utility at the expense of Foreign utility. With strong enough preferences for domestic goods, the switching effect is greater than the shifting effect and Foreign utility falls. That is, with enough home bias monetary policy is beggar-thy-neighbor.”

Table 1: A Typical Home Producer/Consumer’s Problem

\[
U_t = \sum_{s=1}^{\infty} \beta^{s-t} \left[ \log C_t + \frac{\alpha}{1-\delta} \left( \frac{M_t}{P_t} \right)^{1-\delta} - \frac{\kappa_2}{2} y_t^2 \right] 
\]

\[
P_t B_t + M_t = P_t (1 + r_{1-t}) B_{t-1} + M_{t-1} + p_t \gamma_t(\theta) - P_t C_t - P_t F_t
\]

\[
G_t = T_t + \frac{M_t - M_{t-1}}{P_t}
\]

\[
y^d(\theta) = n(c_t^d(\theta) + g_t^d(\theta)) + (1-n)(c_t^s(\theta) + g_t^s(\theta))
\]

\[
= n \left( \frac{P(\theta)}{P} \right)^{-\theta} \alpha (C+G) + (1-n) \left( \frac{P(\theta)}{EP^*} \right)^{-\theta} (2-\alpha^*)(C^*+G^*)
\]
\[
C = \left[ \int_0^1 c_H^{1/\alpha}(z) \frac{\delta-1}{\delta} \, dz + \int_n^1 (2-\alpha)^{1/\alpha}(c_F(z))^\frac{\delta-1}{\delta} \, dz \right]^{\frac{\delta}{\delta-1}}
\]
home bias \( \alpha \in (0,2) \)

where \( c_H(z), z \in [0,n], \) and \( c_F(z), z \in (n,1], \) are a Home individual's consumption of a product \( z \)

\[
e_H(z) = \alpha(p(z)/P)^{-\theta}C, \quad z \in [0,n]
\]

\[
e_F(z) = (2-\alpha)\left[ (EP^*_F(z))^\theta P \right] \frac{\delta}{\delta-1}C, \quad z \in (n,1]
\]

\[
\frac{c_H}{c_F} = \left( \frac{\alpha}{2-\alpha} \right) \left( \frac{P}{EP^*_F} \right)^{-\theta}
\]

\[
P = \left( \int_0^n \alpha p(z)^{1-\theta} \, dz + \int_n^1 (2-\alpha)(EP^*_F(z))^{1-\theta} \, dz \right)^{1/(1-\theta)}
\]

Table 2: First-Order Conditions

\( C_{\cdot i} = \beta C_t(1+r) \) \hspace{1cm} (T2.1)

\[
\frac{M_i}{P_t} = \left( \frac{C_t^{1+i_t}}{i_t} \right)^{1/\theta}
\]

\( y_t^{*+1} = \left( \frac{\theta-1}{\theta r_t} \right)^{\theta} C_t^{*+b} [n a(C_t^* + G_t^*) + (1-n) \left( \frac{EP^*_F}{P_t} \right)^{\theta} (2-\alpha^*)(C_t^* + G_t^*)] \) \hspace{1cm} (T2.3)

Table 3: Steady State Solutions equal sizes \((n = 1 - n = \frac{1}{2})\) and identical bias parameters \((\alpha = \alpha^*)\)

\( \bar{r} = \frac{1-\theta}{\beta} \) \hspace{1cm} (T3.1)

\( \bar{C} = \bar{r} \bar{B} + \bar{P} \bar{y} - \bar{G} \) \hspace{1cm} (T3.2)

\( \bar{C}^* = -\bar{r} \left( \frac{n}{1-n} \right) \frac{\bar{P}}{EP^*_F} \bar{B} + \bar{P}^* \bar{y}^* - \bar{G}^* \) \hspace{1cm} (T3.2*)

Initial Steady State Assumptions

\( \bar{B}_0 = \bar{B}_0^* = 0 \)
\( \bar{G}_0 = \bar{G}_0^* = 0 \)
\( \alpha = \alpha^* \)
Initial Steady State Levels

\[ \tilde{y}_0 = \tilde{y}_0^* = \left( \frac{\theta - 1}{\theta \bar{K}} \right)^{\frac{\kappa}{\delta_a}} \]  

\[ \tilde{M}_0 \tilde{P}_0^* = \tilde{L}_0 \tilde{p}_0^* = \left( \frac{\chi}{1 - \delta} \right)^{\frac{\kappa}{\delta_a}} \tilde{y}_0^* \]  

(T3.3)

(T3.4)

Table 4: Long-Run Equations

\[ \bar{Q}_t = \frac{E_t \tilde{p}_t^*}{P_t} - t_t = \frac{p_t}{p_t^* \bar{E}_t} \]  

real exchange rate, \( Q \), and terms of trade, \( t \)

\[ \bar{Q} = (1 - \alpha) \bar{E} \]  

(T4.1)

\[ \bar{y} - \bar{y}^* = -\alpha (2 - \alpha) \bar{E} + (\alpha - 1) (\bar{C} - \bar{C}^*) + (\bar{C} - \bar{C}^*)/\bar{y}_0 \]  

(T4.2)

\[ (\theta + 1) (\bar{y} - \bar{y}^*) = -\theta (\epsilon - \bar{K}^*) + (\alpha - 1) (\bar{C} - \bar{C}^*) - (2 - \alpha) (\alpha - 1) \bar{E} \]  

(T4.3)

\[ (\bar{C} - \bar{C}^*) = \frac{\beta \bar{D} \bar{B}}{\bar{y}_0} + (2 - \alpha) \bar{E} + (\bar{y} - \bar{y}^*) - \frac{\bar{D} \bar{G}_t - \bar{D} \bar{G}^*}{\bar{y}_0} \]  

(T4.4)

\[ \bar{C} = n \bar{C} + (1 - n) (\bar{E} + \bar{p}_t^* - \bar{p} + \bar{C}^*) \]  

(T4.5)

\[ \bar{P} - \bar{p}_t^* = (\bar{M} - \bar{M}^*) - \frac{1}{\epsilon} (\bar{C} - \bar{C}^*) \]  

(T4.6)

Table 5: Short-Run Equations

\[ \bar{Q} = (\alpha - 1) \bar{E} \]  

(T5.1)

\[ \bar{y} - \bar{y}^* = \alpha (2 - \alpha) \bar{E} + (\alpha - 1) (\bar{C} - \bar{C}^*) + (\alpha - 1) (\bar{D} \bar{G}_t - \bar{D} \bar{G}_t^*)/\bar{y}_0 \]  

(T5.2)

\[ (\bar{C} - \bar{C}^*) = \frac{2 \bar{D} \bar{B}}{\bar{y}_0} - (2 - \alpha) \bar{E} + (\bar{y} - \bar{y}^*) - \frac{\bar{D} \bar{G}_t - \bar{D} \bar{G}_t^*}{\bar{y}_0} \]  

(T5.3)

\[ (\bar{M} - \bar{M}_t^*) = (2 - \alpha) \bar{E} + (\bar{E} - \bar{E}^*) \]  

(T5.4)

Table 6: Long-Run Semi-Reduced Forms

\[ (1 - \alpha + \theta) (\bar{C} - \bar{C}^*) = -\theta (2 - \alpha) (\alpha - 1) \bar{E} + (\theta + 1) \frac{\bar{D} \bar{B}}{\bar{y}_0} - \frac{2 - 2 \alpha + 3 \bar{D} \bar{G}_t - \bar{D} \bar{G}_t^*}{2 \bar{y}_0} + \frac{\theta - 1}{2} (\epsilon - \bar{K}^*) \]  

(T6.1)

\[ \bar{y} - \bar{y}^* = \frac{1}{1 + \theta} \left[ -\theta (\bar{C} - \bar{C}^*) - (2 - \alpha) (\alpha - 1) \theta \bar{E} + (\alpha - 1) \frac{\bar{D} \bar{G}_t - \bar{D} \bar{G}_t^*}{\bar{y}_0} - \theta (\epsilon - \bar{K}^*) \right] \]  

(T6.2)
\[ i = \frac{1}{(2-\alpha)(\alpha \theta + 1)} \left( \alpha(\bar{L} - \bar{L}^*) + (\alpha - 1) \frac{\bar{G} - \bar{G}^*}{\bar{y}_0} + (\bar{r} - \bar{r}^*) \right) \]  
\[ = \frac{1}{(2-\alpha)(1-\alpha + \alpha \theta)} \left( -\alpha(\bar{y} - \bar{y}^*) + (\alpha - 1) \frac{\bar{G} - \bar{G}^*}{\bar{y}_0} - (\alpha - 1)(\bar{r} - \bar{r}^*) \right) \]  

(T6.3)

Table 7: MM and GG Curves

Money Shocks

\[ \bar{E} = \left( \frac{1 + \bar{r}e}{1 + \bar{r}e(2-\alpha)} \right) (\bar{M} - \bar{M}^*) - \left( \frac{(1 + \bar{r}e)(2-\alpha)(1+\alpha \theta) - e(1-\alpha)}{e(1+\bar{r}e(2-\alpha))(2-\alpha)(1+\alpha \theta)} \right) (\bar{C} - \bar{C}^*) \]  
\[ = \left( \frac{1}{\alpha \theta - 1} + \frac{2(1-\alpha + \theta(1+\alpha \theta))}{\bar{r}(1+\theta)(2-\alpha)(\alpha \theta^2 - 1)} \right) (\bar{C} - \bar{C}^*) \]  

(T7.1)

Government Spending Shocks

\[ \bar{E} = \left( \frac{1}{1+\bar{r}e(2-\alpha)} \right) \left( \bar{G} - \frac{1 + \bar{r}e}{e} (\bar{C} - \bar{C}^*) \right) \]  
\[ \bar{E} = \left( \frac{\bar{r}(1+\theta)(2-\alpha) + 2(1-\alpha + \theta)}{\bar{r}(1+\theta)(2-\alpha)(1-\alpha \theta)} \right) (\bar{C} - \bar{C}^*) - \frac{2 \theta}{\bar{r}(1+\theta)(1-\alpha)} \bar{G} + \frac{(1-2\alpha + 3\theta + (\theta + 1)(2-\alpha)}{\bar{r}(1+\theta)(2-\alpha)(1-\alpha \theta)} \left( \frac{\bar{G} - \bar{G}^*}{\bar{y}_0} \right) \]  

(T7.2)

Productivity Shocks

\[ \bar{E} = \frac{- (\bar{C} - \bar{C}^*)(\bar{G}(\theta + 1)(2-\alpha) + 2(1-\alpha + \theta) + 2(\bar{r}(2-\alpha) + \theta - (\theta - 1)(\bar{r} - \bar{r}^*)))}{\bar{r}(\theta + 1)(2-\alpha)(1-\alpha \theta)} \]  

(T7.4)

Exchange Rate Overshooting (if \( e \neq 1 \) and \( \alpha \neq 1 \))

\[ \bar{E} = \bar{E} \frac{1 + \bar{r}e}{1 + \bar{r}(\alpha - 1 + e(2-\alpha))} + \bar{r}(\varepsilon - 1)(\alpha - 1) \bar{i} \]
Figures

Figure 1: A Home Individual’s Intratemporal Consumption Choice

Figure 2: An unanticipated permanent relative Home money supply increase
Small Open Economy Model (Gali & Monacelli, NBER 8905)  
(see also Clarida, Gali, & Gertler, AER, May 2001)  
Consumers maximize utility of consumption, including home and foreign goods, less effort ($\alpha$ is an index of openness):

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U \left( C_t \right) - V \left( N_t \right) \right]$$  \hspace{1cm} (1)

$$C_t = \left( 1 - \alpha \right)^{\frac{1}{\eta}} C_{H,t}^{\frac{\mu+1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\mu+1}{\eta}}$$  \hspace{1cm} (2)

where home and foreign goods are aggregated from

$$C_{H,t} = \left( \int_0^1 C_{H,t} \left( i \right)^{\frac{\mu}{\eta}} di \right)^{\frac{1}{\mu+1}} ; C_{F,t} = \left( \int_0^1 C_{F,t} \left( i \right)^{\frac{\mu}{\eta}} di \right)^{\frac{1}{\mu+1}}$$

$\eta$ is elasticity of substitution between home and foreign goods, $\varepsilon$ is elasticity of substitution within types of goods. Budget constraints are

$$\int_0^1 \left[ P_{H,t}(i)C_{H,t}(i) + P_{F,t}(i)C_{F,t}(i) \right] di + E_t \left\{ Q_{t,t+1}D_{t+1} \right\} \leq D_t + W_tN_t + T_t$$  \hspace{1cm} (3)

where $Q_{t,t+1}$ is the stochastic discount factor for nominal dividends $D_{t+1}$. Complete contingent claims exist, but no durable goods. Optimal allocation of goods within types imply:

$$C_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} ; C_{F,t}(i) = \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}$$  \hspace{1cm} (4)

for all $i$, where $P_{H,t} = \left( \int_0^1 P_{H,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} ; P_{F,t} = \left( \int_0^1 P_{F,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$.

Optimal allocation between domestic and foreign yields

$$C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t ; C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t$$  \hspace{1cm} (5)

where $P_t = \left[ \left( 1 - \alpha \right) P_{H,t}^{1+\eta} + \alpha P_{F,t}^{1-\eta} \right]^\frac{1}{\mu+1}$ is the CPI. Then the budget constraint becomes

$$P_tC_t + E_t \left\{ Q_{t,t+1}D_{t+1} \right\} \leq D_t + W_tN_t + T_t$$  \hspace{1cm} (6)

Assuming $U(C_t) = \frac{C_t^{1+\sigma}}{1+\sigma} ; V(N_t) = \frac{N_t^{1+\phi}}{1+\phi}$ the consumer optimality conditions are:
\[ C_t^\sigma N_t^{\phi} = \frac{W_t}{P_t} \]

\[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = Q_{t,t+1} \]  \hspace{1cm} (7,8)

Expectation of (8) gives Euler equation

\[ \beta R_t E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = 1 \]  \hspace{1cm} (9)

where \( R_t^{-1} = E_t \{ Q_{t,t+1} \} \) is the price of a riskless one-period bond, \( R_t \) is its gross return in domestic currency. Money does not appear in the model.

In log-linearized form (7) and (9) can be written as

\[ w_t - p_t = \sigma c_t + \phi n_t \]
\[ c_t = E \{ c_{t+1} \} - \frac{1}{\beta} (r_t - E_t \{ \pi_{t+1} \} - \rho) \]

where \( \rho \equiv -\log \beta \) is the time discount rate and \( \pi_t \equiv \Delta p_t \equiv \Delta \log P_t \) is CPI inflation. Log linearization of the CPI formula around a steady state with \( P_{H,t} = P_{F,t} \) gives

\[ p_t = (1 - \alpha) P_{H,t} + \alpha P_{F,t} = p_{H,t} + \alpha s_t \]  \hspace{1cm} (10)

where \( s_t = P_{F,t} - P_{H,t} \) is the terms of trade. (10) holds exactly if \( \eta = 1 \). The rate of inflation of domestic goods is

\[ \pi_{H,t} \equiv P_{H,t+1} - P_{H,t} = \pi_t - \alpha \Delta s_t \]

Assume that the small open economy doesn’t affect world inflation, so \( \pi_t^* = \pi_{F,t}^* \). Also assume law of one price, so \( P_{F,t}(i) = \varepsilon_t P_{F,t}^*(i) \), \( i \), or \( P_{F,t} = \varepsilon_t P_{F,t}^* \), where \( \varepsilon_t \) is the nominal exchange rate. Hence \( P_{F,t} = \varepsilon_t + P_{F,t}^* \), so the terms of trade will be

\[ s_t \equiv \varepsilon_t + P_{F,t}^* - P_{H,t} \]

Define the real exchange rate as \( Q_t \equiv \frac{\varepsilon_t P_{F,t}^*}{P_t} \) so that its log is

\[ q_t = s_t + P_{H,t} - p_t = (1 - \alpha) s_t \]

With complete markets, the return on a riskless foreign bond in domestic currency is \( \varepsilon_t R_t^{-1} = E_t \{ Q_{t,t+1} \} \), which combines with

\[ R_t^{-1} = E_t \{ Q_{t,t+1} \} \]  \hspace{1cm} (10)

\[ E_t \left\{ Q_{t,t+1} \left[ R_t - R_t^* (\varepsilon_{t+1} / \varepsilon_t) \right] \right\} = 0 \]
Linearizing(!),
\[ r_t - r_t^* = E_t \{ \Delta e_{t+1} \} \]  \hspace{1cm} (17)

Using the terms of trade in (17) yields
\[ s_t = (r_t^* - E_t \{ \pi_{t+1}^* \}) - (r_t - E_t \{ \pi_{t+1} \}) + E_t \{ s_{t+1} \} \]  \hspace{1cm} (18)

which can be solved forward in the usual way.

