1. Computational Ex. 8.17. \( \tilde{G}(s) = \frac{\lambda}{s + \lambda}, \frac{\mu}{s + \mu} \)

\[
\tilde{M}(s) = \frac{\tilde{G}(s)}{1 - \tilde{G}(s)} = \frac{\lambda\mu}{s(s + \lambda + \mu)} = \frac{\lambda\mu}{\lambda + \mu s} - \frac{\lambda\mu}{\lambda + \mu s + \lambda + \mu}.
\]

\[
M(t) = \frac{\lambda\mu}{\lambda + \mu} t - \frac{\lambda\mu}{(\lambda + \mu)^2} (1 - e^{-(\lambda + \mu)t}).
\]

2. Computational Ex. 8.18. Let \( H(t) = E(S_{N(t)} + k) \). Then

\[
E(S_{N(t)} + k \mid S_1 = x) = \begin{cases} 
  x + E(S_{k-1} \mid S_1 = x) & \text{if } x > t \\
  x + E(S_{N(t-x)} + k) & \text{if } x \leq t
\end{cases}
\]

Hence

\[
H(t) = \int_{0}^{\infty} x dG(x) + \int_{t}^{\infty} (k-1)\tau dG(x) + \int_{0}^{t} H(t-x) dG(x)
\]

Taking LSTs,

\[
\tilde{H}(s) = \tau + (k-1)\tau (1 - \tilde{G}(s)) + \tilde{H}(s)\tilde{G}(s)
\]

which yields

\[
\tilde{H}(s) = \tau \left( k + \frac{\tilde{G}(s)}{1 - \tilde{G}(s)} \right) = \tau (k + \tilde{M}(s)).
\]

Hence \( H(t) = \tau (k + M(t)) \).

3. Computational Ex. 8.22. Let \( H(t) = E(A(t)B(t)) \). Conditioning on \( X_1 = x \), we get

\[
E(A(t)B(t) \mid X_1 = x) = \begin{cases} 
  H(t-x) & \text{if } x \leq t \\
  H(t-x) & \text{if } x > t
\end{cases}
\]

Unconditioning gives
\[
H(t) = \int_0^t H(t - x) dG(x) + \int_t^\infty t(x - t) dG(x).
\]

Thus \(H\) satisfies the following renewal type equation
\[
H(t) = D(t) + \int_0^t H(t - x) dG(x)
\]
where
\[
D(t) = \int_t^\infty t(x - t) dG(x) = \tau - t \int_0^t (1 - G(x)) dx.
\]

Then \(D\) is a difference of two monotone functions and bounded. Assuming \(G(\cdot)\) is aperiodic, we get
\[
\lim_{t \to \infty} H(t) = \frac{1}{\tau} \int_0^\infty (1 - G(t)) dt = \frac{E(X^3)}{6\tau}.
\]

Hence the limiting covariance of \(A(t)\) and \(B(t)\) is given as
\[
\lim_{t \to \infty} \text{Cov}(A(t), B(t)) = \lim_{t \to \infty} \left[ E(A(t)B(t)) - E(A(t))E(B(t)) \right] = \frac{E(X^3) - 3E(X^2)}{6E(X)}.
\]

4. Computational Ex. 8.23. We have
\[
P(N(t) \text{ is odd} | X_1 = x) = \begin{cases} 
1 - p(t - x) & \text{if } x \leq t \\
0 & \text{if } x \geq t
\end{cases}
\]

Hence, unconditioning yields
\[
p(t) = \int_0^t (1 - p(t - x)) dG(x) + \int_t^\infty dG(x) = 1 - \int_0^t p(t - x) dG(x).
\]

This is not a renewal equation due to the minus sign on the right hand side. For a PP(\(\lambda\))
\[
p(t) = \frac{1}{2}(1 - e^{-2\lambda t})
\]

5. Computational Ex. 8.24. Let \(U_1 =\) first up time, \(D_1 =\) first down time and \(S_1 = U_1 + D_1\). Use renewal argument by conditioning on \(S_1\).
\[
E(W(t) \mid S_1 = x) = \begin{cases} 
E(U_1 \mid S_1 = x) + H(t - x) & \text{if } x < t \\
E(\min(U_1, t) \mid S_1 = x) & \text{if } x \geq t
\end{cases}
\]
Unconditioning

\[ H(t) = \int_0^t E(U_1 | S_1 = x) \, dG(x) + \int_t^\infty E(\min(U_1, t) | S_1 = x) \, dG(x) + \int_0^t H(t-x) \, dG(x) \]

\[ = E(\min(U_1, t)) + \int_0^t H(t-x) \, dG(x) \]

\[ = \frac{1}{\lambda} (1 - e^{-\lambda t}) + H * G(t). \]

Taking LSTs, we get

\[ \tilde{H}(s) = \frac{1}{s + \lambda} + \tilde{H}(s) \frac{\lambda}{s + \lambda} \frac{\mu}{s + \lambda + \mu}. \]

This yields,

\[ \tilde{H}(s) = \frac{\mu + s}{s(s + \lambda + \mu)} = \frac{\mu}{\lambda + \mu} \frac{1}{s} + \frac{\lambda}{\lambda + \mu} \frac{1}{s + \lambda + \mu}. \]

Inverting, we get

\[ H(t) = \frac{\mu}{\lambda + \mu} t + \frac{\lambda}{(\lambda + \mu)^2} (1 - e^{-(\lambda+\mu)t}). \]