Firms produce differentiated products with linear technology
\[ Y_t(i) = A_i N_t(i) \]

Log marginal cost is common across firms
\[ mc_t^n = -\nu + w_t - a_t; \text{ where } a_t = \rho_o a_{t-1} + \varepsilon_t \]

Linearizing aggregated output,
\[ \dot{y}_t = n_t + a_t \]  \hspace{1cm} (20)

Prices are set by a staggered Calvo (1983) process, as \( 1 - \theta \) randomly selected firms reset prices each period, resulting in
\[ \bar{p}_{H,t} = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \{ mc_{t+k}^n \} \]  \hspace{1cm} (21)

World output is determined by market clearing \( \dot{y}_t = c_t^* \) and the consumer Euler equation:
\[ y_t^* = E_t \{ y_{t+1}^* \} - \frac{1}{\sigma} (r_t^* - E_t \{ \pi_{t+1}^* \} - \rho) \]  \hspace{1cm} (22)

Because of risk-sharing under complete markets, domestic consumption will be cointegrated with world consumption via
\[ C_t = \mathcal{Q}_t^* \]  \hspace{1cm} (15)

or
\[ c_t = c_t^* + \left( \frac{1 - \alpha}{\sigma} \right) s_t \]  \hspace{1cm} (16)

Market clearing in the small economy gives
\[ Y_t(i) = C_{H,t}(i) + C_{H,t}^*(i) \]
\[ = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left[ \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} (1 - \alpha) C_t + \left( \frac{P_{H,t}}{\varepsilon_i P_t^*} \right)^{-\eta} \right]^* \]  \hspace{1cm} (23)
\[ = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \mathcal{Q}_t^* \left[ \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} (1 - \alpha) \mathcal{Q}_t^* + \left( \frac{P_{H,t}}{\varepsilon_i P_t^*} \right)^{-\eta} \right]^* \]
Aggregating via
\[ Y_i = \left[ \int_0^1 Y_i(i) \frac{\partial Y_i}{\partial i} \right]^{\frac{1}{\alpha}} \]
leads to

\[ Y_i = 9Y_i^* S_i^\epsilon \left[ (1 - \alpha) \mathcal{Q}_i^{\frac{1}{\epsilon}} + \alpha^* \right] \quad (24) \]

approximated as

\[ y_i = y_i^* + \frac{w_a}{\sigma} s_i \quad (25) \]

where \( w_a \equiv 1 + \alpha(2 - \alpha)(\sigma \eta - 1) > 0 \) depends on openness \( \alpha \). Using (16) to eliminate \( s_i \) leads to

\[ c_i = \Phi_a y_i + (1 - \Phi_a) y_i^* \] with \( \Phi_a \equiv \frac{1 - \alpha}{w_a} \quad (27) \]

Substituting (27), (25), (11) into the consumer’s Euler equation gives

\[ y_i = E_i \{ y_{i+1}^* \} - \frac{w_a}{\sigma} \left( r_i - E_i \{ \pi_{H,i+1} \} - \rho \right) + (w_a - 1) E_i \{ \Delta y_{i+1}^* \} \quad (29) \]

The level of domestic output is negatively related to current and expected future real interest rates and to anticipated world output growth with a coefficient \( w_a - 1 \) whose sign is positive if \( \sigma \eta > 1 \).

Trade Balance: Let \( nx_i \equiv \left( \frac{1}{Y} \right) \left( Y_i - \frac{P_i}{P_{H,i}} C_i \right) \) be net exports as a percent of steady state output. If \( \sigma = \eta = 1 \), from (15) and (24), \( P_{H,i} Y_i = PC_i \) so trade is always balanced in this special case. More generally, \( nx_i = y_i - c_i - \alpha s_i \), which with (25) and (27) implies

\[ nx_i = (1 - \Phi_a)(y_i - y_i^*) - \alpha s_i = \frac{\alpha \Lambda}{w_a} (y_i - y_i^*) , \]

where \( \Lambda \equiv (2 - \alpha)(\sigma \eta - 1) + (1 - \sigma) \), which is zero if \( \sigma = \eta = 1 \).

\( \Lambda > 0 \) is the equivalent to the Marshall-Lerner condition in this model.

Supply Side: Marginal Cost and Inflation Dynamics
For the world economy, optimal staggered price setting yields

\[ \pi_i^* = \beta E_i \{ \pi_{i+1}^* \} + \lambda \tilde{mc}_i \quad (31) \]

where \( \tilde{mc}_i \equiv mc_i^* + \mu \) is the (log) marginal cost expressed as a deviation from its steady state value \(-\mu \) and the slope coefficient is \( \lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \).

Real marginal cost is

\[ mc_i^* = v^* + (w_i^* - p_i^*) - a_i^* \]

\[ = v^* + \sigma c_i^* + \phi n_i^* - a_i^* \]

\[ = v^* + (\sigma + \phi) y_i^* - (1 + \phi)a_i^* \quad (32) \]
where $\nu^* \equiv -\log(1 - \tau^*)$ with the optimal subsidy $\tau^* = \frac{1}{\xi}$ defined below.

Marginal Cost and Inflation Dynamics for the Small Open Economy

Domestic inflation follows

$$\pi_{H,t} = \beta E_t \{\pi_{H,t+1}\} + \lambda \hat{mc}_t$$

where

$$mc_t = -\nu + w_t - a_t - p_{H,t}$$
$$= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t$$
$$= -\nu + \sigma c_t + \phi n_t + \alpha s_t - a_t$$
$$= -\nu + \sigma y^*_t + \phi y_t + s_t - (1 + \phi) a_t$$

and $\nu_t \equiv -\log(1 - \tau)$.

Substituting from (25) to eliminate the terms of trade $s_t$,

$$mc_t = -\nu + \left(\frac{\sigma}{w_{\tau}} + \phi\right)y_t + \sigma \left(1 - \frac{1}{w_{\tau}}\right)y_t^* - (1 + \phi)a_t$$

(35)

Canonical Representation of the World Economy:

Let the output gap relative to flexible price equilibrium be $\tilde{y}_t \equiv y_t^* - \bar{y}_t$, and similarly for the world output gap. Under flexible prices, real marginal costs will be constant at $mc^*_t \equiv -\mu$. Evaluating (32), we have

$$\bar{y}_t^* = \Omega_0 + \Gamma_0 a_t^*$$

(36)

where $\Omega_0 \equiv \frac{\nu^* - \mu}{\sigma + \phi}$ and $\Gamma_0 \equiv \frac{1 + \phi}{\sigma + \phi}$. Also, the deviation of real marginal cost from steady state is

$$\hat{mc}_t = (\sigma + \phi)\tilde{y}_t^*$$

which yields the New Keynesian Phillips Curve:

$$\pi_t^* = \beta E_t \{\pi_{t+1}^*\} + \kappa_0 \tilde{y}_t^*$$

(37)

where $\kappa_0 \equiv \lambda(\sigma + \phi)$. Then (22) can be written in terms of the world output gap as

$$\tilde{y}_t^* = E_t \{\tilde{y}_{t+1}^*\} - \frac{1}{\sigma} (r_t^* - E_t \{\pi_{t+1}^*\} - r_{\tau t}^*)$$

(38)

where $r_{\tau t}^* \equiv -\sigma(1 - \rho_a^*)\Gamma_0 a_t^* + \rho$ is the natural real rate of interest.

Equations (37 and (38) with an interest rate rule describe the evolution of the world economy.
Dynamics of the Small Open Economy:

The natural level of output comes from setting $mc_t = \mu$ in (35) and solving for

$$\tilde{y}_t = \Omega + \Gamma a_t + \Theta y_t^*$$  \hspace{1cm} (39)

where $\Omega = \frac{w_0 (v-\mu)}{\sigma w_\mu}, \Gamma = \frac{w_0 (1+\phi)}{\sigma w_\mu} > 0, \Theta = \frac{\sigma (1-w_\mu)}{\sigma w_\mu}$. Also real marginal cost will be related to the output gap by

$$\tilde{mc}_t = \left(\frac{\sigma}{w_\mu} + \phi\right) \tilde{y}_t$$

combined with (33) yields the New Keynesian Phillips Curve

$$\pi_t^{H,t} = \beta E_t \{\pi_{H,t+1}\} + \kappa a_t \tilde{y}_t$$  \hspace{1cm} (40)

where $\kappa = \lambda(\frac{\sigma}{w_\mu} + \phi)$, which depends on openness. Using (29), the “IS” curve of the small open economy becomes

$$\tilde{y}_t = E_t \{\tilde{y}_{t+1}\} - \frac{w_\alpha}{\sigma} \left(\tilde{y} - E_t \{\pi_{H,t+1}\} - \tilde{r}_t\right)$$  \hspace{1cm} (41)

where

$$\tilde{r}_t = \rho - \frac{\sigma (1+\phi)(1-\rho_\alpha)}{\sigma + \phi w_\alpha} a_t - \phi \Theta a_t \left\{\Delta y_{t+1}^*\right\}$$

the open economy’s natural rate of interest. Thus the IS curve for the open economy is analogous to that of the closed economy, but with a coefficient that depends on openness.

Optimal Monetary Policy in the World Economy

Choose an optimal employment subsidy $\tau^* = \frac{1}{\varepsilon}$ to offset the monopolistic distortion in the goods market, and similarly in the small open economy. Hence the optimum is the flexible price equilibrium. In the special case $\sigma = \eta = 1$ where trade is always balanced, Gali & Monacelli show that the utility of the representative consumer, as a percent of steady state consumption, is

$$W \approx -\frac{(1-\alpha)}{2} \sum_{t=0}^\infty \beta^t \left[\frac{\varepsilon}{\lambda} \pi_{H,t}^2 + (1+\phi) \tilde{y}_t^2\right]$$  \hspace{1cm} (52)

Clarida, Gali, and Gertler maximize this subject to (40) and (41) under discretion and derive a “leaning against the wind” rule on inflation of the form

$$\tilde{y} = -\frac{\kappa (\varepsilon / \lambda)}{(1+\phi)} \pi_{H,t}$$
Black Markets and Parallel Exchange Markets

Edwards (Ch. 3)

 Tradable and non-tradable goods, domestic and foreign money, fixed commercial exchange rate $E$, floating financial rate $\delta$.

$$A = M + \delta F, \ a = \frac{A}{E} = \frac{M + \delta F}{E} = m + \rho F, \ \rho = \frac{\delta}{E}$$

Portfolio Balance $m = \sigma\left(\frac{\dot{\delta}}{\delta}\right)\rho F, \ \sigma' < 0$

Capital controls $\hat{F} = 0, \ F_0 > 0$

Goods demand = supply $C_T(e,a) = Q_T(e), \ e = \frac{E}{P_N}, C_T < 0, C_{T_a} > 0, Q_{T_e} > 0$

$$C_N(e,a) = Q_N(e), \ C_{N_e} > 0, C_{N_a} > 0, Q_{N_e} < 0, P_T^* = 1$$

Government Budget $G = P_N G_N + EG_T, \ \frac{EG_T}{G_N} = \lambda, \ G = t + \hat{D}$

$$g \equiv \frac{G}{E} = g_N + g_T, \ g_N \equiv \frac{P_N G_N}{E} = \frac{G_N}{e}$$

External Balance $CA = Q_T(e) - C_T(e,a) - G_T$

International Reserves $\hat{R} = CA, \ \hat{M} = \hat{D} + E\hat{R}, \ R_0 > 0$

Long-run Equilibrium $G = t, \ \hat{D} = 0, CA = 0, \hat{R} = 0$

Internal Balance $C_N(e,a) + eg_N = Q_N(e) \Rightarrow$ ERER $e = v(a, g_N), \ v_a < 0, v_{g_N} < 0$

Portfolio Equilibrium given $E, \ \frac{\dot{\delta}}{\rho} = \frac{\dot{\delta}}{\delta} = L \left(\frac{m}{\rho F}\right), \ L' < 0$

External Balance $\dot{m} = Q_T(e) - C_T(e,a) + \frac{G_N}{e} - \frac{t}{E}$

Given $\rho_0, m_0$, ERER $\tilde{e}_{LR} = v(a_0, g_{N0}) = v(m_0 + \rho_0 F_0, g_{N0})$
At B, \( m_1 > m_0 \), the spread \( \rho = \frac{\delta}{E} \) jumps, followed by appreciation \( \frac{\delta}{\delta} < 0 \). At B, \( e \) is over-valued, since \( de = \frac{\partial v}{\partial a} \cdot (dm + F_0 \partial \rho) < 0 \) appreciates \((v_a < 0)\). So \( P_N \) rises relative to \( E \). During the transition back to equilibrium, \( m \) falls as \( \dot{R} < 0 \) due to BOP deficit. Devaluation (rise in \( E \)) accelerates the return to equilibrium as \( m, \rho \) fall with rise in \( E \).

Suppose reserves are insufficient. Then devaluation will be anticipated at time \( t_1 \), when reserves run out. Therefore \( \delta \) will jump to the unstable path that leads to the saddle path after the devaluation takes place.

At time \( t_0 \), \( m_0 \) jumps to \( m_1 \), \( \rho \) jumps to \( C > B \), then depreciates further to D at time \( t_1 \), when reserves run out. Devaluation drops down to E on SS and so back to equilibrium.
Black Markets (Kharas & Pinto, *Restat*, 1989)

Commercial rate $e$, black market rate $b$. $X_2$ is illegal foreign trade, $X_3$ is legal foreign trade, importable inputs $I$ purchased at black market rate $b$.

Final goods $H$ produced with imported inputs and labor.

Production functions $H = L^a I^{1-a}$, $X_1 = L_i$,

Cost of smuggling $C(X_2), C(0) = 0, C' > 0, C'' > 0$

Maximize $p_H H + bp_x X_2 + ep_x X_3 - bp_x C(X_2) - w(L_1 + L_2 + L_3) - bI$

$bp_x (1 - C'(X_2)) = ep_x = w$

let $c = C'^{-1}$, $\phi = b/e$, $X_2 = c\left(1 - \frac{1}{\phi}\right)$

BOP $p_x (X_2 + X_3) = I + g + \hat{F}$, $g =$ government imports

Portfolio Balance $M = \frac{\lambda}{1 - \lambda}\frac{b F}{\hat{b}/b}$, $\lambda = \frac{M}{M + b F}$, $\lambda' < 0$

Assume $\alpha = 0$, so $H = I$, all production is exported

Private Spending $P_H H = a(M + b F)$

let $m = M/e$, Then $\hat{F} = p_x \bar{L} - g - a\left(\frac{m}{\phi} + F\right)$

$\dot{M} = \dot{D} = e(g - t) \Rightarrow \dot{m} = g - t - m \cdot \hat{e}$

Portfolio Balance implies $\frac{m}{\phi} + F = \frac{F}{1 - \lambda}$

Steady-state Equilibrium $\hat{F} = 0 \Rightarrow p_x \bar{L} - g = m/\phi + F = \frac{F}{1 - \lambda}$

$\dot{m} = 0 \Rightarrow m = \frac{g - t}{\hat{e}}$

$\therefore F = (1 - \lambda) \cdot \frac{p_x \bar{L} - g}{a}$ and $\frac{m}{\phi} = \lambda \cdot \frac{p_x \bar{L} - g}{a}$

Hence $\phi = \frac{m \cdot a}{\lambda \cdot \left(p_x \bar{L} - g\right)} = \frac{a \cdot g - t}{\lambda \cdot p_x \bar{L} - g}$ or $\hat{e} \cdot \phi = \frac{a \cdot (g - t)}{\lambda \cdot (p_x \bar{L} - g)}$
Observe government deficit is financed by inflation tax. Let $\theta \equiv \lambda \cdot \hat{e}$ be the unit inflation tax.

Keep inflation tax on the positive side of the Laffer curve. Note tradeoff between black market tax on exports $\varphi$ and inflation tax $\hat{e}$, for given deficit $g - t$. Therefore, rise in inflation tax reduces need for export tax revenue.
Balance of Payments Crisis Models

Type I Crisis – Fundamentals are inconsistent with pegged exchange rate. Krugman (1979), O&R (Ch. 8.4.2)

Demand for money
\[
\ln \left( \frac{M_t}{P_t} \right) = a - \eta \bar{e}_t, \; \bar{e}_t = i_t^* + \bar{e}_t^*, \; i_t^* \equiv 0, \; P_t = E_t + P_t^*, \; P_t^* \equiv 1
\]
\[m_t - e_t = -\eta \bar{e}_t.\] and with fixed exchange rate  \[\bar{m} = \bar{e}.\]

Money supply \[M_t = B_{H,t} + \xi B_{F,t} \] expanding because of growth in government debt, bought by central bank and held as domestic asset \[B_H.\]

\[\frac{\dot{B}_H}{B_H} = b_H = \mu,\]

As long as the exchange rate is pegged and output is fixed, the demand for money remains constant. Thus rise in \(B_H\) requires fall in \(B_F\). \[\xi \dot{B}_F = -\dot{B}_H.\] The inconsistent fundamentals will become evident as reserves fall towards zero. Prior to the time of collapse, it will become evident that \(E\bar{e} > 0\) and therefore \(i_t > i_t^*\) so the demand for \(M\) will fall, as speculators attack the currency. To find the effect on behavior prior to collapse, find the “shadow” exchange rate that would prevail if the attack has already occurred and \(m_t = b_{H,t} \) and \(\bar{e}_t = \mu:\)

\[\bar{e}_t = b_{H,t} + \eta \mu,\] The collapse occurs when \(\bar{e}_T = \bar{e}.\) Otherwise there would be a perfectly foreseen jump in the exchange rate, which would either bring the collapse forward or back.
Time of attack $T$ from substituting $b_{H,t} = b_{H,0} + \mu t$, into the shadow rate equation at time $T$ to get 

$$\bar{\epsilon} = b_{H,0} + \mu T + \eta \mu .$$

whence

$$T = \frac{\bar{\epsilon} - b_{H,0} - \eta \mu}{\mu} .$$

or

$$T = \frac{\log(B_{H,0} + B_{F,0}) - b_{H,0} - \eta \mu}{\mu} .$$

Larger initial reserves $B_{F,0}$ postpone the attack. There is a regime shift at time $T$.

Type II Crisis – Fundamentals ok, speculators anticipate a regime change induced by the attack itself. (Obstfeld, 1996, O&R Ch. 9.5.4)

Government objective function

$$\mathcal{L}_t = (y_t - \bar{y})^2 + \chi \pi_t^2 + C(\pi_t) ,$$

Public behavior

$$y = \bar{y} + (\pi - \pi^e) - z \quad \text{and} \quad \bar{y} - \bar{y} = k > 0$$

$$\mathcal{L} = (\pi - \pi^e - k - z)^2 + \chi \pi^2$$

$$2(\pi - \pi^e - k - z) + 2 \chi \pi = 0$$

$$\pi = \frac{k + \pi^e + z}{1 + \chi} \quad y = \bar{y} + \frac{k - \chi \pi^e - \chi z}{1 + \chi} ,$$

$$\mathcal{L}^{\text{FLEX}} = \frac{\chi}{1 + \chi} (k + \pi^e + z)^2 . + C(\pi)$$

$$\mathcal{L}^{\text{FIX}} = (k + z + \pi^e)^2 > \mathcal{L}^{\text{FLEX}} .$$

Devalue if $\mathcal{L}^{\text{FLEX}} + C(\pi) < \mathcal{L}^{\text{FIX}} \text{ for } z > \bar{z}$

Revalue if $\mathcal{L}^{\text{FLEX}} + C(\pi) < \mathcal{L}^{\text{FIX}} \text{ for } z < \bar{z}$

$$\bar{z} = \sqrt{c(1 + \chi) - k - \pi^e} , \quad \bar{z} = -\sqrt{c(1 + \chi) - k - \pi^e} .$$
Assume $z$ is uniformly distributed over the interval $[-Z, Z]$. Then

\[
E\pi = E\{\pi \mid z < \bar{z}\}Pr(z < \bar{z})
+ E\{\pi \mid z > \bar{z}\}Pr(z > \bar{z}),
\]

Also, $E\pi = \pi^e$, so that $\pi^e$ has possible multiple equilibria, as shown by O&R:

![Graph showing multiple equilibria](image)

Additional types of crisis behaviors:


“Convertibility Risk: The Precautionary Demand for Foreign Currency in a Crisis” IMF WP 01/210
Stanley W. Black, Charis Christofides, and Alex Mourmouras

Outline of Paper

• Background – the Korean Crisis of December 1997
• Exchange Market Pressure
• A Model of Currency Inconvertibility
  – Model Structure
  – Effects of Convertibility Risk on Private Behavior
  – Interest Rate Defense of Domestic Currency
  – Devaluation as an Alternative
• A Cross-section Estimate of Convertibility Risk
• An Empirical Estimate of Demand for Money in Korea allowing for Convertibility Risk
• Implications for Exchange Rate Policy Management

Background - the Korean Crisis of December 1997

• Pressure on reserves from banks’ unwillingness to roll over dollar loans after Thai baht collapse.
• 5% daily limit on exchange rate movements, up from 3%.
• Dec. 6 IMF Agreement limited loss of reserves for month.
• Combined price and quantity limits led to shortened market days, effective market closure, rationing of dollars.
• Panic dollar buying as response of public to rationing.
• Shift in demand for money offset gain from rationing.
• New IMF Agreement Dec. 24 dropped 5% limit as of Dec. 16. Exchange rate dropped sharply, then recovered.

Figure 3. Foreign Exchange Holdings and Exchange Rate
Exchange Market Pressure

Girton-Roper (1977)

- Money Demand: \( m - p = y - v \)
- \( m = d \log(M/M^*) \), \( p = d \log(P/P^*) \), \( y = d \log(Y/Y^*) \)
- Money Supply: \( m = d + r \)
- \( d = (D/M)-(D^*/M^*) \), \( r = (R/M)-(R^*/M^*) \)
- Purchasing Power Parity: \( e = p (\ p^*) \)
- \( M^d = M^s \): \( d + r = e + y - v \)
- Exchange Market Pressure \( = r - e = y - v - d \)

Figure 2. Exchange Market Pressure

Figure 4. Exchange Market Pressure
January 1997 - December 1998
A Model of Currency Inconvertibility

- Two periods
- Two nonstorable goods
- Tradable domestic good $x$ with price $p$
- Imported good $y$
- Two assets:
  - domestic currency $h$
  - foreign currency $f$, exchangeable for $h$ at 1 today
- Many agents, each with endowment $W$ units of $x$
- utility of consumption: $u(x, y)$ in next period
- gross rate of return on domestic currency $R > 1$
- $\alpha =$ probability agents are rationed out of forex market
- $b =$ forex purchase if not rationed next period

- State 1: Convertibility
  - Probability $1 - \alpha$
  - $h/p + f = W$
  - $x^1 = Rh/p - b$
  - $y^1 = f + b$
  - If $\alpha = 0$, set $f = 0$. When $\alpha > 0$, set $f > 0$. 

  $EU = (1 - \alpha) u[(W-f)R-b,f+b] + \alpha u[(W-f)R,f]$ 

  $b$: $u_1(x^1, y^1) = u_2(x^1, y^1)$
  $f$: $(1 - \alpha)(R-1) u_1(x^1, y^1) = \alpha - R u_1(x^2, y^2) + u_2(x^2, y^2)$

  $\frac{df}{d\alpha} = \frac{1 - R}{\alpha} \times \frac{u_2^1}{\alpha [R^2 u_1^2 - R (u_1^2 + u_2^2) + u_2^2] + (1 - \alpha) (R - 1)^2 u_2^2} > 0$

- State 2: Inconvertibility
  - Probability $\alpha$
  - $h/p + f = W$
  - $x^2 = Rh/p$
  - $y^2 = f$

$EU = (1 - \alpha) u[(W-f)R-f,b] + \alpha u[(W-f)R,f]$ 

$b$: $u_1(x^1, y^1) = u_2(x^1, y^1)$

$f$: $(1 - \alpha)(R-1) u_1(x^1, y^1) = \alpha - R u_1(x^2, y^2) + u_2(x^2, y^2)$

$\frac{df}{d\alpha} = \frac{1 - R}{\alpha} \times \frac{u_2^1}{\alpha [R^2 u_1^2 - R (u_1^2 + u_2^2) + u_2^2] + (1 - \alpha) (R - 1)^2 u_2^2} > 0$

Figure 1. Share of Assets Held in Foreign Currency

![Figure 1](image-url)
**Interest Rate Defense**

- Raise domestic interest rates to increase attractiveness of domestic vs. foreign assets.
- Possible offset to increased precautionary demand or foreign currency.

*The Interest Rate defense is ineffective if $\alpha$ is high*

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**Devalue Exchange Rate vs. Rationing**

- Alternative option of devaluation to $e < 1$ to avoid market closing.
- Precautionary demand for foreign currency depends on expected return vs. domestic:

  $$\alpha(1/e - R) + (1- \alpha)(1 - R)$$

- So interest rate is a defense against devaluation risk (higher R offsets risk of $e < 1$).
- But interest rate less effective against rationing risk.

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**Tradeoff of Interest Rate vs. Devaluation**

*(for given $f/W$)*

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[Diagram: Graph showing the tradeoff]
Cross-section Demand for Money

- $V_i = \alpha + \beta \pi_i + \gamma Y_i + \delta D_i + u_i$
- $V_i =$ velocity of M2, 1991-1995
- $\pi_i =$ consumer price inflation, 1991-1995
- $D_i =$ current account inconvertibility, IMF
- Sample of 135 countries
- $V_i = 0.762 + 0.367 \pi_i - 0.058 Y_i + 0.103 D_i, R^2 = 0.77$
- Upper bound for estimated effect: 10.3% of M2

Time Series Demand for Money in Korea


$$\log(\frac{h_i}{p_i}) = 7.17 + 0.85\log(y_t) - 0.01i_t + 0.01\pi_t + 0.08\log(e_t) + 0.11 \frac{R_t}{M_t} + u_t$$

Short run demand for money

$$\Delta \log(\frac{h_t}{p_t}) = 0.02 + \text{seasonals} - 0.015u_{t-1} + 0.1\Delta \log(\frac{h_{t-1}}{p_{t-1}}) +$$

$$0.09\Delta \log(y_t) - 0.0004\Delta i_t - 0.004\Delta \pi_t - 0.09\Delta \log(e_t) + 0.01\Delta \left(\frac{R_t}{M_t}\right) + v_t$$

Figure 1. Money Demand Equation

(in changes; logarithmic scale)

Actual vs. Forecast Money Demand

Residuals from Estimated Money Demand Equation
Simulation of Korean Forex Crisis of December 1997

- Actual decline in money stock: -3 %
- Predicted decline in money stock: -1.7 %
- In absence of crisis, M2 would have risen 1.9 %
- Decline due to crisis variables: -3.5%
- Alternative estimate - 4.5% (based on actual decline vs. predicted in absence of crisis)
- Magnitude of predicted reserve loss: $5.6-7.1 bil., vs. total reserves end-Nov. 1997 $24.4 bil.

Conclusions:

- Importance of avoiding simultaneous price and quantity rationing of foreign exchange.
- Interest rate not effective to offset decline in demand for money due to convertibility risk.
- Devaluation or float option may be preferable.
- Interest rate can be effective against devaluation risk.
Central Bank Intervention

Henderson Model (1984). Portfolio approach, no UIP, risk averse agents

IS Curve \( Y = y_0 + y_1(Y + Y^*) - y_2r - y_3r^* + y_4(e + p^* - p) + y_5e - y_6p + \alpha \)
real interest rate \( r = i - (\bar{q} - q) \)
CPI \( q = hp + (1 - h)(e + p^*) = q^* + e \)
Production \( Y = x_0 + x_1L + x_2\beta \)
Labor Demand \( w - p = \ell_0 - \ell_1L + \beta \)
Wage indexing \( w - \bar{w} = \mu(q - \bar{q}) \), where \( \bar{w} = \bar{p} + \ell_0 - \ell_1L_f \)
Money \( M = m_0 + m_1p + m_2Y - m_3i - m_4(i^* + \bar{e} - e) + \gamma + \delta \)
Bonds \( B = b_0 - b_1p - b_2Y + b_3i - b_4(i^* + \bar{e} - e) + b_5(i - e + \bar{e}) - b_6i^* + b_7e - b_8p^* - b_9Y^* - \gamma + \varepsilon \)
Assume \( \mu = 0 \), foreign country uses monetary and fiscal policy to keep \( Y^*, p^*, i^* \) constant
If shocks are temporary, \( Ep = \bar{p}, Ee = \bar{e} \), etc.
Wages are set at \( \bar{w}, p = \bar{w} - \ell_0 + \ell_1L - \beta \), eliminate \( Y \) with production function to get
\( XX : y_1\dot{\hat{L}} = \alpha - y_1\dot{\hat{i}} + y_2\dot{\hat{e}} + y_3\beta \)
\( MM : \dot{\hat{M}} = m_1\dot{\hat{L}} - m_1\dot{\hat{i}} + m_2\dot{\hat{e}} - m_3\beta + \gamma + \delta \)
\( BB : \dot{\hat{B}} = -b_1\dot{\hat{L}} + b_1\dot{\hat{i}} + b_2\dot{\hat{e}} + b_3\beta - \gamma + \varepsilon \)
Pegged regime: \( i, e \) fixed, \( B, M \) variable. Floating regime: \( i, e \) variable, \( B, M \) fixed.

Demand shock \( \alpha \) shifts \( XX \). Given \( M & B \), \( i \) rises, creates excess demand for \( B \) at given \( e \), so \( e \) falls, shifting \( XX \) back, \( MM \) down, \( BB \) up to point \( x \).
Output [employment] moves to \( L_1 > L_f \). If rates are fixed, \( L \) moves to \( L_2 \).
Flexible rates give better stabilization of demand shocks.
Financial shock $\varepsilon$ shifts $BB$ up. Keeping rates constant implies central bank offsets demand shifts with $\Delta B + \Delta F = 0$. Flexible rates implies rise in $i$, $e$, fall in $L$, in response to fall in demand for $B$. Fixed rates give better stabilization of financial shocks.

Feedback rule based on response to current variable $e$:

**Instruments: $M+B$ and $i$, Target: $L_f$. Add together $MM$ and $BB$:**

\[
B'B' : \hat{B}' = \hat{B} + \hat{M} = b'_L \hat{L} + b'_e \hat{e} + \delta + \varepsilon
\]

\[
X'X' : y_L \hat{L} = y_e \hat{e} + \theta, \quad \theta = \alpha + y_\beta \beta
\]

Policy Rule: $\hat{B}' = \psi \hat{e} = b'_L \hat{L} + b'_e \hat{e} + \delta + \varepsilon$

\[
(\psi - b'_e) \cdot \hat{e} = b'_L \hat{L} + \delta + \varepsilon, \text{ substitute in } X'X'
\]

\[
y_L \hat{L} = \theta + y_e \left( \frac{b'_L \hat{L} + \delta + \varepsilon}{\psi - b'_e} \right), \text{ solve for } \hat{L}
\]

\[
\left( y_L - \frac{y_e b'_L}{\psi - b'_e} \right) \cdot \hat{L} = \theta + \frac{y_e}{\psi - b'_e} (\delta + \varepsilon)
\]

Minimize $\hat{L}^2 = \frac{\sigma^2_\theta + \left( \frac{y_e}{\psi - b'_e} \right) (\sigma^2_\delta + \sigma^2_\varepsilon)}{\left( y_L - \frac{y_e b'_L}{\psi - b'_e} \right)^2}$

\[
\psi = b'_e - \left( \frac{y_e}{y_L b'_L} \right) \left( \frac{\sigma^2_\delta + \sigma^2_\varepsilon}{\sigma^2_\theta} \right)
\]

\[
e | B' \quad \psi X(\alpha + \beta')
\]

Shocks to $XX$, set $\psi = b'_e$ to make $B'B'$ vertical. Shocks to $B'B'$, set
ψ \rightarrow -\infty \text{ or } B'B' \text{ horizontal. In general, use managed intervention in response to mix of shocks. More intervention if shocks are mainly financial.}

Episodes of Central Bank Intervention:

6. Resumption of intervention by Clinton Administration, 1993-95.


\[ rp_i = i^*_i + \Delta s_{t+1} - i_i = \beta_0 + \beta_1 v_i + \beta_2 v_t x_i + u_i \left[ \rho \sigma^2 x \right] \]

\( v_i = \) moving variance of past \( \Delta s_{t-i} \), \( x_i = \) cumulative intervention
\( \Delta s_{t+1} = \) survey expectation of \( \Delta s_{t+1} \)
\( s^e_t - s^e_{t-1} = \alpha_0 + \alpha_1 (s_t - s^e_{t-1}) + \alpha_2 (s_t - s^e_{t-1}) + \alpha_3 AN + \alpha_4 INT \)

\( AN = \) announcements of intervention
\( INT = \) actual intervention

\( INT = $100 \text{ m} \)

\( AB = .08\% \)

\( BC = 2.7\% \)

Dealer transacts with liquidity traders and central bank, prior to the announcement of the fundamental value $f$ of the exchange rate, which is known to the central bank, but not to others. Its mean value $s_0$ is known.

$$f \sim N(s_0, \Sigma_0^f), \text{central bank order } x, \text{ liquidity orders } \varepsilon \sim N(0, \sigma_0^2)$$

Competitive dealer makes zero profits, so $s_i = E[f | I_i]$.

Central bank minimizes $c = (s_i - f)x + q(s_i - \bar{s})^2$, where $q$ is degree of commitment to target $\bar{s}$

Suppose the central bank's target $\bar{s}$ is known, as is $q$. Let $s_i = s_0 + \lambda[x + \varepsilon - h(\bar{s} - s_0)]$

where $\lambda = \frac{\beta \Sigma_0^f}{(\beta)^2 \Sigma_0^i + \sigma_i^2}$. Substitute $s_i$ into $c$ and optimize to derive rule for $x$.

Then $x = \beta(f - s_0) + \gamma(\bar{s} - s_0)$, depends on misalignment and deviation from target

Then $s_i = s_0 + \lambda[\beta(f - s_0) + \varepsilon]$, where $\lambda$ is the positive root of $4\lambda^2 (1 + \lambda q)^2 \sigma_i^2 = (1 + 2\lambda q)\Sigma_0^f$

$$\beta = \frac{1}{2\lambda(1 + \lambda q)}, \quad \gamma = 2q$$

The conditional variance is $\Sigma_i^f = E[(f - s_i)^2 | I_i] = \frac{1 + 2\lambda q}{2(1 + \lambda q)} \Sigma_0^f$

The liquidity coefficient $\lambda$ determines the responsiveness of the exchange rate to the order flow. The market maker filters out the target component $\gamma(\bar{s} - s_0)$, but cannot do so if the target is secret. If it is secret, but the central bank announces falsely that $\bar{s} = s_0$, the dealer’s pricing rule will then be $s_i = s_0 + \lambda(x + \varepsilon)$ and the central bank order $x = (\bar{s} - s_0)/\lambda$ can achieve the target exactly. Also $\lambda$ will be smaller if the target is secret, so the variance of the exchange rate will be smaller. This is a signaling equilibrium, where the central bank provides a signal about its target through its intervention. More recent paper includes Barro-Gordon type objective for central bank. Signaling intervention then can reduce the variance of the real variables.
Target Zone Model (due to Krugman, see Svensson, JEP, 1992) EMS

\[ m_t - p_t = \alpha_0 + \alpha_i y_t - \alpha_i v_t \]

\[ m_t^* - p_t^* = \alpha_0 + \alpha_i y_t^* - \alpha_i v_t^* \]

\[ s_t = p_t - p_t^* + q_t \]

\[ E_t \frac{ds_t}{dt} = i_t - s_t^* - \rho_t \]

\[ s_t = q_t + m_t - m_t^* - \alpha_1 (y_t - y_t^*) + \alpha E_t \frac{ds_t}{dt} + \alpha \rho_t - (v_t - v_t^*) \]

\[ s_t = f_t + \alpha E_t \frac{ds_t}{dt} \text{ where } f_t = q_t + m_t - m_t^* - \alpha_1 (y_t - y_t^*) + \alpha \rho_t - (v_t - v_t^*) \]

\[ s_t = \alpha^{-1} \int_{s_0}^{s_t} e^{(-s-t)} a E_t [f(s)] ds \text{ solution for floating exchange rate} \]

Suppose \( df_t = \sigma dz(t) \), random walk without drift. \( Edz(t) = 0, Edz(t)^2 = 1 \)

\( E_t df_t = 0, s_t = f_t \)

Suppose \( df_t = \eta dt + \sigma dz(t) \), random walk with drift.

\[ s_t = \alpha^{-1} \int_{s_0}^{s_t} e^{(-s-t)} (f_t + \eta(s-t)) ds = f_t + \alpha \eta \]

Float bounded by interventions. Suppose \( s_t = S(f_t) \)

Ito's lemma \( ds_t = S, df_t + \frac{1}{2} S_{ff} df_t^2 \)

\[ ds_t = S \left[ \eta dt + \sigma dz(t) \right] + \frac{1}{2} S_{ff} \sigma^2 dt \]

\[ E_t \frac{ds_t}{dt} = \eta S_f + \frac{\sigma^2}{2} S_{ff} \]

\[ S(f) = f + \alpha \left[ \eta S_f + \frac{\sigma^2}{2} S_{ff} \right] \text{ with solution} \]

\[ S(f) = f + \alpha \eta + A_1 e^{\lambda_1 f} + A_2 e^{\lambda_2 f} \]

\( \lambda_{1,2} \text{ roots of } \frac{\lambda^2 a^2 - a^2}{2} + \lambda a \eta - 1 = 0 \), if \( \eta = 0, \lambda_{1,2} = \pm \sqrt{2 / a \sigma^2} \)

Target zone boundaries \( \{ f, f \} \Rightarrow S = S(f), \overline{S} = S(\overline{f}) \)

"smooth pasting" \( S_f (\overline{f}) = 0, S_f (f) = 0 \)

\[ 1 + A_1 \lambda_1 e^{\lambda_1 f} + A_2 \lambda_2 e^{\lambda_2 f} = 0, 1 + A_1 \lambda_1 e^{\lambda_1 f} + A_2 \lambda_2 e^{\lambda_2 f} = 0 \]

If \( \eta = 0 \) and \( \phi(f) = \frac{1}{f - \overline{f}}, A_{1,2} = \frac{1}{2 \lambda \cosh(\lambda f)}, S(f) = f - \frac{\sinh(\lambda f)}{ \lambda \cosh(\lambda f)} \)
At $\bar{s}$, $Eds \leq 0$, $\text{Prob}(ds^+)=0$, $\text{Prob}(ds^-)=1$ "Honeymoon" effect, flatter than free float. Implications: i. U-shaped distribution of $s$ rejected.

ii. Rotated S distribution for $i-i^*$ rejected.

iii. Nonlinearity rejected.

Modifications:

1. Imperfect credibility of zone- realignment of parities (Svensson)

$$s_i = x_i + c_i, \quad x_i = \text{within band, } c_i = \text{central parity}$$

$$E_i ds_i = E_i dx_i + E_i dc_i$$

$$x_i = f_i - c_i + \alpha E_i \frac{dc_i}{dt} + \alpha E_i \frac{dx_i}{dt} = h_i + \alpha E_i \frac{dx_i}{dt}$$

$$i_i - i_i^* = E_i \frac{dc_i}{dt} + E_i \frac{dx_i}{dt}$$

composite fundamental $h_i$ does appear to have "honeymoon" effect.

Use regression to predict $E_i \frac{dx_i}{dt} = a(i_i - i_i^*) + bx_i$

Estimate $E_i \frac{d\hat{c}_i}{dt} = i_i - i_i^* - E_i \frac{dx_i}{dt}$ Relate to size of reserves.

2. Intra-marginal intervention at $\bar{s} < \bar{s}$ or mean-reverting fundamental $E_i \frac{dh_i}{dt} = -\rho h_i$ will center distribution.

3. Sticky price target zone model (Miller & Weller, EJ, 1992) Note that monetary model assumes UIP, real exchange rate and output constant, removes reasons for central bank intervention. Modify latter two assumptions to get Dornbusch-type target zone model.
Costs and Benefits of Currency Union

- Costs of Currency Union
- Benefits of Currency Union
- EC Snake – 1973-78
- EMS –1979-99

Costs of Currency Union

- Loss of Independent Monetary Policy
- Costs of transition to common inflation rate
- Differences in labor market institutions
- Different uses of inflation tax
- Openness of Economy
  - Pass-through of exchange rate into costs
  - Fiscal policy impact on BOP or GDP

Loss of Independent Monetary Policy

- Demand Shocks: F-, G+ → F deficit, unemployment; G surplus, boom
- Adjust via wage flexibility or labor movement
- Immobile labor or inflexible wage requires exchange rate change
- Effect temporary, since shifts S as well as D

[Diagram showing the effects of independent monetary policy loss with demand shocks and their consequences for France and Germany.]
Costs of transition to common inflation rate

• Phillips curve: $\pi - \pi^e = f(u - u_n)$, $f' < 0$
  – $\pi^e =$ expected inflation
  – $u_n =$ “natural” rate of unemployment
• Italy vs. Germany

Differences in labor market institutions
• highly centralized bargaining internalizes the inflation externality
• decentralized bargaining internalizes the firm-specific increase in cost and loss of market share
Different uses of inflation tax

- high reliance on seigniorage in Southern Europe
  - $\tilde{M}/PY = 2.5-3.5\%$ vs. $< 1\%$ in North
  - effect of fall in $\delta$ on Italian budget deficit
- Seigniorage 1979-87

<table>
<thead>
<tr>
<th>% tax rev</th>
<th>% of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portugal</td>
<td>10.0</td>
</tr>
<tr>
<td>Greece</td>
<td>9.6</td>
</tr>
<tr>
<td>Spain</td>
<td>9.3</td>
</tr>
<tr>
<td>Italy</td>
<td>6.4</td>
</tr>
<tr>
<td>Ireland</td>
<td>2.3</td>
</tr>
<tr>
<td>France</td>
<td>1.3</td>
</tr>
<tr>
<td>Germany</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Openness of Economy

- Speed of pass-through into costs faster if economy is highly open (import share $m$)
- Impact of fiscal policy on BOP higher if open
  - $Y-A = B, \quad C=(1-s)Y, \quad M=mY$
  - $Y - (C+I+G) = X-M$
  - $Y^* = (X+I+G)/(s+m)$
  - $dY/dG = 1/(s+m), \quad dB/dG = -m/(s+m)$

- Lower cost of union if highly open
Benefits of Currency Union

- Reduced Transactions Costs
  - EU estimates ½% of GDP
- Less Price Discrimination
  - market less segmented
- Reduced uncertainty of prices – gain vs. loss
- Reduced Misalignment of exchange rates
- Gain in Credibility of Monetary Policy
- Effects on Growth

Credibility of Monetary Policy

- Gain for “wet” govt. such as Italy
- Steep preference trade-off (π, u) gives high π, same u_n; reduce π by tying to low inflation currency with flat trade-off

Benefits vs. Costs

- Benefits (transactions cost, risk, credibility) increase with openness.
- Costs (loss of monetary policy, slow pass-through) decrease with openness.
Benefits vs. Costs

- Relatively closed prefer flexible, open prefer fixed exchange rate.
- EC \( m_i = .25-.75 \), \( m_{EC} = .123 \), \( m_{US} = .10 \), \( m_{JP} = .114 \)
- Monetarists believe costs are low, Keynesians believe they are high.

EC “Snake”

- Werner Report (1970) - Monetary Union by 1980
  - Narrow margins (± 2½%) after Smithsonian (Dec 1971) ±4½% allowed crossrates ±9%. Float March 1973.
  - Membership reduced to DE, BeNeLx, DK, SW.
  - Weak financial support mechanism

EMS – 1979-99

- Schmidt/Giscard d’Estaing proposal
  - reduce dependence on then-weak $, spread adjustment burden
  - Intervention Rules
    - ECU = \( \Sigma w_i e_i \), with ECU parity rates \( e_i \) and bilateral parities \( e_{ij} = e_i/e_j \).
    - Parity Grid ±2½%. Both CB’s intervene at margins.
    - If \( e_i \) is EXR/$ & \( g_i \) is DM/CU, \( g_i = e_{dm}/e_i \) then using Greek letter for parity:
      \[
      |(g_i - \gamma_i)/\gamma_i| = |(e_{dm} - \epsilon_{dm})/\epsilon_{dm} - (e_i - \epsilon_i)/\epsilon_i| \leq .0225
      \]
    - Unlimited Short-Term Credit for 75 days. EMCF credit up to 6 mos. $42 bil of gold & $ reserves.

EMS Grid

<table>
<thead>
<tr>
<th>ECU Sept.’89 amt</th>
<th>EXR/$ amt</th>
<th>EXR/EU amt in $</th>
<th>EXR/EU</th>
<th>wt.</th>
<th>DM/CU</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(1)/(2)</td>
<td>(2)x1.05</td>
<td>(1)/(4)</td>
<td>(4)/i</td>
</tr>
<tr>
<td>Deutschmark</td>
<td>.6242</td>
<td>1.9619</td>
<td>0.318</td>
<td>2.059</td>
<td>30.3</td>
</tr>
<tr>
<td>£ sterling</td>
<td>.0878</td>
<td>0.6859</td>
<td>0.128</td>
<td>0.72</td>
<td>12.2</td>
</tr>
<tr>
<td>French franc</td>
<td>1.332</td>
<td>6.5714</td>
<td>0.203</td>
<td>6.904</td>
<td>19.3</td>
</tr>
<tr>
<td>Italian lira</td>
<td>151.8</td>
<td>1412.93</td>
<td>0.107</td>
<td>1483.58</td>
<td>10.2</td>
</tr>
<tr>
<td>ECU</td>
<td>wi</td>
<td>ei</td>
<td>$1.05</td>
<td>1.00</td>
<td>100.0</td>
</tr>
</tbody>
</table>

- DM/FF=2.059/6.904=29.82 pfennig ±2½%.
- Mutual Agreement for parity changes - to be frequent if needed 79-83(5); 84-87(3); 88-91(1)
- Basel-Nyborg Agreement 1987: change interest rates more frequently.
Effects of EMS

- reduced short-run variation in bilateral nominal & real exchange rates
- no change in SR variation of multilateral RER
- increased LR variation of multilateral RER, except DE.
- reduced inflation in FR, DK, others? increased inflation in DE.

<table>
<thead>
<tr>
<th>LR VAR RER</th>
<th>1960-79</th>
<th>1979-85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deutschemark</td>
<td>12.3%</td>
<td>10.9%</td>
</tr>
<tr>
<td>Belgian franc</td>
<td>3.4%</td>
<td>14.1%</td>
</tr>
<tr>
<td>Dutch guilder</td>
<td>8.1%</td>
<td>9.2%</td>
</tr>
<tr>
<td>French franc</td>
<td>5.2%</td>
<td>11.1%</td>
</tr>
<tr>
<td>Italian lira</td>
<td>3.8%</td>
<td>6.1%</td>
</tr>
</tbody>
</table>

- Capital controls used by FR, IT to stay in EMS, until 1990 Single Market rules.
- Credibility
  - up to 1987, frequent devaluations, low credibility.
  - Speculative crisis implied run on currency.
  - Assymmetry - German as hegemon set money growth, others peg to DM. Germany sterilizes intervention. Magnifies business cycle in periphery. Spreads shocks from center.

Convergence of Inflation

- Convergence of Inflation Rates - 1980’s
  - French & Italian $\pi$ fell sharply 1983-87. But also in non-EMS countries.
  - Unemployment rose to ’87, fell to ’90, then rose.
  - Unemploy. rose less, fell sooner, in non-EMS.
  - Mon-Fiscal Pol. tighter in EMS.
  - Slow reversal of $\pi$,$U$ spiral in EMS suggests low credibility.

Collapse of EMS

- end of capital controls in 1990
- rise in German interest rates after Unification
- failure of Danish referendum on Maastricht
- recession outside Germany
  - Sept. 16, 1992, UK, IT withdrew, SP, PT devalue.
- Fundamentals or Speculative attack?

Appendix A

Bayoumi’s Model of Currency Union

In Bayoumi (1994), each region has the same production structure but produces its own goods with a fixed amount of labor:

\[ Y_i = L_i e^{\varepsilon_i} \]  
(A.1)

where \( Y_i \) is the output of region i, \( L_i \) the labor input in region i, \( \varepsilon_i \) is a disturbance with mean zero and independent of the exchange rate regime. The capital stock is normalized at 1. In logs

\[ y_i = \alpha l_i + \varepsilon_i \]  
(A.2)

In a competitive market, labor is employed up to the point where real wages equal the marginal product of labor in the common currency:

\[ w_i - p_i = \log \alpha - (1 - \alpha) l_i + \varepsilon_i \]  
(A.3)

where \( e_i \) = the (log of the) bilateral exchange rate with region 1. The level of wages and prices are \( W_i \) and \( P_i \) with their log counterparts in (A.3). To incorporate wage stickiness, assume that full employment wages, \( \bar{w} = \log(\alpha) \), hold when there is full employment (\( L_i = 1 \)) and no shocks (\( \varepsilon_i = 0 \)), and that the exchange rate is at its parity value (\( E_i = 1 \)). If there is excess demand for labor when \( W_i = \alpha \), then wages will be raised until the demand falls to \( L_i = 1 \). But if there is excess supply at \( W_i = \alpha \), then wages remain at this level and unemployment results.

Each region has to choose its own exchange rate regime. If regions i and j
form a currency union then their exchange rate ratio, \( E_i/E_j \), is fixed at unity and they have a common currency. If they choose separate currencies then \( E_i/E_j \) may vary, but there is a transactions cost between the two currencies. This cost implies that, in value terms, goods exported from region i "shrink" by a factor \( (1-T_i) \) when they arrive in region j. For simplicity, let \( T_i = T \) for all regions.

Finally on the demand side, if production is owned locally, region i's income will be \( P_i Y_i \). Utility levels are given by the Cobb-Douglas function

\[
U_j = \sum_{i=1}^{N} \beta_{ji} \log C_{ji} - \phi
\]

(A.4)

where \( C_{ji} \) is the consumption of good i in region j, and \( \phi = \sum_{j} \beta_{ji} \) is a constant. The \( \beta_{ji} \) parameters are subject to the normalizations \( \sum_{i} \beta_{ji} = 1 \), \( \sum_{j} \beta_{ji} = 1 \).

Since \( \beta_{ji} \) is the proportion of region j's income spent on goods produced in region i, these restrictions ensure total income is spent and that aggregate demand exhausts income spent on each good (before payment of transactions costs \( T_j \)). Under these conditions the demand for good i from region j is

\[
Y_{ji} = (\beta_{ji} P_j Y_j / P_i) e^{v_{ji}}
\]

(A.5)

where \( v_{ji} \) is another normally distributed disturbance with zero mean arising on the demand side. Note that production in region i will be expected to equal unity in the absence of shocks \( y_i = 0 \) in (A.2) at full employment). Now let prices be normalized such that \( P_i = 1 \), so that \( E(P_i, Y_i) = 1 \) where E denotes expectations. That
means all other national incomes are also unity in expectation since, if \( P_j Y_j = 1 \), then

\[
Y_i = \sum_j Y_j = (\sum \theta_j e^{\gamma_j P_j})
\]

(A.6)

which implies \( P_i Y_i = 1 \). But \( P_i Y_i = 1 \), so \( P_i Y_i = 1 \) for \( i = 2 \ldots N \). Hence, in the long run, output in each region will be independent of the real exchange rate, but in the short term actual output will depend on relative prices. Thus if region \( j \) and region \( i \) do not form a currency union, the equilibrium consumption of good \( i \) in region \( j \) will be

\[
C_{ij} = (\beta_i (1 - T_j)/P_j)e^{\gamma_i}
\]

(A.7)

where the production of good \( i \) is given by (A.6) and, for simplicity, \( T_j = T \). This implies

\[
c_{ij} = \log \beta_{ij} + \log (1 - T) + d_{ij}
\]

(A.8)

where \( d_{ij} = \nu_{ji} + \epsilon_{ij} \) is a composite disturbance term, since full employment with equilibrium wages and exchange rates implies \( y_i = -p_i = \epsilon_i \). Each region's utility level is therefore

\[
U_j = \sum_i \beta_{ij} d_{ij} - \sum_{i \neq j} \beta_{ij} \epsilon_{ij}
\]

(A.9)

where \( \tau = \log (1 - T) \).

Now suppose region \( j \) and region \( k \) decide to form a currency union. Equations (A.6) to (A.9) hold with \( k \) replacing \( j \) and \( T_k = 0 \). The external exchange rate will be the mean of the free float exchange rates which, under the normalization of \( E_j = E_k = 1 \) vs. region 1, implies \( \epsilon_{jk} = e_j - e_k = (\epsilon_j + \epsilon_k)/2 \) and \( \bar{\epsilon}_{jk} = 0 \). Suppose wages adjust to provide full employment in the high demand region (region \( j \) say) but are sticky downwards.
in the other region. Then

\[ y_j = s_j, \quad w_j = \log(\bar{w}) + (e_j - e_k)/2 \]  \hspace{1cm} (A.10)

using (A.3) and the results above for \( e_j \) and \( p_j \), where \( \bar{w} \) is the log equilibrium real wage with no shocks and \( E_j = 1 \). Substituting back into (A.3), using (A.2) for \( y_k \), yields

\[ y_k = s_k - \alpha(e_j - e_k)/(2(1 - \alpha)), \quad w_k = \log(\bar{w}) \]  \hspace{1cm} (A.11)

for region \( k \). Now evaluating the difference between (A.9) and the utilities generated by (A.10) and (A.11), we find the utilities gained by being in the currency union are

\[
\Delta U_j = \beta_{jk}v_i - \beta_{ik}v_j - \beta_{jk}a(e_j - e_k)/(2(1 - \alpha)) + \sum_{i} \beta_{ik} \Delta v_{ji} \\
\Delta U_k = \beta_{kj}v_i - \beta_{ik}v_j - \beta_{kj}a(e_j - e_k)/(2(1 - \alpha)) + \sum_{j} \beta_{kj} \Delta v_{ij}
\]

\hspace{1cm} (A.12)

while the loss to region 1 (say) staying outside the Union is

\[
\Delta U_1 = -\beta_{1k}a(e_j - e_k)/(2(1 - \alpha)) + \sum \beta_{1k} \Delta v_{ji}
\]

where \( \Delta v_{ji} \) is the increase in region \( j \)'s demand for goods produced in region \( i \) as a result of joining the currency union; that is the trade creation and trade diversion effects of the monetary union itself.

Since the shocks are symmetrically distributed around zero, with \( e_j < e_k \) with probability one-half and \( e_j > e_k \) with probability one-half, the \textit{ex ante} expected benefit of being in a currency union is

\[
E(\Delta U_j) = \beta_{jk}v_i - \beta_{ik}v_j - \beta_{jk}a(e_j - e_k)/(2(1 - \alpha))P(e_j < e_k) \\
- \beta_{kj}v_i - \beta_{ik}v_j - \beta_{kj}a(e_j - e_k)/(2(1 - \alpha))P(e_j > e_k) \\
= \beta_{jk}v_i - \gamma(\beta_{jk} + \beta_{kj})\Phi(0)\sqrt{\sigma_j^2 - 2\rho_{jk}\sigma_j\sigma_k + \sigma_k^2}
\]

\hspace{1cm} (A.14)
For the non-members, the expected welfare costs are

\[ E(\Delta U_j) = -\gamma(\beta V + \beta B)\phi(0)\sqrt{\sigma_j^2 - 2\rho\sigma_{j}\sigma_k + \sigma_k^2} \]  

(A.15)

where \( \phi(0) \) is the density function of a standard jointly normal distribution of random variables and

\[ \gamma = \alpha/(2(1-\alpha)). \]

Extending this analysis to a Union of several countries, 1 \( \ldots k \), region \( j \) will have full employment if its supply (productivity) shock is above the average for the remainder of the Union; wages adjust up to give full employment if demand is above the level of the other members since, if wages did not adjust, labor would move to region \( j \) (not possible under our current assumptions) or wages would be bid down elsewhere (also not possible given inflexible labor costs). Hence the expected output loss in region \( j \), without full employment, is

\[ E(\Delta y_j) = -\gamma\phi(0)\sqrt{\bar{\sigma}_j^2 - 2\rho\bar{\sigma}_j\bar{\sigma}_j + \bar{\sigma}_j^2} \]  

(A.16)

where \( \bar{\epsilon}_j, \bar{\sigma}_j \) are the mean and standard deviation of the disturbances of the Union excluding region \( j \). Since each region will have this output loss, and the disturbances are symmetrically distributed so that each region has an equal chance of receiving a shock below or above the average of its partners, the expected welfare changes are:

\[ E(\Delta U_j) = \sum_{k \neq j} \beta_{jk} - \gamma \sum_{k \neq j} \beta_{jk}\phi(0)\sqrt{\sigma_j^2 - 2\rho\sigma_j\sigma_k + \sigma_k^2} \]  

(A.17)

for members \( j \) and \( k \), and
It is possible to decompose the second term of (A.17) to show the costs to region j when it decides to join an existing union, and the costs to region i in an existing union when region j joins (Bayoumi, 1994). We do not use those terms in this paper, but they imply a periphery country sufficiently similar to the core average will certainly want to join when it can, but the core countries will probably want to keep it out unless it is a major trading partner with the existing core. Alesina and Grilli (1993) make a similar point in a political economy context.

\[ B(\Delta U_j) = -\gamma \sum_k \beta_{jk} \phi(0) \sqrt{\sigma_j^2 - 2\rho \sigma_j \sigma_k + \sigma_k^2} \]

for nonmembers, k is a member. The conclusion is the same; the costs inside and outside the Union diminish only if the shocks are of similar size and well correlated within the core.  

**Introducing Market Flexibility**

(a) Labor mobility:

All this analysis has been conducted assuming no labor mobility. However, suppose instead that a proportion \( \delta \) of the number remaining unemployed (X) in a depressed region migrates to a full employment region. If the latter had had full employment of 1, then a migration equilibrium implies employment of \( 1-\delta X \) and \( 1+\delta X \) respectively. If X is small, \( \log(1\pm \delta X) \approx \pm \delta X \) and (A.10) and (A.11) become

\[ y_k = s_j + \gamma \delta (s_j - s_k), \quad w_j = \log(\bar{w}) + (1-\delta)(s_j - s_k)/2 \]

\[ y_k = s_k - \gamma \delta (s_j - s_k), \quad w_k = \log(\bar{w}) \]

(A.19)

respectively, where region j is the excess demand region (\( s_j > s_k \)). Aggregate output has simply been reallocated between members of the Union. Hence repeating the steps to (A.15) we get the *ex ante* expected welfare benefit from being in the currency union

\[ B(\Delta U_j) = \beta_{jk} \tau - \gamma (1-\delta)(\beta_{jk} + \beta_{kj}) \phi(0) \sqrt{\sigma_j^2 - 2\rho \sigma_j \sigma_k + \sigma_k^2} \]

(A.20)
and the cost of being left outside is

\[ E(\Delta U_j) = -\gamma (1-\delta) (\beta_j + \beta_k) \Phi(0) \sqrt{\sigma_j^2 - 2\rho \sigma_j \sigma_k + \sigma_k^2} \quad (A.21) \]

under the usual symmetric distributions assumption. Hence the costs of forming a currency union have fallen for both insiders and outsiders (0<\delta<1), but the interpretation is exactly the same. Comparison with (A.14) and (A.15) shows that the costs still vary with \( \rho \), \( \sigma_j \) and \( \sigma_k \), but will vanish if \( \delta \rightarrow 1 \). Hence full mobility will offset the costs of a monetary union for both insiders and outsiders.

(b) Wage Flexibility:

A similar kind of outcome can be achieved if wages and non-wage costs are flexible downwards in region k. Suppose wages fall sufficiently to re-employ the \( \delta X \) employees, who might otherwise have migrated to region j. That leaves employment of 1 and \((1-(1-\delta)X)\) in regions j and k respectively. Evidently region j is unaffected:

\[ y_j = \epsilon_j \quad w_j = \log \bar{w} + (\epsilon_j - \epsilon_k) / 2 \quad (A.22) \]

But region k will now achieve

\[ y_k = \epsilon_k - \gamma (1-\delta) (\epsilon_j - \epsilon_k), \quad w_k = \log \bar{w} - \delta (\epsilon_j - \epsilon_k) / 2 \quad (A.23) \]

These expressions are identical to (A.10) and (A.11), except for the fact that \( \gamma (1-\delta) \) has been substituted for \( \gamma \) in (A.11) and \( w_k \) has fallen by \( \delta (\epsilon_j - \epsilon_k) / 2 \). Consequently the costs of forming a monetary union, and the costs for outsiders, are reduced. But the interpretation remains the same; the costs depend on \( \rho \), \( \sigma_j \) and \( \sigma_k \), and will vanish if \( \delta \rightarrow 1 \). Hence fully flexible wages restore full employment in region k and remove the costs of a currency union.
Exchange Rate Policy
Outline of Lecture

• Analytical Framework
  – Real Targets Approach (Internal-External Balance)
  – Anchor Approach
  – Optimal Currency Area Approach
• Key Issues
  – Volatility and Trade
  – Independence and Credibility of Monetary Policy
  – Asymmetric Shocks and Openness
  – Capital Mobility
• Exchange Rate Management

  **Real Targets Approach**

• Targets: Internal and External Balance
  – Mundell-Fleming or Traded-Nontraded Model
  – Maintain Equilibrium Real Exchange Rate
  – Allow ER to offset shocks to BOP
  – Use ER to Promote Exports
• Offset RER effect of domestic inflation
  – Passive Crawling Peg – RER target
• Internal Balance also a target

**Internal and External Balance**

\[ e \]

\[ e^* \]
Anchor Approach – Hard Peg option

- Inflation Control as Primary Objective
- Target Exchange Rate, Inflation or Money Growth
- Hard Peg excludes consideration of internal target
- Active Crawling Peg as anchor - tablita
- Currency Board as option when credibility is lacking
- Inflation Targeting allows more flexibility
- Dollarization as extreme case

Optimal Currency Area – Exchange Stability

- Benefits
  - Reduced Transactions Costs
  - Price Stability if within-bloc trade share large
  - Increased Credibility for Weak Monetary Authority
- Costs
  - Loss of Independent Monetary Policy
  - Loss of Seigniorage Revenue
  - Need for Political Control of Area Central Bank
  - Fiscal Flexibility and Fiscal Transfers

ER Regimes: Range of options

- **Currency union or dollarization**
- **Currency board**
  - Peg
    - Fixed
      - Horizontal bands
    - Crawling peg
      - Without bands
      - With bands
  - Floating
    - Managed
    - Independent
**Key Issues: Exchange Rate Volatility & Trade**

- **Does Volatility Reduce Trade?**
  - Are firms risk averse?
  - Can hedging offset risk?
- **Evidence on Industrial Countries Mixed & Weak**
- **Emerging Market Economies**
  - Due to foreign currency denomination of exports?
  - Less opportunity to hedge risks.
Independence of Monetary Policy

• Pegged exchange rate loses monetary independence with high capital mobility
• Flexible rate restores monetary independence
• Desirability of monetary independence depends on responsibility and credibility of monetary authority
• Fiscal dominance - need for seigniorage revenues
• Monetary dominance – control of inflation

Nature of shocks

Suppose the objective is to stabilize real output in an environment of high capital mobility.

*If the prevalent shocks are...*  *Then the desirable ER regime is...*

- Real Flexible
- Nominal
  - Domestic Fixed
  - External Flexible

Response to Asymmetric Shocks

• ER flexibility useful if economy not too open.
• High pass-through coefficient undermines exchange rate adjustment process
• Pass-through coefficient higher for EMC’s than IC’s, but still low in most countries (Calvo & Reinhart)

Openness vs. Size

• Openness raises benefits of OCA
• Fiscal effects on BOP stronger, pass-through higher, so exchange rate adjustment less effective
• Effect of reduced transactions costs and price stability stronger
• Small economies more open than large ones
• Large economies more diversified than small, so have fewer asymmetric shocks
Impact of Openness

- Let \( Y-A - G = sY - G = X - mY \)
- Then \( Y^* = (X+G)/(s+m) \)
- \( dY/dG = 1/(s+m) \), lower if \( m \) large
- Given \( B = X - mY \), \( dB/dG = -m/(s+m) \), higher if \( m \) large
- Pass-through effect of exchange rate on prices proportional to openness \( m \), so effectiveness of exchange rate adjustment inverse to \( m \).
- Conclusion: peg or CU for more open economy

Costs vs. Benefits of Currency Union

![Diagram showing the costs vs. benefits of currency union](attachment:image.png)
Capital flows


The Impossible Trinity

**FULL CAPITAL CONTROLS**

Monetary independence

Monetary independence

Increasing capital mobility

Increasing capital mobility

Exchange rate stability

Exchange rate stability

**PURE FLOAT**

Pure financial integration

Pure financial integration

**MONETARY UNION**

Monetary union

Monetary union

(Percents of GDP; sample of developing & transition countries)
Corner Solutions vs. Institution Building

- Increased capital mobility leads to base of triangle
- Middle options: crawling peg, fixed but adjustable rate, managed float
- Middle ruled out if central bank lacks credibility
- Either choose corners or build credibility or limit capital mobility
- How to build credibility? Central bank independence, fiscal restraint, political stability

Active vs. Passive Crawling Peg

- Active Crawl
  - Pre-announced devaluation path to promote disinflation
  - Liable to over-valuation of RER
  - Invites “carry-trade” capital inflows if i>i*+Δe
  - Unsterilized capital inflows raise money growth
- Passive Crawl
  - Keeps RER stable
  - Feeds inflation process
  - Uncertain Δe keeps capital inflows down

Government Credibility

- High Credibility allows Inflation Targeting option
  - Flexible response to shocks
- Low Credibility may be overcome via OCA or Currency Board
  - Requires sound banking system as no bailouts allowed
  - High risk for large country of RER misalignments
  - Especially if asymmetric shocks likely – Argentina
  - Exit option from CB hard to arrange unless to OCA
Bayoumi & Eichengreen study

- Compare shocks in EU & US 1960-90:
  - EC members growth & inflation correlation 0.58 w/DE.
  - US regions growth & inflation correlation 0.68 w/mid-East region.
  - Decompose demand shocks & supply shocks, permanent & temporary
  - Because EC permanent shocks are larger & more asymmetric than US, lack of exch. rate adjust. is worse.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Type of shock</th>
<th>EC/Ger.</th>
<th>US/East</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td>Permanent</td>
<td>0.33</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>Temporary</td>
<td>0.18</td>
<td>0.37</td>
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</tbody>
</table>

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<thead>
<tr>
<th>Measure</th>
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<th>US/East</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of Shocks</td>
<td>Permanent</td>
<td>2.1</td>
<td>1.5</td>
</tr>
<tr>
<td>(std. Dev.)</td>
<td>Temporary</td>
<td>1.7</td>
<td>2.1*</td>
</tr>
</tbody>
</table>

* due to specialization in US

**Correlation of Shocks vs. Trade Shares**

Optimal Currency Area

- US States
- Belgium, Netherlands
- EU member countries
- Japan
- US, EU
- East Asia

Correlation of income shocks vs. Extent of trade among members
European Monetary Union

- Political goals explicit in EU Treaties
- Within-bloc trade shares high
- Asymmetric shocks remain important
- Correlation of shocks may rise endogenously after EMU created (Frankel)
- Flexibility of labor markets low, but may rise after EMU
- Labor mobility included in EU goals

Exchange Rate Management

- Regime-specific Aspects of Management
  - Choice of Peg or Basket
  - Intervention Currency
  - “Fear of Floating” vs. Dangers of Pegging
- Policy Problems
  - Capital Flows and other shocks
  - Moral hazard
- Policy Options
  - Central Bank Intervention
  - Interest rate policies

Choice of Peg or Basket

- Real vs. Nominal Stability
  - Anchor or Real Targets
- Nominal stability choose $ peg
  - Allows one-way bet, moral hazard, capital flows
- Real stability choose basket peg
  - Promotes trade, but less transparent than nominal peg
- Trade Shares (Asia: $ 40%, ¥ 30%, € 30%)
- Debt Shares (Asia: $ > ¥ > €)
Exchange Rate Responses: Asia

= \alpha + \beta \Delta \log(DM/\$) \text{ over 125 day window, (McCauley, BIS)}
**Intervention Currency**

- US Dollar the largest forex market (91% in April 1998)
- Other currencies mainly traded vs. US dollar
  - Because of low costs & high volumes in bilateral $ markets (economies of scale and competition)
- Euro growing as an alternative, but slowly, and more in European markets

Forex Market Transactions

Currency Pair Volumes, April 1998

US Dollar 91%
Deutsche mark 6%
Japanese Yen 1%
Other 2%

Source: Bank for International Settlements, 1999

“Fear of Floating”

- Evidence shows volatility of floaters not very different from peggers (ex. US, Japan)

[Average probability that monthly percent change <±2.5%]
- Floating Rate Currencies: 79.27
- Managed Float Currencies: 87.54
- Limited Flexibility Currencies: 92.02
- Fixed Rate Currencies: 95.88

Source: Calvo and Reinhart (2000)

Explanations for “Fear of Floating”

- Lack of Credibility of Monetary Authority
- Contractionary devaluations
- Liability dollarization
- Banking instability
- Negative effects of volatility on trade
Dangers of Pegging

- One-way bet against peg
- Danger of misalignment due to third country effects ($/€ or $/$ or real/$)
- Reliance on wage flexibility or fiscal policy in response to shocks
- Encouragement of foreign borrowing

Policy Problems

- Capital Inflows if short-term and denominated in foreign currency can create serious problems:
  - Maturity mismatch – short vs. long
  - Currency mismatch – foreign vs. domestic
  - Reversibility – possible rapid outflow
  - Monetary expansion or currency appreciation
  - Unsustainable domestic expansion – especially if sectoral
- Capital Outflows can lead to sudden collapse in investment and imports, loss of reserves, currency depreciation.

Policy Responses to Capital Flows

- Sterilize capital inflows
  - Can lead to quasi-fiscal deficit if $i > i^*$
  - Pegged rate invites further inflows
- Allow exchange rate appreciation
  - Over-values real exchange rate
  - Flexible rate increases uncertainty of returns
- Tax-subsidy to promote long-term flows
  - FDI vs. short-term flows
- Financing Consumption or Investment?

Moral Hazard and Agency Costs

- Pegged exchange rate offers:
  - Guaranteed return to foreign investor, $i > i^*$
  - Cheap borrowing rate to domestic borrower, $i^* < i$
- Currency mismatch shifts risk to government
- Deposit insurance (implicit or explicit) can also create moral hazard
- Government or private managers may fail to represent public or shareholders’ interests
Policy Options to Manage Exchange Rate

• Central Bank Intervention
  – By rule (peg) or discretionary (disorderly mkts)
  – Spot or Forward
  – Sterilized or Unsterilized
• Interest Rate Policy (unsterilized intervention)
• Capital Controls
  – Prohibition on offshore markets
  – Limits on capital flows

Central Bank Intervention

• FA = Foreign Assets
• DA = Domestic Assets
• MB = Monetary base
• Sterilized: \( \Delta FA + \Delta DA = 0 \), Sale of dollars vs. purchase of domestic bonds or other domestic currency assets. Buying dollars may generate quasi-fiscal costs if \( i > i^* \).
• Unsterilized: \( \Delta FA = \Delta MB \),
Sale of dollars reduces domestic monetary base.

Policy Coordination

• Coordinated Interest Rate Policies
• Swap Agreements: G10 ($30 bn), US-Mexico ($6 bn), EMS
  – Reciprocal borrowing arrangements
  – G10 (expired) & US-Mexico to support intervention
  – EMS (expired) to support pegged rate system
• ASEAN+3 Swap Agreement – May 2001
  – $9.5 bn swaps: Japan, China, Korea+ASEAN
  – 10% available without strings
  – To be used for Central Bank Intervention

Further Reading

INTERDEPENDENCE

Mundell (Monetary Theory, Ch. 16)

\( n-1 \) problem: with fixed exchange rates among \( n \) countries, \( \sum_{i=1}^{n} B_i = \Delta R \)

only \( n-1 \) of the targets \( B_i \) are independent. Mundells’ solution: assignment of targets and instruments. Example: US vs. Europe targets: inflation and BOP \( \pi = m + m^* \), \( B = m^* - m \) US sets \( m \) to control world inflation, Europe uses \( m^* \) to peg to US dollar, control BOP, converge at point A. Equivalent \( n-1 \) story available for exchange rates, Europe controls \( e \), US controls \( \pi \).

Alternative analysis of conflict over inconsistent targets (Hamada, 1985). Given preferences for each country, suppose US targets \( B < 0 \) and Europe targets \( B^* < 0 \). Then US wants \( m^* < m \) and Europe wants \( m^* > m \). Nash-Cournot reaction functions lead to excess money growth at point \( N \). Cooperation could lead to improved outcome for both at point A on contract curve \( BB \).
Canzoneri and Henderson Analysis of Exchange Rate Policy

\[ y = (1 - \alpha)n - x, x \] is a negative productivity shock

\[ w - p = -\alpha n - x \]

\[ m - p = y \]

\[ p = m - y = m - (1 - \alpha)n + x \]

\[ m - p = n - \alpha n - x = n + (w - p + x) - x \Rightarrow m = w = n \]

\[ \min En^2 = E(m - w)^2 \Rightarrow w = m^\epsilon & \& n = m - m^\epsilon \]

Let \( z = e + p^* - p, y - y^* = \delta z, q = p + \beta z \)

\[ \max \Omega = -\sigma(n - k)^2 - q^2 \]

\[ q = p + \beta z = m - (1 - \alpha)n + x + \frac{\beta}{\delta} (y - y^*) \]

\[ = m - (1 - \alpha)(m - m^\epsilon) + x + \frac{\beta}{\delta} (1 - \alpha)(n - n^*), m = (1 - \alpha)(m - m^\epsilon) + x + \varepsilon(m - m^\epsilon) - \varepsilon(m^* - m^\epsilon) \]

\[ \max \Omega = -\sigma(m - m^\epsilon - k)^2 - q^2 \]

\[ \Omega_m = -2\sigma(m - m^\epsilon - k) - 2q(\alpha + \varepsilon) = 0 \]

\[ E\Omega_m = \sigma k - q(\alpha + \varepsilon) \text{ and } q^* = m^\epsilon \Rightarrow m^\epsilon = \frac{\sigma}{\alpha + \epsilon} - k, \ m^* = \frac{\sigma}{\alpha + \epsilon} - k^* \]

Let \( m - m^\epsilon = \delta m \) or \( m = m^\epsilon + \delta m \) Then

\[ -\frac{1}{2} \Omega_m = \sigma(\delta m - k) + (\alpha + \varepsilon) \left[ \frac{\sigma}{\alpha + \varepsilon} - k + \delta m - (1 - \alpha)\delta m + x + \varepsilon\delta m - \varepsilon\delta m^* \right] = 0 \]

\[ \left[ \sigma + (\alpha + \varepsilon)^2 \right] \delta m = -(\alpha + \varepsilon)x + \varepsilon(\alpha + \varepsilon)\delta m^* \]

Find Nash-Cournot reaction function as

\[ R : \delta m = \frac{\varepsilon(\alpha + \varepsilon)}{\sigma + (\alpha + \varepsilon)^2} \delta m^* - \frac{\alpha + \varepsilon}{\sigma + (\alpha + \varepsilon)^2} x, \text{ slope } \frac{\delta m^*}{\delta m} > 1, \ R^* : \text{ slope } \frac{\delta m^*}{\delta m} < 1 \]

Joint solution \( \delta m = \delta m^* = -\Phi x, \Phi = \frac{\alpha + \varepsilon}{\sigma + (\alpha + \varepsilon)^2} \left[ 1 - \frac{\varepsilon(\alpha + \varepsilon)}{\sigma + (\alpha + \varepsilon)^2} \right]^{-1} \)

Finding focal point \( U^{opt} : \Omega_m = 0 \) as above and \( \Omega_m = \varepsilon q = 0 \Rightarrow \)

\[ q = m^\epsilon + \delta m - (1 - \alpha)\delta m + x + \varepsilon\delta m - \varepsilon m^* = 0 \]

\[ (\alpha + \varepsilon)\delta m = \varepsilon m^* - x - \frac{\sigma}{\alpha + \varepsilon}k \text{ and } \delta m = \frac{\varepsilon(\alpha + \varepsilon)}{\sigma + (\alpha + \varepsilon)^2} \delta m^* - \frac{\alpha + \varepsilon}{\sigma + (\alpha + \varepsilon)^2} x \Rightarrow \]

\[ U^{opt} : \delta m = k, \delta m^* = \frac{\sigma + (\alpha + \varepsilon)^2}{\varepsilon(\alpha + \varepsilon)} \left[ k + \frac{x}{\sigma} \right] = \left[ 0, \frac{x}{\sigma} \right] \text{ if } k = 0 \]
Fixed exchange rate requires $\delta m = \delta m^*$ on 45 degree line, imposes cooperative solution, avoids excessively contractionary monetary policy at point $N$.

Asymmetric shock $u$ causes shift away from domestic goods onto foreign goods.

$y - y^* = \delta z - u$ Let $k = 0$ so $m^e = m^{*e} = 0$

$R: \delta m = \frac{\varepsilon(\alpha + \varepsilon)}{\sigma + (\sigma + \varepsilon)^2} \delta m^* + \frac{\alpha + \varepsilon}{\delta[\sigma + (\sigma + \varepsilon)^2]}u$

since shock is asymmetric, set $\delta m^* = -\delta m$ so

$$\left[1 + \frac{\varepsilon(\alpha + \varepsilon)}{\sigma + (\sigma + \varepsilon)^2}\right] \delta m^* = \frac{\alpha + \varepsilon}{\delta[\sigma + (\sigma + \varepsilon)^2]}u \Rightarrow$$

$$\delta m = \frac{\alpha + \varepsilon}{\delta[\sigma + (\sigma + \varepsilon)^2 + \varepsilon(\alpha + \varepsilon)]} = -\delta m^* > 0$$

Note change in exchange rate required in response to asymmetric shock.
Credibility Effect of Currency Union

Closed economy $\varepsilon = 0 \Rightarrow q = m = \frac{\sigma}{\alpha} k$ assuming no shocks

Flexible rate open economy $q = \frac{\sigma}{\varepsilon + \alpha} k$

Inflationary effect of depreciation acts to discipline monetary policy.

Suppose that German $k^* < $ Italian $k$. For Italy, joining EMS with $q^* = \frac{\sigma}{\alpha} k^*$

will be a gain if and only if $\frac{\sigma}{\alpha} k^* < \frac{\sigma}{\varepsilon + \alpha} k \iff \frac{\varepsilon}{\alpha} < \frac{k - k^*}{k^*}$,
incentive to depreciate (loss of discipline) is less than credibility gap.
Sachs & McKibben Static Model – includes UIP and IS curve, sticky prices

<table>
<thead>
<tr>
<th>Home</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $m - p = \phi q - \beta i$</td>
<td>(1*) $m^* - p^* = \phi q^* - \beta i^*$</td>
</tr>
<tr>
<td>(2) $q = \delta(e + p^* - p) - \sigma i$</td>
<td>(2*) $q^* = -\delta(e + p^* - p) - \sigma i^*$</td>
</tr>
<tr>
<td>(3) $p^c = \alpha p + (1 - \alpha)(p^* + e)$</td>
<td>(3*) $p^{c*} = \alpha p^* + (1 - \alpha)(p - e)$</td>
</tr>
</tbody>
</table>

Assume $p = p^* = p_0$, $i = i^*$

(2) + (2*) $\Rightarrow i = -(q + q^*) / 2\sigma$, substitute in (1) + (1*)

$$q + q^* = \frac{m + m^*}{\phi + \beta / \sigma} - \frac{2p_0}{\phi + \beta / \sigma}$$

(1) $-(1*) \Rightarrow m - m^* = \phi(q - q^*)$, substitute in (2) - (2*)

$$e = \frac{q - q^*}{2\delta} = \frac{m - m^*}{2\delta \phi}, \quad q - q^* = \frac{m - m^*}{\phi}$$

(4) $q = \sqrt{ \left[ (q + q^*) + (q - q^*) \right] } = \Omega_1 m - \Omega_2 m^* - (\Omega_1 - \Omega_2) p_0$

$$\Omega_1 = \frac{2\sigma \phi + \beta}{\phi(2\sigma \phi + 2\beta)}, \quad \Omega_2 = \frac{\beta}{\phi(2\sigma \phi + 2\beta)}$$

$$MinU = q^2 + \mu (p^*)^2, \quad p^c = p_0 + \gamma(m - m^*), \quad \gamma = \frac{1 - \alpha}{2\delta \phi}$$

Noncooperative Equilibrium

$$q \left( \frac{\partial q}{\partial m} \right) + \mu p^c \frac{\partial p^c}{\partial m} = 0$$

(5) $$\left[ \Omega_1 m - \Omega_2 m^* - (\Omega_1 - \Omega_2) p_0 \right] \Omega_1 + \mu \left[ p_0 + \gamma(m - m^*) \right] \gamma = 0$$

Reaction function $m = \Gamma_1 m^* + \Gamma_2 p_0$, $\Gamma_1 = \frac{\Omega_1 \Omega_2 + \mu^2 \gamma}{\Omega_1^2 + \mu^2 \gamma^2} < 1$

similarly for $R^*$, solve jointly for $m = m^*$

$$\Rightarrow p^c = p_0, \quad q = -\frac{\mu \gamma}{\Omega_1} p_0 < 0$$

Cooperative Equilibrium, take account of $m = m^*$

$$\frac{\partial q}{\partial m} = \Omega_1 - \Omega_2, \quad \frac{\partial p^c}{\partial m} = 0 \Rightarrow (\Omega_1 - \Omega_2)m = (\Omega_1 - \Omega_2)p_0$$

$m = m^* = p_0, \quad q = 0$
Symmetric Inflationary Shock

- Home Reaction $R$
- Foreign Reaction $R^*$
- 45 degree line
- Axis

Legend:
- Red: Home Reaction $R$
- Blue: Foreign Reaction $R^*$
- Green: 45 degree line
- Pink: Axis
Dynamic Model (McKibbin & Sachs, Ch. 7)

(1) \( m_t - p_t = \phi q_t - \beta i_t \)
(1') \( m + m^* = p + p^* + \phi (q + q^*) - 2 \beta r \)
(2) \( q_t = \delta (e_t + p_t^* - p_t) - \sigma r_t \)
(2') \( q + q^* = -2 \sigma r \)
(3) \( r_t = i_t - \epsilon_{t+1} + p_t \)
(4) \( i_t^* = i_t + \epsilon_{t+1} - e_t \)
(5) \( p_t = w_i + \theta q_i \)
(6) \( w_{t+1} = \zeta p_{t+1}^e + (1 - \zeta) p_t^e \)
substitute (2') + (3') into (1') \( \Rightarrow \)
(7) \( p_t^e = \alpha p_t + (1 - \alpha) (p_t^* + e_t) \)
(4') \( q + q^* = \frac{(m + m^*) - (w + w^*)}{\theta + \phi + \beta / \sigma} \)
(8) \( p_t^e = p_t^* - p_{t-1}^* \)
(1') (1') & (5) - (5') \( \Rightarrow q - q^* = \frac{(m - m^*) - (w - w^*)}{\theta + \phi} \)

\[ q = \frac{1}{2 \Theta} \left[ (m + m^*) - (w + w^*) + \frac{\Theta}{\theta + \phi} (m - m^*) - (w - w^*) \right] \]

(9) \( q = \beta_1 (m - w) + \beta_2 (m^* - w^*), \) where \( \beta_1 + \beta_2 = 1 / \Theta = 1 / (\theta + \phi + \beta / \sigma) \)
solve (2)-(2') for \( e \) and substitute into (7) and then (8) to get
(10) \( \pi^e = w + \gamma_1 (m - w) + \gamma_2 (m^* - w^*) - p_{t-1}^e \)
where \( \gamma_1 + \gamma_2 = \theta / \Theta \)

Welfare \( \max \sum_{s=0}^{\infty} (1 + \eta)^{-s} \left\{ (q_{t+s} - q_0)^2 + \mu \pi_{t+s}^2 \right\} \)

Noncooperative Equilibrium \( (q_T - q_0) \frac{\partial q}{\partial m} + \mu \pi_T \frac{\partial \pi}{\partial m} = 0 \)
in equilibrium, \( m = m^*, \) \( w = w^*, \) using (9) and (10)
\[
\left[ (\beta_1 + \beta_2) m_T - (\beta_1 + \beta_2) w_T - q_0 \right] \beta_1 + \mu \gamma_1 \left[ w_T + (\gamma_1 + \gamma_2) m_T - (\gamma_1 + \gamma_2) w_T \right] = 0
\]
\[
\left[ m_T - w_T - \Theta q_0 \right] \beta_1 + \mu \gamma_1 \left[ \Theta w_T + \theta m_T - \theta w_T - \Theta p_{T-1}^e \right] = 0
\]
\[
(\beta_1 + \theta \mu \gamma_1) m_T = (\beta_1 + \mu \gamma_1 \Theta) w_T - \mu \gamma_1 \Theta w_T + (\beta_1 + \mu \gamma_1 \Theta) p_{T-1}^e + \mu \gamma_1 \Theta p_{T-1}^e
\]
\[
m_T = \left[ \frac{1 - \mu \gamma_1 \Theta}{\beta_1 + \mu \gamma_1 \theta} \right] w_T + \frac{\beta_1 \Theta}{\beta_1 + \mu \gamma_1 \theta} q_0 + \frac{\mu \gamma_1 \Theta}{\beta_1 + \mu \gamma_1 \theta} p_{T-1}^e
\]

If \( m = w = p^e \) with rational expectations \( [\zeta = 1 \text{ in (6)}] \) then solution is
\[
\mu \gamma_1 \Theta (p_T^e - p_{T-1}^e) = \beta_1 \Theta q_0 \Rightarrow \pi_T^e = \frac{\beta_1}{\mu \gamma_1} q_0, \quad q_T = 0
\]
Cooperative equilibrium \( \frac{\partial q}{\partial m} = \beta_1 + \beta_2 = 1/\Theta, \frac{\partial \pi}{\partial m} = \gamma_1 + \gamma_2 = \theta / \Theta \)

Solution is \( \pi_T^c = \frac{1}{\theta \phi} q_0 \), \( q_T = 0 \) \( \Rightarrow \pi_T^{\text{Noncoop}} \gg \pi_T^{\text{Coop}} \Leftrightarrow \frac{\beta_1}{\mu \gamma_1} > \frac{1}{\theta \phi} \)

But \( \gamma_1 = \theta \beta_1 + \frac{(\beta_1 - \beta_2)(1-\alpha)}{2\delta} \Rightarrow \frac{\gamma_1}{\beta_1} > \theta \) or \( \frac{\beta_1}{\gamma_1} < \frac{1}{\theta} \)

\( \therefore \pi_T^{\text{Noncoop}} < \pi_T^{\text{Coop}} \) if \( \phi < \mu \) (Rogoff - due to 3rd party wage setters)

But if wage setters look backward \( (\zeta = 0) \),

Noncooperative \( \pi^c = \frac{\theta}{1 + \theta \mu \gamma_1 / \beta_1} q_0 \), \( q = \frac{1}{1 + \theta \mu \gamma_1 / \beta_0} q_0 \)

Cooperative \( \pi^c = \frac{\theta}{1 + \theta^2 \mu} q_0 \), \( q = \frac{1}{1 + \theta^2 \mu} q_0 \)

\( \pi_T^{\text{Noncoop}} > \pi_T^{\text{Coop}} \) if \( \frac{\gamma_1}{\beta_1} > \theta \) which it is.

Policy Simulations with McKibbin-Sachs Model

Social Welfare Function Weights

<table>
<thead>
<tr>
<th></th>
<th>Employment</th>
<th>Inflation</th>
<th>Govt. Deficit</th>
<th>Current A/C</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>.4</td>
<td>.8</td>
<td>.3</td>
<td>.1</td>
</tr>
<tr>
<td>Japan</td>
<td>.4</td>
<td>.8</td>
<td>.8</td>
<td>.4</td>
</tr>
<tr>
<td>Germany</td>
<td>.3</td>
<td>1.2</td>
<td>.2</td>
<td>.1</td>
</tr>
</tbody>
</table>

Global Disinflation – early 1980’s

<table>
<thead>
<tr>
<th></th>
<th>Noncooperative</th>
<th>Cooperative</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>-3.08</td>
<td>-2.33</td>
<td>24.35</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.67</td>
<td>-0.60</td>
<td>10.45</td>
</tr>
<tr>
<td>Germany</td>
<td>-1.26</td>
<td>-0.88</td>
<td>30.16</td>
</tr>
<tr>
<td>Rest of OECD</td>
<td>-3.69</td>
<td>-3.53</td>
<td>4.34</td>
</tr>
<tr>
<td>Rest of EMS</td>
<td>-0.63</td>
<td>-0.49</td>
<td>22.22</td>
</tr>
<tr>
<td>LDC’s</td>
<td>-36.10</td>
<td>-5.52</td>
<td>84.71</td>
</tr>
</tbody>
</table>

Note large welfare gains, especially for LDC’s because of lower interest rates from less restrictive monetary policies.
Trade Balance Adjustment – Fiscal Changes with Monetary Policy to Minimize Loss

<table>
<thead>
<tr>
<th>Year</th>
<th>US Fiscal Δ (%GDP)</th>
<th>Ger. &amp; Jap. Fiscal Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.5</td>
<td>+0.5</td>
</tr>
<tr>
<td>2</td>
<td>-1.0</td>
<td>+1.0</td>
</tr>
<tr>
<td>3</td>
<td>-1.5</td>
<td>+1.5</td>
</tr>
<tr>
<td>4</td>
<td>-2.0</td>
<td>+1.5</td>
</tr>
<tr>
<td>5</td>
<td>-2.5</td>
<td>+1.5</td>
</tr>
</tbody>
</table>

Welfare losses are small for all groups, no difference between Cooperative and Noncooperative solutions.
Monetary Politics of the ECB (Alesina & Grilli, in Canzoneri, et al, Establishing a Central Bank, 1992) A central bank without a political union will choose a different policy from national policies chosen separately.

First, consider behavior of “Europe” as a unit:

ECB

$$\Omega = \frac{1}{2} E \left[ \pi^2 + b(x - \bar{x})^2 \right], \quad \bar{x} > 0$$

subject to

$$x = \pi - \pi^e + \varepsilon \quad Ex = 0$$

$$\frac{\partial \Omega}{\partial \pi} = \pi + b(\pi - \pi^e + \varepsilon - \bar{x}) = 0$$

Biased

$$\pi = \bar{x} - \frac{b}{1 + b} \varepsilon, \quad x = \frac{1}{1 + b} \varepsilon$$

vs. "Optimal"

$$\pi = -\frac{b}{1 + b} \varepsilon, \quad x = \frac{1}{1 + b} \varepsilon$$

$$\sigma^2_x = \frac{\sigma^2_\varepsilon}{(1 + b)^2}$$

Voting on preference parameter $b$

$$\Omega^l = \frac{1}{2} E \left[ \pi^2 + b^l (x - \bar{x})^2 \right]$$

Median voter prefers action according to $\Omega^m$:

$$\min_b \frac{1}{2} E \left[ \left( b \bar{x} - \frac{b}{1 + b} \varepsilon \right)^2 + b^m \left( \frac{1}{1 + b} \varepsilon - \bar{x} \right)^2 \right]$$

$$\frac{\partial \Omega^m}{\partial b} = \frac{d}{db} \bar{x}^2 + \frac{b}{1 + b} \frac{\sigma^2_{\varepsilon}}{(1 + b)^2} - \frac{b^2}{(1 + b)^3} \sigma^2_{\varepsilon} - \frac{b^m}{(1 + b)^3} \sigma^2_{\varepsilon} = b \bar{x}^2 - \frac{\sigma^2_{\varepsilon}}{(1 + b)^3} (b^m - b) = 0$$

Optimal $b \in (0, b^m)$, $\frac{db}{d\sigma^2_{\varepsilon}} > 0, \frac{db}{db^m} > 0$

Median voter prefers a “conservative” central banker.
**ECB without Political Union:**

\[ y_i = \pi - \pi^e + \mu, \text{ Outputs differ, but } \pi_i \equiv \pi \]

National welfare functions \( \Omega^i = \frac{1}{2} E \left[ \pi^2 + \beta_i (y_i - \bar{y}_i)^2 \right] \)

ECB policy \( \Rightarrow \ (*) \) \( \Omega^i_{ECB} = \frac{1}{2} E \left[ \left( b\bar{x} - \frac{b}{1+b} \epsilon \right)^2 + \beta_i \left( \mu_i - \frac{b}{1+b} \epsilon - \bar{y}_i \right)^2 \right] \)

National policy would be \( \pi_i = \beta_i \bar{y}_i - \frac{\beta_i}{1+\beta_i} \mu_i, y_i = \frac{1}{1+\beta_i} \mu_i \Rightarrow \)

\( \Omega^i_N = \frac{1}{2} E \left[ \left( \beta_i \bar{y}_i - \frac{\beta_i}{1+\beta_i} \mu_i \right)^2 + \beta_i \left( \frac{1}{1+\beta_i} \mu_i - \bar{y}_i \right)^2 \right] \)

Difference between ECB and National policies (if \( \bar{x} = \bar{y}_i \))

\[ \Omega^i_{ECB} - \Omega^i_N = \frac{1}{2} \left[ \bar{x}^2 (b^2 - \beta^2_i) + (1 + \beta_i) \left( \frac{b}{1+b} \sigma_{\epsilon}^2 - \left( \frac{\beta_i}{1+\beta_i} \right)^2 \sigma_{\mu}^2 \right) - 2 \beta_i \left( \frac{b}{1+b} \sigma_{\epsilon \mu} - \left( \frac{\beta_i}{1+\beta_i} \right)^2 \sigma_{\mu}^2 \right) \right] \]

Political Differences only: \( b \neq \beta_i, (\mu = 0, \sigma_{\mu}^2 = \sigma_{\epsilon}^2 = \sigma_{\epsilon \mu} = \sigma^2) \)

\[ \Omega^i_{ECB} - \Omega^i_N = \frac{1}{2} \left[ \bar{x}^2 (b^2 - \beta^2_i) + \sigma^2 \left( \frac{b}{1+b} - \frac{\beta_i}{1+\beta_i} \right)^2 (1 + \beta_i) \right] < 0 \text{ if } b < \beta_i \]

Economic Differences only: \( (\beta_i = b) \)

\[ \Omega^i_{ECB} - \Omega^i_N = \frac{1}{2} \left[ \frac{b^2}{1+b} \left( \sigma_{\epsilon}^2 + \sigma_{\mu}^2 - 2 \rho \sigma_{\epsilon} \sigma_{\mu} \right) \right] \]

(i) high correlation: \( \rho_i = 1, \sigma_{\epsilon}^2 \neq \sigma_{\mu}^2 \) \( \Omega^i_{ECB} - \Omega^i_N = \frac{1}{2} \left[ \frac{b^2}{1+b} \left( \sigma_{\epsilon}^2 - \sigma_{\mu}^2 \right) \right] > 0 \)

(ii) low correlation \( \rho_i < 1, \sigma_{\epsilon}^2 = \sigma_{\mu}^2 \) \( \Omega^i_{ECB} - \Omega^i_N = \frac{1}{2} \left[ \frac{b^2}{1+b} \sigma_{\epsilon}^2 (1 - \rho_i) \right] > 0 \)

Voting on Governors of ECB, choose \( b \) to minimize \( (*) \)

\[ b\bar{x}^2 + (1 + \beta_i) \frac{b}{(1+b)^3} \sigma_{\epsilon}^2 - \frac{\beta_i}{(1+b)^2} \sigma_{\epsilon \mu} = 0 \Rightarrow \frac{db}{d\sigma_{\epsilon}} > 0, \frac{db}{d\rho_i} > 0 \]

**Loss in national welfare due to ECB depends on political and economic differences.** Since \( \Omega^i_{ECB} < \Omega^i_N \) if \( b < \beta_i \), credibility gain from political differences. But if \( \sigma_{\epsilon} \ll \sigma_{\mu} \), ECB stabilizes either too much or too little.

\( \Omega^i_{ECB} > \Omega^i_N \) if \( \rho_i < 1 \), so there is a loss due to asymmetric shocks.
Transactions Costs and Vehicle Currencies (Black, *JIMF*, 1991)

Model of Bid-Ask Spreads – based on economies of scale, zero profit for bank dealers

Liquidity Traders Buy $\tilde{Q}_d$, Sell $\tilde{Q}_s$ Dealers Sell at $P_a$, Buy at $P_b$

Speculators buy $a(\bar{P} - \tilde{P})$, sell $b(\tilde{P} - \bar{P})$

Market equilibrium $\tilde{Q}_s + b(\tilde{P} - \bar{P}) = \tilde{Q}_d + a(\bar{P} - \tilde{P})$, $\tilde{P} =$ market rate, $\bar{P} =$ expected rate

Dealer's profit $\tilde{\pi} = \tilde{Q}_d (P_a - \tilde{P}) + \tilde{Q}_s (\tilde{P} - P_b) = (\tilde{Q}_s - \tilde{Q}_d)(\tilde{P} - P_b) + \tilde{Q}_d (P_a - P_b)$

Expected profit $\bar{\pi} = \text{Cov}(\tilde{Q}_s - \tilde{Q}_d, \tilde{P} - \bar{P}) + \tilde{Q}t = -(a + b)\sigma_p^2 + \tilde{Q}t$

Bid-ask spread $t = \frac{(a + b)\sigma_p^2}{\tilde{Q}}$

Estimated on panel data for 7 major currencies vs. $\$, 1980, 83, 86, 89

\[
t = 0.023 + 0.000605 \frac{\sigma_p}{\tilde{Q}} + 0.038 \sigma_p \bar{R}^2 = 0.52
\]

(2.66) (4.56) (2.58)

\[
\tilde{P} = P_a + (a + b)(\tilde{P} - \bar{P})
\]

\[
\tilde{Q}_d - \tilde{Q}_s
\]
Model of Vehicle Currency Use – direct trade transactions between currencies $i,j$ can be mediated through vehicle currency $n$. Fraction mediated is $s$.

Transaction costs between $(i,j)$ \( T = \{ t_{ij} \}, t_{ij} > t_{in} + t_{nj} \)

\[
U = \{ u_{ij} \} \text{ Direct exchange}
\]

\[
q_{ij} = \frac{u_{ij}}{1+s}, \quad q_{in} = \frac{u_{in} + s}{1+s}
\]

\[
t_{ij} = \frac{(a+b)\sigma_{ij}}{q_{ij}} = \frac{(a+b)\sigma_{i} (1+s)}{u_{ij}}
\]

\[
t_{in} = \frac{(a+b)\sigma_{in} (1+s)}{u_{in} + s}
\]

\[
\tau = \frac{\sum_{i,j=1}^{n}(u_{in} + u_{nj})t_{in}}{\sum_{i,j=1}^{n}u_{ij}t_{ij}}
\]

\[
\tau = \phi(s) = \frac{\sum_{i,j=1}^{n}\sigma_{ij} \left[ \left( \frac{1+s}{u_{in}} \right)^{-1} + \left( \frac{1+s}{u_{ai}} \right)^{-1} \right]}{\sum_{i,j=1}^{n}\sigma_{ij}}, \quad \phi' < 0, \phi'' > 0, \quad \tau_{\max} = \phi(0)
\]

\[
s = \theta(\tau), \quad \theta' < 0
\]

\[
\dot{s} = \lambda \left[ \theta'(\phi(s)) - s \right], \quad 0 < \lambda < 1
\]

\[
\frac{\partial \dot{s}}{\partial s} = \lambda \theta' \phi' - \lambda < 0 \quad \text{iff} \quad \theta' < \frac{1}{\phi'}
\]
Currency area reduces $\sigma_{ij}$, raises $\sigma_{nj}$, so $t_{ij}$ falls relative to $t_{in}$, $\tau = \phi(s)$ shifts up, $s^*$ falls.

Communication costs or imperfect competition imply $c_{ij} > 1$ in

$$t_{ij} = \frac{(a + b)\sigma_{ij}c_{ij}}{q_{ij}},$$

so falling costs in non-vehicle currencies or increased competition shifts $\phi(s)$ shifts up, $s^*$ falls.

Recent study by Hartmann confirms these findings.
International Monetary System

Arrangements for a payments *system* among countries with different currencies to promote international trade and investment via *convertibility*. Requires: (1) vehicle currencies and convertible reserve assets, (2) exchange rate arrangements – pegged or floating, (3) adjustment mechanisms – to promote external balance, (4) financing – access to reserve assets to avoid excessive adjustment.

Demand for Reserve Assets – opportunity cost of holding asset

\[ r = MP_K - i^* \] vs. reduced need for adjustment (Heller, Hamada-Ueda, Kelley-Clark)

Heller – assumes constant marginal cost of adjustment, postpone adjustment to the last minute.

\[ \Delta R \sim \text{binomial with step size } \pm h \text{ with probability } \frac{1}{2}. \]

Given initial \( R \), probability of exhaustion in \( R/h \) steps = \( \left( \frac{1}{2} \right)^{R/h} \)

Cost of adjustment = \( \frac{1}{m} \) so expected cost = \( 1/m \left( \frac{1}{2} \right)^{R/h} = r = \text{marginal cost of holding} \)

\[ R^* = \frac{\log (rm)}{\log (0.5)} h = -\frac{\log (rm) h}{0.69} \]

Ignores all other than straight-line paths. Hamada-Ueda find

probability of exhaustion \( \frac{1}{2 \left( \frac{R}{h} - 1 \right)} > \left( \frac{1}{2} \right)^{R/h} \) Minimize expected cost

over entire path \( \frac{rR}{2} + \frac{1}{2m \left( \frac{R}{h} - 1 \right)} \Rightarrow R^* = \left[ 1 + \frac{1}{\sqrt{rm}} \right] h \)
Kelley-Clark assume increasing marginal cost of adjustment due to a welfare function that depends on mean and variance of income, so partial adjustment makes more sense. Analysis shows the choice between adjustment and financing.

Partial adjustment $\Delta R = \gamma (R - R^*) + u$, $u \sim N(0, \sigma_u^2)$

$$\Rightarrow \sigma_R = \frac{\sigma_u}{\sqrt{\gamma (2 - \gamma)}}$$

Tchebychev inequality $P(R \geq 0) = \frac{\sigma_R^2}{2R^{*2}} = \frac{\sigma_u^2}{2\gamma (2 - \gamma) R^{*2}}$

Income variability $\sigma_y = \frac{\gamma \sigma_R}{m} = \frac{\gamma \sigma_u}{m \left[ \gamma (2 - \gamma) \right]^{1/2}}$

Mean income $E_y = Y_0 - rR^*$

Welfare function $Max U(E_y, \sigma_y)$, $U_1 > 0, U_2 < 0$

Lower opportunity cost $r = MP_K - i^*$ implies higher $R^*$, lower $\gamma$.

Lower $P(R > 0)$ implies higher $R^*$ and $\gamma$.

Expectation that floating rates would reduced demand for reserves was disappointed, since “fear of floating” and “dangers of pegging” plus increased capital mobility have raised demand for reserve assets.
Seigniorage – if marginal cost of issuance of reserve asset is $\epsilon$, then seigniorage is $MP_K - i^* - \epsilon$.


**Alternative Regimes:**

<table>
<thead>
<tr>
<th></th>
<th>Floating Rates</th>
<th>Pegged Rates</th>
<th>Managed Rates</th>
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<tbody>
<tr>
<td>Symmetrical</td>
<td>Monetary Growth Targets (73-85)</td>
<td>Gold Standard</td>
<td>Louvre Target Zones (85-89)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>McKinnon*</td>
<td>Williamson Target Zones**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bretton Woods</td>
<td></td>
</tr>
<tr>
<td>Hegemonic</td>
<td></td>
<td>Dollar Standard (68-73)</td>
<td>EMS (79-93)</td>
</tr>
</tbody>
</table>

*McKinnon: Global monetary growth targeted to $\pi = 0$, national monetary policies targeted to peg exchange rates.

**Williamson Target Zones: Global monetary growth targeted to Nominal Income target, national monetary policies and central bank intervention aimed at real exchange rate target zones.*
Model   Home  
LM  \[ m - p = ky - \lambda i + \varepsilon_m \]  \[ m^* - p^* = ky^* - \lambda i^* + \varepsilon^*_m \]

IS  \[ y = -\gamma Er + \delta c + s + \eta y^* + \varepsilon_y \]  \[ y^* = -\gamma Er^* - \delta c^* + s^* + \eta y^* + \varepsilon^*_y \]

PC  \[ Dp = \phi y + \pi + \varepsilon_p \]  \[ Dp^* = \phi y^* + \pi^* + \varepsilon^*_p \]

UIP Arbitrage + Fad  \[ EDe = i - i^* + EDf \]

Fad w/ mean regression  \[ Df = -\psi f + \omega \]

[\[ r = i - Dp, \quad r^* = i^* - Dp^*, \quad Er = i - \phi y - \pi, \quad Er^* = i^* - \phi y^* - \pi^* \]

\[ c = e + p^* - p, \quad s = \text{fiscal policy}, \quad \varepsilon \sim N(0, \sigma_e^2), \quad \omega \sim N(0, \sigma_\omega^2) \]

Define  \[ y_a = \frac{y + y^*}{2}, \quad \varepsilon_m = \frac{\varepsilon_m + \varepsilon^*_m}{2}, \quad y_d = y - y^*, \quad \hat{\varepsilon}_m = \varepsilon_m - \varepsilon^*_m \]

Global economy

LM  \[ m_a - p_a = ky_a - \lambda i_a + \varepsilon_m \Rightarrow i_a = \lambda^{-1} \left( p_a - ky_a + \varepsilon_m - m_a \right) \]

IS  \[ y_a = -\gamma \left( i_a - \phi y_a - \pi_a \right) + s_a + \eta y_a + \varepsilon_y \]

PC  \[ Dp_a = \phi y_a + \pi_a + \varepsilon_p \]

Differences

\[ m_d - p_d = ky_d - \lambda i_d + \hat{\varepsilon}_m \]

\[ y_d = -\gamma Er_d + 2\delta c + s_d - \eta y_d + \hat{\varepsilon}_y \]

\[ Dp_d = \phi y_d + \pi_d + \hat{\varepsilon}_p \]

\[ EDe = i_d - \psi f \]

Global solution with given monetary targets \( m_a \) growing at rate \( \mu_a \) with  
\( \pi_a = \mu_a \), substitute LM into IS:

\[ y_a = \frac{1}{\Delta_a} \left( -\gamma \lambda^{-1} p_a + \gamma \lambda^{-1} m_a + \gamma \mu_a + s_a - \gamma \lambda^{-1} \varepsilon_m + \varepsilon_y \right), \quad \Delta_a = 1 + \gamma \lambda^{-1} k - \phi \gamma - \eta \]

\[ Dp_a = \phi y_a + \mu_a + \varepsilon_p \]

Let \( m_a = \mu_a = s_a = 0 \), fixed money supply, neutral fiscal pol.

\[ Dp_a = -\frac{\phi \gamma \lambda^{-1}}{\Delta_a} p_a - \frac{\phi \gamma \lambda^{-1}}{\Delta_a} \varepsilon_m + \frac{\phi}{\Delta_a} \varepsilon_y + \varepsilon_p \Rightarrow \rho_s = -\frac{\phi \gamma \lambda^{-1}}{\Delta_a} \]

\[ \sigma^2_{pa} = \frac{1}{2|\rho_s|} \left\{ \rho_s^2 \sigma^2_{\varepsilon_m} + \left( \frac{\phi}{\Delta_a} \right)^2 \sigma^2_{\varepsilon_y} + \sigma^2_{\varepsilon_p} \right\} \]
Solution for Variance:

\[ Dx = axdt + \sigma dz \implies x(t) = \int_0^t e^{a(t-s)} \sigma dz \]

\[ x^2 = \int_0^t e^{2a(t-s)} \sigma^2 ds \]

\[ \sigma_x^2 = Ex^2 = \lim_{t \to \infty} \int_0^t e^{2a(t-s)} \sigma^2 ds = \sigma^2 \lim_{t \to \infty} \frac{1-e^{2at}}{-2a} = \frac{\sigma^2}{2|a|} \]

National Differences: \( \rho_s = -\phi \left( \gamma \lambda^{-1} + 2\delta \theta \right) / \Delta_1, \Delta_1 = 1 + \gamma \lambda^{-1} k - \phi \gamma + \eta \)

McKinnon Rule: \( i_a = \beta_M p_a \) with \( \beta_M = \lambda^{-1}, k = 0 \) in LM and omit \( \sigma_{x_m}^2 \)

Williamson Rule: \( i_a = \beta_W (p_a + y_a), k = 1, \beta_W = \lambda^{-1}, \) omit \( \sigma_{x_m}^2 \)

\[ \rho_s \] and \( \Delta_1 \) change to \( \left\{ \begin{array}{l} \text{McKinnon: } \rho_s = -2\phi \delta / \Delta_2, \Delta_2 = 1 - \phi \gamma + \eta \\ \text{Williamson: } \rho_s = -\phi \sigma / \Delta_3, \Delta_3 = 1 + \sigma + \eta \end{array} \right\} \)

Floating Rates with National Monetary Targets:

Let \( \ell = m - p, \ell^* = m^* - p^*, \ell_d = \ell - \ell^* \implies D\ell_d = -\phi y_d - \hat{\epsilon}_p \)

\[ Dc = De + Dp^* - Dp, EDc = Ei_d - \psi f - \phi Ey_d - \mu_d \]

\[
\begin{bmatrix}
D\ell_d \\
EDc \\
Df
\end{bmatrix} = \frac{1}{\Delta}
\begin{bmatrix}
\phi \gamma & 2\phi \lambda \delta & 0 \\
1 + \eta & 2\delta (\phi \lambda - k) & -\Delta \psi \\
0 & 0 & -\Delta \psi
\end{bmatrix}
\begin{bmatrix}
\ell_d \\
c \\
f
\end{bmatrix} + \frac{1}{\Delta}
\begin{bmatrix}
\phi \gamma & \phi \lambda & \phi \gamma \lambda & -\Delta & 0 \\
0 & 0 & \lambda (1 + \eta) & 0 & 0 \\
0 & 0 & 0 & 0 & \Delta
\end{bmatrix}
\begin{bmatrix}
\hat{\epsilon}_m \\
\hat{\epsilon}_y \\
\mu_d \\
\hat{\epsilon}_p \\
\omega
\end{bmatrix}
\]

\[ \Delta = -k \gamma - \lambda (1 - \phi \gamma + \eta) < 0, s_d = 0 \implies \text{two stable roots: } \rho_s, -\psi \]

If \( \psi = 0 \), saddle path is SS. If \( -\psi = \rho_s \), trajectory with fads is TT.

A fad which raises the value of the currency (lower \( c \)), reduces competitiveness and slows adjustment to origin (\( c \) is below SS). A fad which lowers the value of the currency (higher \( c \)), raises competitiveness and speeds adjustment to origin (\( c \) is above SS).
McKinnon and Williamson proposals change the speed of adjustment and $\sigma^2_p$. Empirical estimates based on standard parameter values:

<table>
<thead>
<tr>
<th></th>
<th>Supply shocks</th>
<th>Demand shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Averages:</strong></td>
<td>$\Delta$</td>
<td>$\rho_s$</td>
</tr>
<tr>
<td>Money or Nominal Income Target</td>
<td>0.90</td>
<td>-0.14</td>
</tr>
<tr>
<td>Price Level Target</td>
<td>0.65</td>
<td>-0.19</td>
</tr>
<tr>
<td><strong>Differences:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Floating w/ Money Targets</td>
<td>1.10</td>
<td>-0.54</td>
</tr>
<tr>
<td>McKinnon: Fixed Rates, M Targets</td>
<td>0.85</td>
<td>-0.59</td>
</tr>
<tr>
<td>Williamson: Fixed Real Rates, PY</td>
<td>1.60</td>
<td>-0.16</td>
</tr>
</tbody>
</table>
Obstfeld & Rogoff Stochastic Model “Global Implications of National Monetary Rules” (QJE May, 2002) – Individuals maximize

$$U_i = \frac{(C_i)^{1-\gamma}}{1-\gamma} + \chi \log \frac{M_i}{P} - KL_i$$

where labor is used in production of Nontraded goods and Home traded goods. Goods produced use imperfectly substitutable labor

$$Y_H(i) = \left[ \int_0^1 L_H(i,j)^{\phi-1}\phi\, dj \right]^\frac{1}{\phi(\phi-1)}$$

Firm $i$'s demand for use of labor of type $j$ is

$$L(i,j) = \left[ \frac{W(j)}{W} \right]^{-\phi} Y(i)$$

K in the utility function is a country-wide productivity shock. Consumption is

$$C = C_T^\gamma C_N^{1-\gamma} / \gamma \gamma (1-\gamma)^{1-\gamma}$$

with constant shares for traded and nontraded goods and $C_T = 2C_H^{1/2}C_F^{1/2}$. The optimal preset money wage is

$$W(i) = \left( \frac{\phi}{\phi - 1} \right) \frac{\mathbb{E}[KL_i]}{\mathbb{E}[L^i(C_i)^{-\gamma}/P]},$$

a markup over the expected marginal disutility of work. Price markups are the same in both countries

$$P_H = [\theta/(\theta - 1)]W = \mathcal{E}P_H^*.$$ 

The real exchange rate

$$\frac{\mathcal{E}P_H^*}{P} = \frac{\mathcal{E}P_T^*P_N^{(1-\gamma)}}{P_T^* P_N^{1-\gamma}} = \left( \frac{\mathcal{E}W^*}{W} \right)^{1-\gamma}.$$ 

The terms of trade

$$\frac{\mathcal{E}P_H^*}{P_H} = \frac{\mathcal{E}W^*}{W}$$

Real expenditure denominated in traded goods is

$$Z = C_T + (P_N/P_T)C_N,$$

and since $P_N/P_T = (1 - \gamma)C_T/\gamma C_N$, we have

$$Z = C_T/\gamma = C_T^*/\gamma = Z^*.$$ 

Assume the (log) shocks $\{m,m^*,\kappa,\kappa^*\}$ are jointly normal, with identical means and variances for the productivity shocks and let
Then the (log) terms of trade are

\[ E_T = E e + w^* - w = f(\sigma_{ze}, \sigma_{kae}, \sigma_{ke}), \]

\[ = \frac{1}{1 - (1 - \gamma)(1 - \rho)} \times [(1 - (1 - \gamma)(1 - \rho)^2)\sigma_{ze} + \sigma_{kae} + 2\sigma_{ke}], \]

Note that \( \sigma_{ke} \) is affected by the monetary policy rule and so affects labor effort.

The expected level of expenditure in terms of tradables is

\[ E_z = g(\sigma_z^2, \sigma_e^2, \sigma_{kae}, \sigma_{ke}), \]

\[ = \frac{1}{\rho} \left\{ \omega + \lambda - \frac{1}{2\rho} \sigma_k^2 \frac{1}{2} [1 - (1 - \rho)^2] \sigma_z^2 - \frac{1}{8} [1 - (1 - \gamma)^2(1 - \rho)^2] \sigma_e^2 - \sigma_{kae} + \frac{1}{2} \sigma_{ke} \right\}, \]

\( \omega \) and \( \lambda \) are constants depending on the moments of \( \kappa \) and \( \kappa^* \). Since \( w \) and \( w^* \) are predetermined, O&R find the effects of shocks on \( z \) and \( e \), through \( m \) and \( m^* \). The innovations to spending and the exchange rate are then

\[ \hat{z} = \frac{1}{2\rho} (\hat{m} + \hat{m}^*), \quad \hat{e} = \frac{\hat{m} - \hat{m}^*}{1 - (1 - \gamma)(1 - \rho)}, \]

In the case of log utility

\[ EU = Ez + \left( \frac{1 - \gamma}{2} \right) E_T - \psi, \quad \psi = \frac{(\phi - 1)(\theta - 1)}{\phi \theta}. \]

And foreign expected utility is

\[ EU^* = EU - (1 - \gamma)E_T. \]

Cooperative monetary policy would be chosen to maximize

\[ EV = \gamma_1 EU^* + \gamma_2 EU, \]

by choosing policy rules.
\[ \dot{m} = -\delta_d \hat{k}_d - \delta_w \hat{k}_w, \]
\[ \dot{m}^* = \delta_d^* \hat{k}_d - \delta_w^* \hat{k}_w. \]

where carets over the variables denote innovations.

\[ \dot{m} = m - \mathbb{E}m \quad \text{and} \quad \kappa_w = \frac{K + K^*}{2}, \quad \kappa_d = \frac{K - K^*}{2}. \]

Under flexible wages with log utility, monetary policy is irrelevant and
\[ \mathbb{E}U = \log (\psi) - \psi - \mathbb{E}\kappa = \mathbb{E}\tilde{U}^* \]
if \( \rho = 1. \)

Under sticky wages with log utility
\[ EU = \mathbb{E}\tilde{U} - \frac{\sigma_{k_e}^2 + \sigma_{k_d}^2}{2} + \lambda - \frac{1}{2} \sigma_z^2 - \frac{1}{8} \sigma_e^2 - \frac{1}{2} \sigma_{k_e} - \frac{1}{2} \sigma_{k_d} - \frac{(1 - \gamma)}{2} \left( \sigma_{z_e} + \sigma_{k_e} + 2 \sigma_{k_d} \right) \]
\[ EU^* = \mathbb{E}\tilde{U}^* - \frac{\sigma_{k_e}^2 + \sigma_{k_d}^2}{2} + \lambda - \frac{1}{2} \sigma_z^2 - \frac{1}{8} \sigma_e^2 - \frac{1}{2} \sigma_{k_e} - \frac{1}{2} \sigma_{k_d} + \frac{\nu(1 - \gamma)}{2} \left( \sigma_{z_e} + \sigma_{k_e} + 2 \sigma_{k_d} \right) \]

Note that \( \sigma_{k_e} > 0 \) raises demand for Home output when disutility of work is high, thus lowering Home utility (and raising Foreign utility).

O&R show that optimal cooperative monetary policies under sticky
prices \( \delta_d^{flex} = \delta_d^{flex} = 1 \), and \( \delta_w^{flex} = \delta_w^{flex} = 1 \) mimic the flexible price equilibrium. With log utility, there are no gains from cooperation and the optimal policy is the same either with or without cooperation. If \( \rho \) is not equal to one, there are (small) gains to cooperation.

### TABLE 1
GAINS FROM STABILIZATION AND COORDINATION (PERCENT OF OUTPUT)

<table>
<thead>
<tr>
<th>Measure</th>
<th>( \rho = 0.5 )</th>
<th>( \rho = 1 )</th>
<th>( \rho = 2 )</th>
<th>( \rho = 4 )</th>
<th>( \rho = 8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Stabilization gain</td>
<td>3.11</td>
<td>1.01</td>
<td>0.33</td>
<td>0.11</td>
<td>0.03</td>
</tr>
<tr>
<td>(ii) Coordination gain</td>
<td>0.02</td>
<td>0.00</td>
<td>6.3 \times 10^{-3}</td>
<td>9.0 \times 10^{-3}</td>
<td>5.8 \times 10^{-3}</td>
</tr>
<tr>
<td>(iii) Ratio (ii)/(i)</td>
<td>7.9 \times 10^{-3}</td>
<td>0.00</td>
<td>1.9 \times 10^{-2}</td>
<td>0.08</td>
<td>0.18</td>
</tr>
</tbody>
</table>
Canzoneri, Cumby, & Diba, “The Need for International Policy Coordination” (2004) point out some restrictive conditions in the O&R model that limit the applicability of its conclusions:

1. balanced current account
2. constant expenditure shares in consumption
3. log utility of money
4. No differential productivity shocks in traded and nontraded goods.

The exchange rate completely insulates the economy from any foreign shocks, so that cooperative and non-cooperative equilibria coincide. Changes in any of the four assumptions remove the insulation property and leads to tradeoffs between inflation and output that provide ample room for differences between cooperative and non-cooperative equilibria. In particular, assume asymmetry in productivity shocks to sectors producing tradable goods for domestic consumption (D) and export (E).
Figure 1: Gains from Nash & Cooperative Policies

Figure 2: Relative Gain from Coordinated Policies
Nash Non-Cooperative Equilibrium:
Fiscal authority chooses subsidy rate \( \tau \) to satisfy
\[
(1 - \tau)(1 + \mu^w)(1 + \mu^p)(1 - \gamma) = 1
\]
where \( \mu^w \) is the wage markup over the marginal disutility of labor and \( \mu^p \) is the price markup over marginal cost and \( I - \gamma \) is the share of home goods in consumption. The central bank’s objective function is
\[
W^H = -\frac{(1 - \gamma)}{2} \Lambda E_0 \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \alpha \gamma_t^2 \right]
\]
subject to \( \pi_t = \beta E_t \{ \pi_{t+1} \} + \lambda \tilde{y}_t + u_t \),
it chooses \( \tilde{y}_t \) and \( \pi_t \) each period as \( \tilde{y}_t = -\frac{\lambda}{\alpha} \pi_t = -\xi \pi_t \) which leads to
\[
\pi_t = \psi u_t \quad \text{and} \quad \tilde{y}_t = -\xi \psi u_t \quad \text{where} \quad \psi = [(1 - \beta \rho) + \lambda \xi]^{-1} > 0 \quad \text{and} \quad \rho \quad \text{is the autocorrelation in the shocks. And similarly for the foreign economy. Combining this with the IS curve}
\]
\[
\tilde{y}_t = E_t \{ \tilde{y}_{t+1} \} - \sigma^{-1} \left( r_t - E_t \{ \pi_{t+1} \} - \bar{r}_t \right)
\]
yields an interest rate rule
\[
r_t = \bar{r}_t + \vartheta E_t \{ \pi_{t+1} \} \quad \text{with} \quad \vartheta = 1 + \xi \sigma (1 - \rho) \rho^{-1} > 1
\]
Same for the other country. They show that if \( \gamma = \frac{1}{2} \), the terms of trade
\[
s_t = -\omega \xi \psi (u_t - u_t^*) + (a_t - a_t^*)
\]
depends on the price shocks and the productivity shocks. The exchange rate will also respond as
\[
e_t = e_{t-1} + \Delta s_t + \pi_t - \pi_t^* = e_{t-1} - \omega \xi \psi (\Delta u_t - \Delta u_t^*) + \Delta a_t - \Delta a_t^* + \psi (u_t - u_t^*)
\]
which implies appreciation in response to a price shock, since \( \omega \xi > 1 \).

Cooperative Equilibrium:
Fiscal authorities in each country set \( (1 - \tau)(1 + \mu^w)(1 + \mu^p) = 1 \) and joint maximization of world welfare leads to policy rules
\[
\tilde{y}_t = -\xi \left( \pi_t + \frac{K_0}{K} \pi_t^* \right)
\]
and similarly for the other country, where \( \kappa \equiv \sigma (1 - \gamma) + \gamma + \phi, \kappa_0 \equiv \gamma (\sigma - 1) \)
Note the consideration of both countries’ inflation rates iff \( \sigma \neq 1 \).